Importance Assessment in PSA with Advanced Calculation Approaches: RiskSpectrum Perspective

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Abstract: This paper will discuss implications of calculation approaches and applications for mainly the importance measures Fussel-Vesely (FV) and Fractional Contribution (FC). Specifically, the following topics will be discussed; 1) dependencies between basic events, for instance introduced by applying conditional quantification where one event is transferred into several events. 2) how to estimate FV in the context of a Minimal Cut Set list quantification by Boolean Decision Diagrams (MCS BDD) 3) what is the relation between FC and FV, especially in the context of a more precise calculation approach like the MCS BDD. 4) and finally, we deal with the difficulties caused by a dynamic quantification of cut sets.

Keywords: Importance Factors, Dependent Events, Boolean Decision Diagrams, Dynamic Calculations.

1. INTRODUCTION

Importance assessments are an essential part of any evaluation of a Probabilistic Safety/Risk Assessment (PSA/PRA) study, as it helps the analyst to see through the large amount of data. They are also a fundamental part of PSA/PRA applications, in order to assess which components are important in a configuration risk management (risk monitors) or for further risk application assessments (e.g. risk informed categorization).

The calculation methods are, however, not static. The approaches evolve to include new modeling features and new quantification approaches are developed. The improvements range from smaller additions, like treatment of mutual exclusivity, to larger improvements like MCS BDD (Binary Decision Diagram built based on the Minimal Cut Sets) up to new approaches like dynamic quantification of minimal cut sets based on Markov Chains. Another important aspect is the application of the results, as that may determine how especially Common Cause Failures shall be treated.

In this paper we first compare and discuss Fussell-Vesely and Fractional Contribution importance. Then we discuss how some types of dependencies between events will affect the importance calculation. Finally, we also touch upon dynamic calculations and importance.

2. FUSSELL-VESELY AND FRACTIONAL CONTRIBUTION

The two frequently calculated and used importance measures for basic events are Fussell-Vesely and Fractional Contribution, defined on a homogeneous fault tree decomposed into a minimal cut set list. Fussell-Vesely [1] for an event A in a Minimal Cut Set (MCS) list M is calculated by

$$FV(A) = P(M_A)/P(M) \qquad (1)$$

where M_A is the subset of M of cut sets containing the event A and P(N) denotes the probability of the MCS list N.

Fractional contribution, sometimes called Criticality Importance [2], is calculated by

$$FC(A) = 1 - P(M[A=0])/P(M) \qquad (2)$$

where P(M[A=0]) denotes the probability of the MSC list M where the value of the event A has been set to zero.

These two importance measures are sometimes used interchangeably, sometimes the Fractional Contribution is also called Fussell-Vesely [3,4], sometimes the Fussell-Vesely importance measure is used to approximate Fractional Contribution [4]. The practical purpose of these two importance measures is to determine the sensitivity of the system to the failure probability of the studied basic event A. They indicate the fractional change of the top value when we double the failure probability of A [5]. How much of the original system failure probability will be added after doubling? Or, how much of the original system failure probability will in best case be removed if I can find a way to make A perfect (practically setting the probability of A to zero)? We can also see it as a characterization of the contribution of the component whose failure is modeled by the basic event A to the top failure probability. The top failure probability will increase by this fraction if we introduce a new component that will have the same characteristic as the original one and will be required in the same way as the original component. In a fault tree, this would correspond to replacing A by an OR-gate with two inputs, A and B, where B is a new basic event with the same reliability model and parameters as A.

The relation between FV and FC varies depending on the way in which we calculate the probability of the MCS lists. If we use the rare event approximation, these definitions coincide. But once we use the Min Cut Upper Bound (MCUB) approximation or even advanced quantification methods such as quantification based on Boolean Decision Diagrams (BDD), the results might differ.

The correct importance measure with respect to the purpose described above is Fractional Contribution. We can illustrate it on a precise quantification of the MCS list which can be achieved, for reasonably large MCS lists, by the MCS BDD implementation in RiskSpectrum [6]. This method gives the same result as we would obtain by summing up values of minterms for satisfying valuations. A formal definition of the criticality importance based on minterms and critical states can be found in [7].

Let us consider a minimal cut set list with three cut sets:

$$\{ \{A, B\}, \{B, C\}, \{A, C\} \}$$

Each basic event is shared by two minimal cut sets. The rare event approximation will over-approximate the value of this minimal cut set list and this conservatism might be considerable if the basic event probabilities are high. The Fussell-Vesely or Fractional Contribution of the event A are calculated by the following formulas, respectively, where P(M) denotes P({ {A, B}, {B, C}, {A, C} }).

$$FV(A) = P(\{\{A, B\},\{A, C\}\})/P(M) \tag{3}$$

$$FC(A) = 1 - P(\{B, C\})/P(M) \tag{4}$$

The exact values of these probabilities are determined by the following minterms:

$$P(\{\{A, B\}, \{B, C\}, \{A, C\}\}) = P(ABC) + P(ABC') + P(AB'C) + P(A'BC) \tag{5}$$
$$P(\{\{A, B\},\{A, C\}\}) = P(ABC) + P(ABC') + P(AB'C) \tag{6}$$
$$P(\{B, C\}) = P(ABC) + P(A'BC) \tag{7}$$

We can see that the minterm ABC belongs to both P({{A, B},{A, C}}) and P({B, C}). This causes the difference between FC and FV in this example. Let us consider what happens to the exact quantification of P(M) when we double the failure probability of A. Value of all minterms will be affected by this change, but the sum of the minterms ABC and A'BC will stay the same. This corresponds to the minimal cut set {BC} which is not affected by updating the probability of A. The minterms ABC' and AB'C will double their value. Therefore, the fractional increase of the probability of M after doubling the value of A is equal to

$$\begin{aligned}(P(ABC') + P(AB'C))/P(M) = (P(M) - P(ABC) + P(A'BC))/P(M) = \\ = 1 - (P(ABC)+P(A'BC))/P(M) = FC(A)\end{aligned} \tag{8}$$

If the component corresponding to the basic event A is perfect and the failure probability of A is equal to zero, then the fractional decrease of the system failure probability will be again the same value – FC(A). The argument for adding another basic event is syntactically more complex, because it doubles the number of minterms, but it follows the same logic.

To generalize this argument, the subset of M containing minimal cut sets which contain the studied event A (denoted M_A) might be satisfied by minterms which also satisfy a cut set from M that does not contain A (denoted $M_{A'}$). In our example, it is the minterm ABC. The minterms satisfying cut sets from $M_{A'}$ contain always pairs of minterms, A·X and A'·X, where X is a minterm on the set of basic events without A. This means that it is not affected by changes of A. The intersection of minterms that satisfy M_A and $M_{A'}$ does not affect the system failure probability when modifying A. Therefore, they should not be included into the importance measure. They are not included in FC (for an argument based on critical states see [7]), but they are included in FV.

In this light, the Fussell-Vesely importance measure defined by the subset of cut sets containing the analyzed event turns out to approximate a measure that can be calculated exactly by, for instance, quantifying the MCS list by a BDD-based algorithm. The imprecision is inherent in the definition and cannot be removed by advanced quantification algorithms. On the other hand, it is not clear how one can derive the FV value from an exact quantification except for extracting the minimal cut sets. The importance measure with a clearly defined meaning is the Fractional Contribution. Also, in some works, the Fussell-Vesely and Fractional Contribution measures are used as synonyms with the definition of the Fractional Contribution.

This leads us to the conclusion that the measure which should be used in prioritization, or risk-informed applications is the Fractional Contribution. This one has the potential to exactly estimate the contribution of the event to the total risk and it has a clear physical/mathematical meaning. It is compatible with advanced quantification methods and can be easily extended to groups of events or components. Additionally, as we show in the next section, dependencies between events and success quantification can be taken into account by Fractional Contribution as well.

The Fussell-Vesely importance measure as a minimal cut set based measure can help approximating Fractional Contribution to provide a fast insight into the importance of a basic event for an analyst. But it should be always used with caution, given its inherent approximative character. With the advances in quantification methods, the additional cost in using Fractional Contribution in connection with the MCS BDD quantification of the MCS list becomes very low compared to the benefit of a better approximation, potentially giving an exact answer.

3. DEPENDENCIES BETWEEN EVENTS

The classical approach to fault trees leaves assumes basic events to be independent of each other. This becomes often a coarse approximation of reality, where failures depend on each other, one failure influences other failures or there is an underlying mechanism connecting failures. Failure probabilities can be correlated. In this section, we describe different types of dependencies between events and show how are they handled by importance measures.

We look at the following dependencies:

- A-priori quantification of one basic event by another one (Basic Event Relations): A very simple form of this dependency allows expressing that one basic event occurs with the probability complementary to the probability of another basic event: P(A) = 1 – P(B).
- Quantification conditional on occurrence of other failures: If an event occurs then another event might increase its failure probability. This type of quantification of events conditionally on occurrence of other events simplifies, for instance, treatment of operator actions.
- Mutually exclusive events: Occurrence of one event excludes occurrence of other events. Basic events themselves keep their failure probabilities, but the combinations of events and minimal cut sets will be quantified differently. Such relations make it possible to specify plant operating states or mutually exclusive maintenance activities as basic events.
- Common cause modeling: Similarly to the conditional quantification of basic events, a simultaneous occurrence of several events might have a higher probability than the product of individual basic event probabilities. This is a classic and standard feature in PSA studies and difficulties for

importance factors related to common cause failures have been explored in the literature, e.g., [1,5,7].

- Dependency caused by success in event trees: In several approaches for quantification of static fault trees consideration of success in the event trees are included.

The importance measures of interest in this section are the Risk Increase Factor (RIF), the Risk Decrease Factor (RDF) and the Sensitivity Factor (SF). We might also compare these calculations with Fussell-Vesely discussed in the previous section. They are defined as follows:

$$RIF(A) = P(M[A=1])/P(M) \quad (9)$$

$$RDF(A) = P(M[A=0])/P(M) \quad (10)$$

$$SF(A) = P(M[A=A*10])/P(M[A=A/10]) \quad (11)$$

For all three factors, we need to modify the value of a basic/CCF event and recalculate the MCS list.

3.1. Basic Event Relations

Basic Event Relations present the simplest class of dependencies for importance factors. The definitions of importance factors do not need to be modified. The algorithm must re-quantify basic events that depend on the studied event after modifying the probability of the studied event. On the other hand, we do not change the probability of the studied event if it depends on other events. One consideration during the Basic Event Relations calculation is that the probability of an event must be within the range of 0 to 1. This treatment makes sure that when we, for instance, study the sensitivity of an event, we also update events that depend on it correspondingly. Assume that we study sensitivity of a plant operating mode and extend its duration 10 times. The duration of other plant operating modes must be then shortened accordingly. The algorithm will take care of this automatically by evaluating the Basic Event Relations. Note that this mechanism will not be applied for the cut set based calculation of Fussell-Vesely. Here again the Fractional Contribution gives a better estimate, because it is based on the Risk Decrease Factor.

3.2. Conditional Quantification

Conditional Quantification defines conditional probabilities for basic events which depend on occurrence of other events. For each conditional probability assigned to an event, the tool creates internally a new basic event with this probability. As an example, consider a basic event A with the following conditional probabilities:

$$P(A')=P(A|B) \quad (12)$$

$$P(A'')=P(A|C) \quad (13)$$

The original event without dependency is denoted as A_0. When we calculate the importance measures, we update all events related to the original event according to the same rule. For example, the Risk Increase Factor for event A is calculated as

$$RIF(A)=P(M[A_0=1 \text{ and } A'=1 \text{ and } A''=1])/P(M) \quad (14)$$

For the Risk Decrease Factor, we set all events to zero:

$$RDF(A)=P(M[A_0=0 \text{ and } A'=0 \text{ and } A''=0])/P(M) \quad (15)$$

The Fussell-Vesely (FV) is typically defined for individual basic events. It is not usually applied to groups of events, such as attributes, components, systems, or event groups. The equivalent event group derived from the conditional probability contains only mutually exclusive events, and their individual FV values can therefore be summed together directly: $FV(A)=FV(A_0)+FV(A')+FV(A'')$. In this case, all importance measures, including Fussell-Vesely, take the conditional quantification into account. There are no additional differences between Fussell-Vesely and Fractional Contribution than those described in Section 3. Details of importance calculations are described in [9].

3.3. Mutually Exclusive Events

Mutual Exclusivity (MUX) is defined between events that are not independent versus each other. For events that are MUX standard calculation approach for independent events cannot be used. P(A+B) = P(A) + P(B) – P(AB). In MUX scenarios P(A+B) = P(A) + P(B).

An observation is that considering MUX and rare event approximation will yield the same result. Which means that if all events in a MCS list are MUX, then FV=FC. But normally, MUX is only considered for a few events in the MCS list. This means, from a cut set list perspective, the MCS list needs to be partitioned based on which events are included in the MCS. Such partitioning can be complex and especially in determining which partitions are MUX versus each other, and often there is an element of conservatism in the calculation of the top when MUX is included.

As the top can be expected to be slightly conservative when MUX is considered, the estimation of importance will be impacted in a similar way. The calculation of MUX is more accurate in the calculation of BDD (see [13] for description of how MUX is included in the BDD). The calculation of FV and FC as well as the other importance measures will work without issues for consideration of MUX. However, as FV is not calculated based on the BDD and BDD is preferred when calculating MUX, then FC has the advantage of being able to use a more exact solution.

3.4. Common Cause Failures

Common cause failures (CCF) are typically modelled by new events under an OR-gate representing simultaneous failures of multiple components. RiskSpectrum tool follows this modeling and calculation pattern. The original basic event is replaced by the OR-gate. This means that the analyzed fault tree does not contain the original event anymore. It contains the new common cause failure events instead. For these events, we can calculate all importance measures without any special treatment. It could be discussed if the importance measures should be calculated for the individual events, and extending it to the impact on the CCF events. For example, FC for event A could be considered by setting both A_i and AB to 0 (where A_i is the independent failure part of event A). Currently the FC is calculated for event A_i by RiskSpectrum.

A similar difficulty with common cause failure events can be observed when we analyze structures, systems or components which include basic events with common cause failures. The question is if we should apply the value change to all common cause failure events that fail the component. These events fail also other components. This creates both theoretical difficulties [7] and issues from the practical perspective of risk-informed applications, e.g., [11,12].

Since Fussell-Vesely does not extend to structures, systems and components, we cannot compare it to Fractional Contribution with respect to common cause failures. Fractional Contribution gives a better estimate for common cause failure events than Fussell-Vesely for the same reason as discussed in the previous section.

3.5. Success in Event Trees

To mitigate the impact of high probability events, or event combinations, like for example in seismic risk analysis it may be relevant to consider the success in the event tree. Leaving the consideration of success may make the assessment overly conservative.

As an example, consider a seismic initiator H. The seismic event in itself is not an initiating event, but the event that causes the trip of the plant is. In this case we have two potential conditional initiators P1 and P2. P1 is a large loss of coolant and P2 is a turbine trip. If we consider this as an event tree with P1 as the first function event (top event in the event tree) and P2 as the second function, we end up with three sequences (P1 happening, P2 happening or none of the above). The sum of the sequences should be H. We hence need to consider, in some way, if P1 has happened or not in the calculation of sequence for P2. In this very simple example, we could use a simple MCUB calculation – but when the situation is expanded and each function event for example contains more events than a more advanced calculation approach is needed.

There are different approaches to consider the success in the event tree. In RiskSpectrum we allow the use of a so-called *Success Module* (SM). A SM represents a MCS list of the failure of the function event and is calculated by estimating the failure probability of the function event(s) failing F, and then calculating the success probability S as 1- F [14].

Inclusion of success will create a challenge to the understanding of cutsets containing A, $P(M_A)$ as expressed in section 3. Event A may be included as success in an MCS, but it is not part of the failing part. Hence, FV will not include the impact of event A in the SM.

In FC, as well as the other importance measures, the changed value of the event will also impact the calculation of the SM. Thereby the other importance measures will consider the dependency by impact of success in event trees.

4. DYNAMIC CALCULATIONS

Static calculations can be characterized by the following properties. Basic events are equipped with a constant failure probability, probability of a conjunction of basic events can be calculated as a product of basic event probabilities. System failures occur instantaneously, at least from the mathematical perspective. There is no notion of time, order of events, no evolution of the system state. For this type of calculations, system failure behaviors are modeled by a description of failure combinations, for instance by Boolean functions.

Dynamic behavior of a system requires a notion of state and its evolution in time. Failure behaviors are (sets of) system runs, characterized by a finite set of transitions. Quantification of failure behaviors requires analysis of these runs. It is typically more involved than the plain multiplication of basic event probabilities. It can require a reachability analysis of, e.g., a Continuous Time Markov Chain, a Petri Net, or a Markov Decision Process. In some cases, the only feasible analysis method is a Monte Carlo simulation. For a certain class of dynamic systems, it is possible to characterize failure behaviors by minimal cut sets, but they are interpreted as dynamic systems that must be quantified by more advanced methods than probability multiplications.

Importance measures should extend also to this class of systems. Generally, the behavior of the system cannot be characterized by cut sets. Hence, a measure based on cut sets will not be practical in this context. But it should be possible to quantify measures like FC, RIF and RDF. The RIF can for example be calculated by running a Monte Carlo simulation where the studied component is unavailable – corresponding to setting the failure probability of a basic event to one. For the RDF, we disable failure possibilities of the studied component – corresponding to setting the failure probability of a basic event to zero.

A special case is the I&AB algorithm in RiskSpectrum. It extends fault trees by a possibility of repairs, grace time and deterministic failures. It is an example of a dynamic calculation method that decomposes the fault tree model into minimal cut sets and quantifies them dynamically. There is an analytical solution to the semantic model based on a subclass of Continuous Time Markov Chains. Because the quantification does not only work with event probabilities, we cannot simply change the probabilities of the events to estimate the importance measures. The exact definitions and algorithms are developed in [10]. For dynamic analyses where an analytical solution can be derived, it is expected that one has to develop specific definitions and algorithms – with the context and purpose of the importance measure in mind.

5. CONCLUSION

In this paper we show that Fussell-Vesely importance measure should be used only for comparisons with values already calculated by Fussell-Vesely, like importance measures from a previous version of the model. Otherwise, Fractional Contribution is the preferable importance measure as it captures the fractional contribution of a basic event more precisely, especially when advanced quantification methods of a minimal cut set list, such as MCS BDD, are used. Fussell-Vesely might be still used as an approximation when the calculation speed is more important than the precision. RiskSpectrum therefore plans to change the presentation of Fractional Contribution and instead call it FC/FV and the previous value presented as FV will be changed for FVH (for FV historical).

The paper also investigates aspects of dependencies between events and how that can impact the calculation of importance. The importance measures, Fussell-Vesely excluded, are properly considering the dependencies discussed.

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