Uncertainty analysis for human reliability analysis with Empirical Bayes method and Kass-Steffey adjustment

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Abstract: Human action is required for reliable operation and mitigations of accidents in the nuclear power plant. However, human actions may fail in any situation, and they can lead to unwanted events in the nuclear power plant. Probabilistic safety assessment (PSA) aims to quantify the risk of complex system and human reliability analysis (HRA) is an essential part of the PSA to quantify the human error probabilities. In the HRA models, the model parameters are typically determined based on expert judgement due to the lack of data. Bayesian approach can provide a framework to combine the prior state-of-knowledge and empirical data. One of challenges in the Bayesian framework for the HRA is constructing prior distribution. Empirical Bayes method, which is one of hierarchical models, is a method to construct prior distribution based on the empirical data and maximum likelihood estimation. Although empirical Bayes method can provide prior knowledge for the parameters, the uncertainties in the hyperparameters are not accounted and the uncertainty in the combined knowledge is underestimated. One of technique to resolve this problem is a Kass-Steffey adjustment. In the PSA for nuclear power plant, the Kass-Steffey adjustment is applied to exponential family because beta and gamma distribution are used as conjugate prior distributions for the binomial distribution and Poisson distribution, respectively. However, in the HRA, the lognormal distribution is widely used to represent likelihood function and model parameter uncertainties.
In this paper, a closed form expression of the Kass-Steffey adjustment for lognormally distributed likelihood function is developed. The developed method is applied to an example HRA situation and compares the results with the results when the Kass-Steffey adjustment is not applied.

Keywords: PRA, Human reliability analysis, Uncertainty analysis, Bayesian inference

1. INTRODUCTION

Human actions are important parts of accident mitigation strategies in the nuclear power plants. In principle, human error can occur during the accident mitigation procedure. Human error has been analyzed as a significant risk contributor in the probabilistic safety assessment (PSA). Human reliability analysis (HRA) identifies potential human failure events and quantifies the occurrence probabilities of the events. There are several factors which affect the human error probabilities. Among the factors, time is a major factor for most HRA methodologies. Technique for Human Error Rate Prediction (THERP) predicts the human error probability in diagnosis process as a function of the time available [1]. A Technique for Human Error Analysis (ATHEANA) treats both the time available and the time required as a performance shaping factor which affects the human error probabilities [2]. Human Cognitive Reliability (HCR) analyzes both the time available and the time required for the human actions, and the human error probability is estimated based on the temporal factors [3]. In general, time has uncertainties because human action depends on multiple sources of variation (e.g. environmental factors, procedures, and so on). Integrated Human Event Analysis System (IDHEAS) provides guidance for estimating the uncertainty distributions of the time required and the time available [4]. In the time-based HRA models, multiple uncertainty factors are analyzed and modeled in the uncertainty distributions. Although multiple uncertainty factors are integrated into the uncertainty distributions, some factors remain excluded from the analysis. These excluded factors affect model parameters of the uncertainty distributions and contribute to the plant-specific characteristics of these model parameters.

In the PSA for nuclear power plants, a Bayesian approach is typically used to analyze uncertainties of model parameters. These uncertainties are represented by empirical data and prior state-of-knowledge within the Bayesian framework. The plant-specific empirical data is used to estimate the current knowledge about plant-specific characteristics. However, the empirical data collected from other plants is used when the plant-specific data is not enough to characterize the model parameters. The data from other plants contribute to the prior knowledge and construct an informative prior distribution for the model parameters. Empirical Bayes is one of methods to construct informative prior distribution when there is a variability between the plants. The empirical Bayes method estimates the parameters of the prior distribution as point values based on the maximum likelihood estimation. However, there are uncertainties in the parameters of the prior distribution because the collected data from other plants is limited. Kass and Steffey proposed an adjustment method to consider the uncertainties in the parameters of the prior distribution [5], and it has been applied to the reliability data for independent failures of components [6, 7]. However, unlike the data for the component failures, the times in HRA are typically modeled as lognormal distributions. For example, Kim et. al. analyzed diagnosis time in digital control room and showed that the lognormal distribution is appropriate for the time required distribution [8]. Therefore, the Kass-Steffey adjustment method for lognormal distribution is required to quantify the uncertainty in the HRA.

In this paper, the Kass-Steffey adjustment method for lognormal distribution is derived. Section 2 describes time-based human reliability model. Section 3 describe the mathematical equations for the Empirical Bayes and the Kass-Steffey adjustment for lognormally distributed data. In Section 4, the human reliability analysis is performed with an example empirical data set. Then, the results with proposed method and conventional method are compared. Section 5 presents the concluding remarks.

2. Time-based human reliability analysis model

The time-based HRA model analyze the human error probability as a function of time available and time required to perform human action. The time required is the time taken for mitigation process. Even if human can correct their actions during the mitigation process, the nuclear power plant may become irreversible state a long time after some events. The time that human can correct their actions and mitigate accident is the time available. When the time required is larger than the time available, the nuclear power plant becomes irreversible state, and the human action is treated as failure. Therefore, the time-based model focuses on analyzing the two times with respect to various conditions of nuclear power plants and events. However, in the real world, the human action and the following time required are uncertain. The uncertainty in the time is represented as uncertainty distribution. The parameters of the uncertainty distribution are estimated based on observed data from multiple data sources. For example, simulator can be used to collect the time required data with respect to various plant conditions or accident scenarios. On the other hand, the time available also have uncertainty depending on the scope of the analysis. Especially, in the accident conditions, the state of plants is very uncertain and hard to estimate the exact time available. Furthermore, the time available may depend on the time that human action is successfully performed. One of method to analyze the time available is the use of thermal-hydraulic code. Various times that human action is performed are used to input data of the thermal-hydraulic analysis. Then, the multiple thermal-hydraulic simulations are performed, and the results determine whether the accident is successfully mitigated or not.

The success or failure probability of accident mitigation can be estimated based on the simulation data. The described uncertainties in the time required and the time available are referred to as aleatory uncertainty. Aleatory uncertainty represents randomness of nature, and it is integrated into the probability in the probabilistic safety assessment. In the time-based HRA model, the human error probability represents the aleatory uncertainty in the process and the analyzed uncertainties are integrated into the human error probability itself. When the time required is analyzed as uncertainty distribution and the time available is analyzed as conditional failure probability given human action time, the human error probability can be calculated based on rule of probability.

$$\boldsymbol{HEP} = \int \boldsymbol{p}(\boldsymbol{t}) \Pr(\boldsymbol{failure}|\boldsymbol{t})\, \boldsymbol{dt} \tag{1}$$

where $\boldsymbol{p}$ is probability distribution function for the time required $\boldsymbol{t}$. Lognormal distribution is widely used uncertainty distribution model to describe natural phenomena. Therefore, the uncertainty distribution for the time required is typically modeled as a lognormal distribution.

$$\boldsymbol{t} \sim \boldsymbol{Lognormal}\left(\ln \boldsymbol{\mu}, \boldsymbol{\sigma}^2\right) \tag{2}$$

3. Uncertainty analysis for human error probability

Even though the human error probability can be calculated when the model parameters of lognormal distribution is determined, there is another source of uncertainty in the distribution. As the model parameters are estimated based on limited observed data, the estimated model parameters cannot represent all the conditions which affect the time uncertainty. This parameter uncertainty is referred to as epistemic uncertainty and it comes from lack of state-of-knowledge of analyst. In contrast to aleatory uncertainty, epistemic uncertainty is not integrated into the human error probability and separately analyzed by uncertainty analysis. In the PSA for nuclear power plants, uncertainty analysis analyzes model parameter uncertainties of events and their impact on the uncertainty of the risk metric using Bayesian framework. As the human error probability is a major event in the PSA, the uncertainty analysis should be performed. When the uncertainties of model parameters of the time required are analyzed, the epistemic uncertainty of human error probability can be quantified. Distribution model for model parameters is an important factor to uncertainty analysis. Due to the mathematical property, conjugate prior distribution is typically used to model the uncertainty distribution for the epistemic uncertainty. Conjugate prior makes current state-of-knowledge distribution same as prior distribution model. In the case of human error probability, the time require follows lognormal distribution. The conjugate prior distribution for the lognormal distribution is another lognormal distribution when the target of analysis is a model parameter related with expectation of logarithm values.

$$\boldsymbol{\mu} \sim \boldsymbol{Lognormal}(\boldsymbol{m}, \boldsymbol{s^2}) \tag{3}$$

where m and s^2 are hyperparameters which are parameters of model parameter μ. If the independently observed time data set, $\boldsymbol{t} = [t_1, \cdots, t_n]$, is collected, the current uncertainty distribution for the model parameters can be expressed as follows:

$$\boldsymbol{\mu}|\boldsymbol{t} \sim \boldsymbol{Lognormal}\left(\frac{s^2 \sum_{i=1}^{n} \ln t_i + m\sigma^2}{s^2 n + \sigma^2}, \frac{\sigma^2 s^2}{s^2 n + \sigma^2}\right) \tag{4}$$

The uncertainty of human error probability can be analyzed based on the derived current uncertainty distribution in Eq. (4).

3.1. Kass-Steffy adjustment for lognormal distribution

In the practical uncertainty analysis of the PSA for nuclear power plant, two stage sampling process is considered. The first stage is plant-to-plant variability for the model parameters. The second stage is plant-specific uncertainty for the model parameters and the empirical data is observed based on the second stage process. In the two-stage model, there are three random variables, sampled empirical data, model parameters for the sampling process, and hyperparameters which are parameters of the uncertainty distributions for the model parameters. Empirical Bayes method is a part of two stage hierarchical models. Empirical Bayes considers marginal likelihood function that all possible model parameters of likelihood function are considered. In the marginal likelihood function, the model parameters are integrated and consider likelihood function for hyperparameters with observed data.

$$L(\xi; D) = \int L(\theta; D) p(\theta|\xi) d\theta \tag{5}$$

where D is empirical data, θ is a set of model parameters, and ξ is a set of hyperparameters.
The hyperparameters are estimated to maximize the marginal likelihood function and represent the prior state of knowledge. The estimated hyperparameters represent the shape of the plant-to-plant variability. However, there are uncertainties in the estimated hyperparameters because the hyperparameters are estimated based on limited observed data. When the uncertainties in the hyperparameters are considered, the current uncertainty distribution has wide distribution compared to that without the uncertainties in the hyperparameters. Therefore, the current uncertainty may be underestimated when the uncertainties in the hyperparameters are not considered. The analytic method to consider the uncertainties in the hyperparameters is constructing uncertainty distributions for the hyperparameters and consider marginal distribution for the model parameters. Then, the variance of model parameter given observed data can be expressed as combination of conditional expectation and variance with the hyperparameters.

$$Var(\theta|D) = E_{\xi|D}[Var(\theta|D, \xi)] + Var_{\xi|D}[E(\theta|D, \xi)] \tag{6}$$

The first term of Eq. (6) represents the estimated variance without the uncertainties in the hyperparameters, and the second term represents the additional variance due to the uncertainties in the hyperparameters. However, if the uncertainty distributions for the hyperparameters are constructed, these distributions have their model parameters, and these parameters also have uncertainties. In this framework, it is not possible to estimate the uncertainty distributions for the sequential of parameters. Kass and Steffey proposed an approximation method to reflect the uncertainties in the hyperparameters. Kass-Steffey adjustment uses delta method to approximate the variance and expected values given the observed data and the hyperparameters as the values that the hyperparameters are the estimated values from empirical Bayes method [5].

$$E_{\xi|D}[Var(\theta|D,\xi)] \approx Var\left(\theta\middle|D,\tilde{\xi}\right) \tag{7}$$

$$Var_{\xi|D}[E(\theta|D,\xi)] \approx \frac{\partial}{\partial\xi}E(\theta|D,\xi)^T|_{\xi=\tilde{\xi}}\tilde{\Sigma}\frac{\partial}{\partial\xi}E(\theta|D,\xi)|_{\xi=\tilde{\xi}} \tag{8}$$

where $\tilde{\xi}$ is the estimated hyperparameters with empirical Bayes method, and $\tilde{\Sigma}$ is negative of Hessian matrix of log-likelihood when the hyperparameters are $\tilde{\xi}$.
The empirical Bayes method considers the plant-to-plant variability and the observed data set from the plant are assumed as independently distributed. When there are *N* number of plants, the likelihood function for the time required can be expressed as

$$L(t;m,s^2) = \prod_{j=1}^{N}\frac{1}{(2\pi)^{n_j/2}\,\sigma^{n_j-1}\sqrt{s^2n_j+\sigma^2}\prod_{i=1}^{n_j}t_{ij}}e^{-\frac{1}{2}\left\{\frac{\sum_{i=1}^{n_j}\left(\ln t_{ij}\right)^2}{\sigma^2}+\frac{m^2}{s^2}-\frac{\left(s^2\sum_{i=1}^{n_j}\ln t_{ij}+m\sigma^2\right)^2}{\sigma^2s^2\left(s^2n_j+\sigma^2\right)}\right\}} \tag{9}$$

where n_{ij} is *i*-th observed data from *j*-th plant. The maximum likelihood estimation can be performed based on the log-likelihood and its partial derivations.

$$\frac{\partial}{\partial m}\ln L(t;m,s^2) = \sum_{j=1}^{N} -\left\{\frac{\left(n_jm-\sum_{i=1}^{n_j}\ln t_{ij}\right)}{(s^2n_j+\sigma^2)}\right\} = 0 \tag{10}$$

$$\frac{\partial}{\partial s^2}\ln L(t;m,s^2) = \sum_{j=1}^{N} -\frac{1}{2}\left\{\frac{n_j}{s^2n_j+\sigma^2}-\frac{\left(n_jm-\sum_{i=1}^{n_j}\ln t_{ij}\right)^2}{\left(s^2n_j+\sigma^2\right)^2}\right\} = 0 \tag{11}$$

To derive the approximated variance in Eq. (8), the partial derivative of the expected values for *j*-th plant should be derived.

$$\frac{\partial}{\partial m}E\left(\mu_j\middle|t_j,m,s\right) = E\left[\mu_j\middle|t_j,m,s\right]\left(\frac{\sigma^2}{s^2n_j+\sigma^2}\right) \tag{12}$$

$$\frac{\partial}{\partial s^2}E\left(\mu_j\middle|t_j,m,s\right) = \frac{E[\mu_j|t_j,m,s]}{s^2n_j+\sigma^2}\left\{\sum_{i=1}^{n_j}\ln t_{ij} - \frac{n_j(s^2\sum_{i=1}^{n}\ln t_{ij}+m\sigma^2)}{s^2n_j+\sigma^2}+\frac{1}{2}\sigma^2-\frac{1}{2}\frac{s^2n_j\sigma^2}{s^2n_j+\sigma^2}\right\} \tag{13}$$

and the Hessian matrix is

$$H = \begin{bmatrix} \sum_{j=1}^{N} -\frac{n_j}{(s^2n_j+\sigma^2)} & \sum_{j=1}^{N}\frac{n_j\left(n_jm-\sum_{i=1}^{n_j}\ln t_{ij}\right)}{\left(s^2n_j+\sigma^2\right)^2} \\ \sum_{j=1}^{N}\frac{n_j\left(n_jm-\sum_{i=1}^{n_j}\ln t_{ij}\right)}{\left(s^2n_j+\sigma^2\right)^2} & \sum_{j=1}^{N}\frac{1}{2}\left\{\frac{n_j^2}{\left(s^2n_j+\sigma^2\right)^2}-\frac{2n_j\left(n_jm-\sum_{i=1}^{n_j}\ln t_{ij}\right)^2}{\left(s^2n_j+\sigma^2\right)^3}\right\} \end{bmatrix} \tag{14}$$

The Kass-Steffey adjustment for lognormal distribution can be calculated based on the Eq. (10-14), and the model parameter uncertainty distribution for specific plant is also derived.

4. Application

To compare the results with Kass-Steffey adjustment with the conventional method, a benchmark problem for human reliability analysis condition is used. Suh et. al. perform human reliability analysis under severe accident condition based on time uncertainty distributions [9]. The target of analysis is the severe accident guideline for total loss of component cooling water (TLOCCW) scenario in the OPR 1000 for preventing reactor vessel failure. During the scenario, both motor-driven pump and turbine-driven pump in the auxiliary feed water system are not available. The operator should perform three severe accident guidelines, injection into steam generator, depressurize reactor coolant system, and injection into reactor coolant system. Then, the time available is analyzed by MAAP code based on the time that the operator performs the three severe accident guidelines. The conditional reactor vessel failure probability which derived from the analysis is used to quantify the human error probability.

$$\mathbf{Pr}(reactor\ vessel\ \boldsymbol{failure}|\boldsymbol{t}) = 0.0337e^{0.0114t} \tag{15}$$

The situations that human actions are performed within 30 minutes are treated as success, and the situations that human actions are performed after 300 minutes are treated as failure no matter what the conditional failure probability in Eq. (15) is.

To simplify the problem, although the analysis should include the three human actions and the times for diagnosis and decision making, the time required is represented as a single lognormal distribution in this paper. Therefore, the assumed data set for the time required is used to quantify the uncertainty distribution for model parameters. Table 1. shows the model parameters of assumed three plants to generate random data for the time required. Figure 1. shows the uncertainty distribution for the time required with respect to the plants.

Table 1. Plant specific model parameters for the time required data

	μ	σ^2
Plant 1	12.1825	1
Plant 2	20.0855	1
Plant 3	33.1155	1

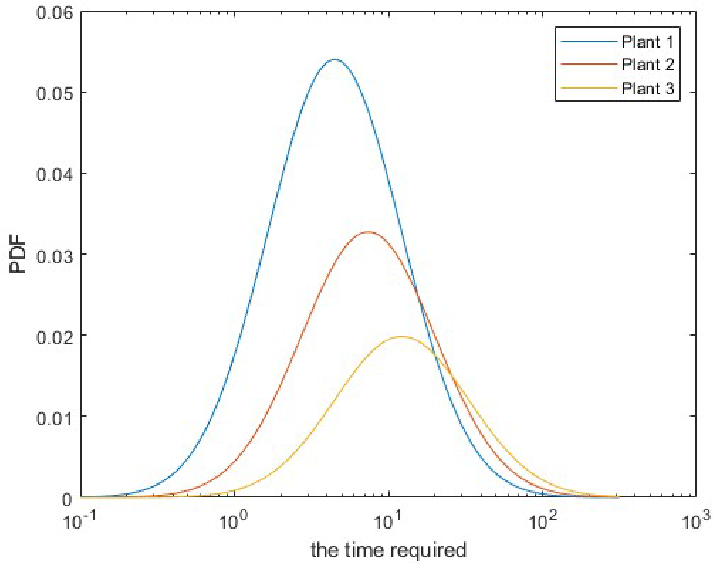
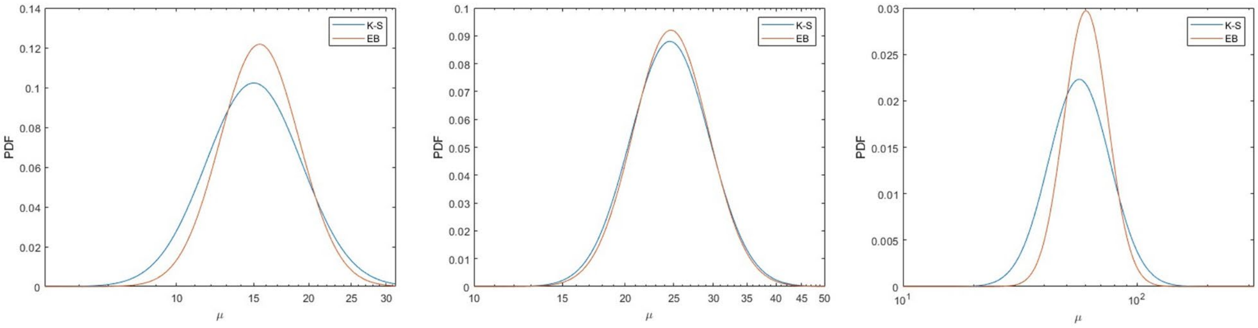


Figure 1. Uncertainty distributions for the time required w.r.t the plants

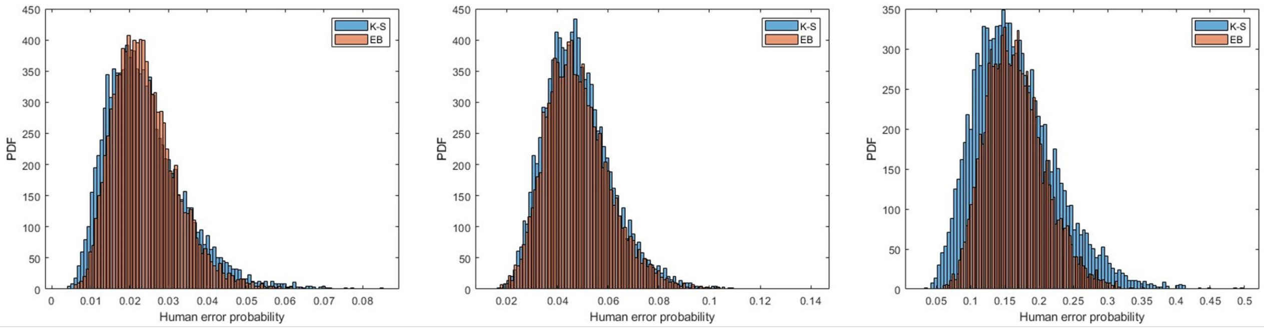
The randomly generated example data set is used to estimate the model parameters of the plants. To compare the results of proposed method with the conventional method, both Kass-Steffey adjustment and the simple empirical Bayes method are used to analyze the model parameter uncertainty. Figure 2. shows the plant specific uncertainty distributions for the model parameters μ. It is shown that the results of plant 2 has similar uncertainty distributions because the model parameter of plant 2 has medium values among the plants. On the other hand, the results of plant 1 and plant 3 has difference, and the results with Kass-Steffey adjustment has large variance due to the uncertainties in the hyperparameters.



(a) (b) (c)

Figure 2. Plant specific model parameter uncertainty distributions (a) plant 1, (b) plant 2, and (c) plant 3

Figure 3. shows the uncertainty distribution for the estimated human error probability and Table 2. shows the mean and variance of the human error probability with respect to the plants. It is shown that the expected values are almost equivalent with respect to the analysis method. However, the variance for the Kass-Steffey adjustment is larger than that of the simple empirical Bayes method because of the uncertainties in the hyperparameters. Therefore, when the Kass-Steffey adjustment is not applied, the uncertainty of human error probability can be underestimated.



(a) (b) (c)

Figure 3. Plant specific uncertainties for the human error probability (a) plant 1, (b) plant 2, and (c) plant 3

Table 2. Plant specific moments of the human error probability

	$E[HEP]$		$Var(HEP)$	
	K-S adjustment	Empirical Bayes	K-S adjustment	Empirical Bayes
Plant 1	0.0240	0.0240	1.040E-4	6.838E-5
Plant 2	0.0482	0.0481	1.731E-4	1.600E-4
Plant 3	0.1656	0.1660	3.600E-3	1.900E-3

4. CONCLUSION

There are several sources of uncertainty in the human reliability analysis. Among the uncertainties, epistemic uncertainty is a major scope of uncertainty analysis because this uncertainty can be reduced as the state-of-knowledge is increased. In the time-based HRA model, the epistemic uncertainty exists in the model parameter for the time required distribution model. The conventional method to analyze the model parameter uncertainty is empirical Bayes which estimate the hyperparameters as point values. However, the estimate is based on the limited empirical data, and there are uncertainties in the hyperparameters. Furthermore, the distribution model for the model parameter is lognormal distribution which is different from the models for the independent component failure model.
The object of this paper extends the Kass-Steffey adjustment method for lognormal distribution to appropriate uncertainty analysis for the human reliability and make consistency in the input data for the PSA model. It is shown that the Kass-Steffey adjustment reflect the uncertainty in the hyperparameters, and the impact becomes significant when the plant-specific characteristic is far from the industry average performance. Furthermore, the uncertainty of model parameter is propagated to human error probability and the uncertainty of human error probability can be underestimated when the Kass-Steffey is not applied. The derived mathematical formula and analysis results are expected to contribute to increase the quality of the PSA model and the analysis result.

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