

## Proposal of Uncertainty Analysis Methodology for L1PRA in Accordance with Markov State-transition Model

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**Abstract:** Digital Reactor Protection System (Digital RPS) are used for the new plants such as ABWR and APWR. Therefore, a new risk assessment method for the Digital RPS is essential to comply with the new regulatory standards and to continuously improve the safety of the plants.

The framework for evaluating the risk of core damage due to Anticipated Transient Without Scram(ATWS) sequence with considering uncertainty and calculate efficiency. And this method is used to evaluate the risk of Digital RPS that has complicated state transitions and variations in failure rates.

By applying this method to complex Digital RPS such as 2 out of 4, we believe that it will be possible to dynamically assess their reliability and evaluate plant risks in more detail, which will be useful for reducing plant risks and continuously improving plant safety.

**Keywords:** Dynamic PRA, ATWS, State transition model, Monte Carlo method

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### 1. INTRODUCTION

Since the accident of Fukushima Daiichi Nuclear Power Plant, Nuclear safety regulations had been revised for the countermeasure. Digital circuits are used in the Reactor Protection System (RPS) of new plants such as ABWR and APWR, although the risk assessment methods including Digital RPS are still under consideration. Development of risk assessment methods for Digital RPS is essential to comply with the new regulatory requirements and to continuously improve safety.

Establishing a Probabilistic Risk Assessment (PRA) model for Digital RPS is an important issue for The Working Group on Risk Assessment (WGRISK). In this context, a dynamic modeling approach can accurately capture the time and probabilistic system behavior specific to digital devices compared to Event Tree Analysis (ETA) and FTA methods. However, fewer PRA has been conducted using this approach, and the development of a PRA model for Digital RPS is in the experimental stage.

In the background and circumstances, FTA and various reliability analysis methods for Digital RPS have been studied, particularly, application of kinds of dynamic methods is introduced because it is necessary to evaluate state probabilities dynamically since Digital RPS has the nature of continuous state transition.

For example, methods using Dynamic Flowgraph Method (DFM), methods using Markov/ Cell-to-Cell Mapping Method (CCMC), methods using statistical techniques, methods using Bayesian Belief Networks (BBN), and Markov model are listed. However, FTA has difficulty for simulating complicated transition model because of its nature. Also, other methods above have problem with handling dynamic state transitions and compatibility with PRA, and are not yet suitable for PRA of plants equipped with digital RPS.

To resolve those issues, Hitoshi Muta et al. had developed Digital RPS reliability assessment method

“Quantitative modeling of digital reactor protection system using Markov State-Transition model,” [1].

Using the paper as a reference, we have developed a reliability assessment method for Digital RPS using a Markov state transition model and that is consistent with conventional PRA. This paper provides an overview of those methods.

## 2. ANALYSIS METHOD BASED ON MARKOV TRANSITION MODEL

### 2.1. ATWS core damage event logic consistent with conventional Level 1 PRA

In the existing frameworks, the FTA method has been used to construct system reliability models. This method logically deploys combinations of equipment failures represented by initiating events and non-deployed events that lead to a top event (Core Damage by ATWS). For example, in the case shown in Figure 1, where a RPS fails after an initiating event (LOCA, LOSP, etc.) occurs, the RPS failure modes such as Independent Failure / Common Cause Failure / Software Failure are connected to the Initiating Event on the right side of each with a Priority AND gate to represent a combination of events that will cause ATWS, taking into account the order of occurrence, and these combinations are connected with an OR gate to represent that any of the combinations will lead to ATWS or even core damage.

Based on this FTA, the probability of the initiating event and each base event can be set to determine the probability of the top event, i.e., the probability of core damage (Core Damage by ATWS).

Although a method has been proposed that expresses order-dependent scenarios by using Priority AND gates in FTA logic and enables dynamic analysis in combination with a Bayesian network, those conventional methods, including FTA method, have difficulty in complicated simulation at component states, such as fluctuations in the probability of various plant states due to periodic function checks, repairs, and manual shutdown operations, or classification of failure modes into detected faults and undetected faults, and state transitions between the detection and repair of corresponding fault states.

Therefore, a reliability assessment method for digital systems that is consistent with the conventional Level 1 PRA framework is still in a development stage. Specifically, it is necessary to develop a method that is consistent with the conventional Level 1 PRA FTA and ETA that engineer safety facilities composed of conventional equipment, and required reliability analysis method considering state transitions and logical model that can contribute to uncertainty analysis and importance analysis is required.

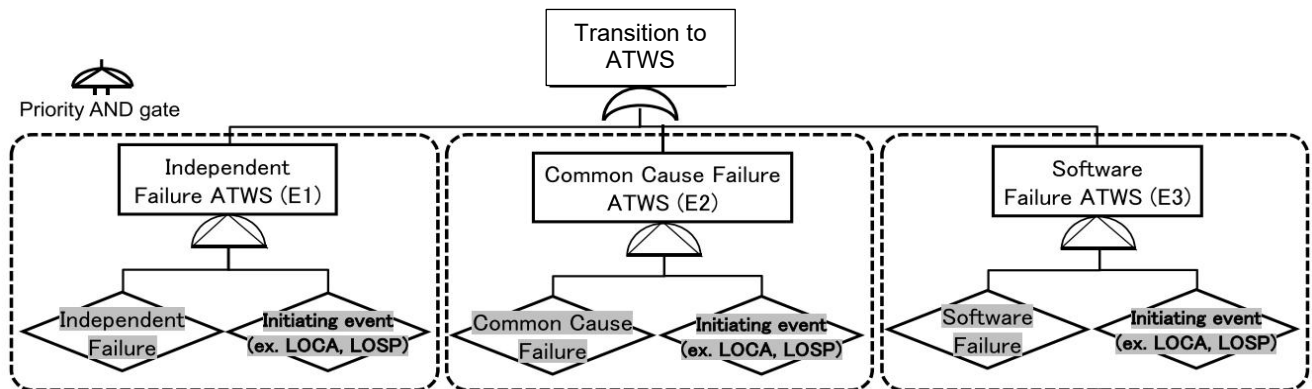
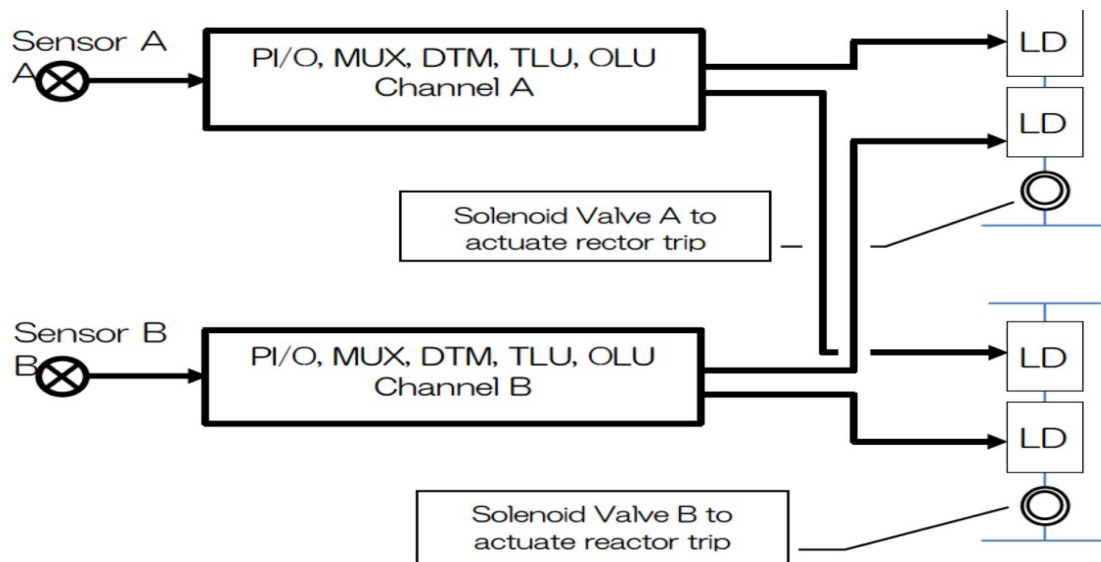


Figure 1. Simplified Logic of Fault Tree Analysis

### 2.2 System description for hypothetical simple digital RPS (1 out of 2 configuration)

The RPS is one of the most important safety equipment that controls reactivity, and loss of the RPS will cause a core damage accident, ATWS that is defined in traditional PRA. An ATWS event sequence can be defined as an event in which the RPS and/or control rod insertion do not function properly following an initiating event. As shown in Figure 2, which is called a “1 out of 2 system”, we assume that two channels each control two trip systems. If either one of the two channels, A and B, operates, a scram shutdown can be achieved.



PI/O: Parallel Input Output  
 TLU: Trip Logic Unit

MUX: Multiplex  
 OLU: Output Logic Unit

DTM: Digital Trip Module  
 LD: Load Driver

**Figure 2. 1 out of 2 System of Digital RPS**

### 2.3 State transition model description for the assumed system and ordinary differential equations (ODEs)

In reliability analysis, the failure states of the Digital RPS hardware are classified first. Taking a 1 out of 2 system as an example, the failure states are classified into a no failures as Normal (A), a Shutdown state (B), 1 Undetected failure state (C), 1 Detected failure state (D), 2 Undetected failure states (E), 1 Undetected and 1 Detected failure state (F), 2 Detected failure states (G), and ATWS (H) when both of the two components fail and an initiating event occurs. The initial state is defined as a Normal state with a probability of 1.0, and an environment is assumed in which these states transition due to failures and repairs. In other words, it is assumed that the probability of each state changes time-dependently based on the transition rate between states.

Digital devices that are constantly monitored, such as logic circuits, are reset when a failure is detected, and devices that are periodically functionally tested are repaired when a failure is detected. If it is confirmed that the redundant system is functioning normally, repairs are made while operation continues, and if it is confirmed that both channels are not functioning, the plant is manually shut down. If a causative event occurs when both channels are not functioning, the reactor will lose its shutdown function, resulting in ATWS and core damage. Causative events and each failure mode occur statistically independent and randomly.

Transitions from a Normal state (A) to 1 Undetected failure state (C) or 1 Detected failure state (D), or transitions from C and D to E, F and G, are caused by failures, so the numerical values  $\lambda_D$  and  $\lambda_U$  based on the failure rate that are used for the transition rate. Scram stop and ATWS are caused by causative events, so  $\lambda_M$  based on their occurrence rate is used for the transition rate.

Transitions other than failure, scram stop and ATWS are caused by repairs or manual shutdowns, so numerical values based on the respective occurrence rates are assigned. Based on this idea, the relationship between the transitions between each state and the probability of being in each state is expressed by simultaneous differential equations, and a quantitative evaluation in a non-steady state is performed numerically, and a dynamic analysis taking into account time-dependent changes is performed. We propose two calculation methods: a method using an ODE solver and a method using the Monte Carlo method.

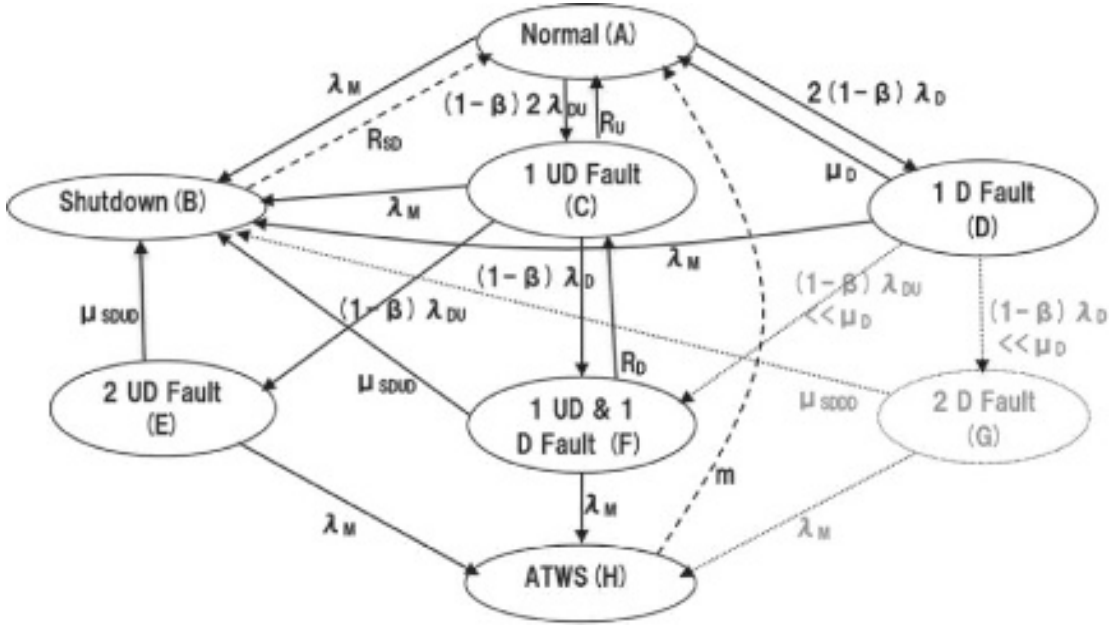


Figure 3. State-transition model for 1 out of 2 System

### 3. ODE QUANTIFICATION METHODS

#### 3.1 Using ODE solver (Point estimation)

In the State transition model (Section 2.3), the expected transition rate is expressed as a differential equation. A code using an ODE solver is created in the numerical calculation software MATLAB to simulate the state transition. The transition rate can be thought of as the rate of movement per unit time to the destination relative to the state probability of the source. For the change rate in each states as  $P'$ , A Subtraction term for a state is total rates of movement to multiple destinations. Addition terms for a state is movements from multiple sources to the state. Complete  $P'$  for all states to create simultaneous differential equations. Note that because the sum of the state probabilities is 1, so the sum of the initial values, which are parameters for ODE solver, is 1, and that the sum of the right-hand sides of the simultaneous differential equations, which are the change amounts, is 0. In the case of the 1 out of 2 system given as an example in State transition model (Section2.3), the following simultaneous differential equations are hold with referring to Ref. [4].

$$\frac{dP_A}{dt} = \{-\lambda_M - 2\lambda_U - 2\lambda_D\} \cdot P_A + \{R_{SD} \cdot P_B + \mu_U \cdot P_C + \mu_D \cdot P_D + m \cdot P_H\}, \quad (1)$$

$$\frac{dP_B}{dt} = -R_{SD} \cdot P_B + \{\lambda_M \cdot P_A + \lambda_M \cdot P_C + \lambda_M \cdot P_D + \mu_{SDUD} \cdot P_E + \mu_{SDDD} \cdot P_F + \mu_{SDDD} \cdot P_G\}, \quad (2)$$

$$\frac{dP_C}{dt} = -\{\lambda_M + \mu_U + \lambda_D + \lambda_U\} \cdot P_C + 2\lambda_U \cdot P_A, \quad (3)$$

$$\frac{dP_D}{dt} = -\{\lambda_M + \mu_D + \lambda_D + \lambda_U\} \cdot P_D + 2\lambda_U \cdot P_A, \quad (4)$$

$$\frac{dP_E}{dt} = -\{\lambda_M + \mu_{SDUD}\} \cdot P_E + \lambda_U \cdot P_C, \quad (5)$$

$$\frac{dP_F}{dt} = -\{\lambda_M + \mu_{SDDD}\} \cdot P_F + \lambda_D \cdot P_C + \lambda_U \cdot P_D, \quad (6)$$

$$\frac{dP_G}{dt} = -\{\lambda_M + \mu_{SDDD}\} \cdot P_G + \lambda_D \cdot P_D, \quad (7)$$

$$\frac{dP_H}{dt} = -m \cdot P_H + \lambda_M \cdot P_E + \lambda_M \cdot P_F + \lambda_M \cdot P_G, \quad (8)$$

Here,  $P_{A(t=0)} = 1.0$  and  $P_{B\sim H(t=0)} = 0.0$ . (9)

The numerical calculation software MATLAB has a command called ODE solver that solves simultaneous differential equations. The simultaneous differential equations mentioned above are created in a file that stores arguments according to the format, and the ODE command is executed by specifying the argument file along with other arguments such as continuous operation time (mainly the periodic inspection interval) and the normal state probability as the initial value of 1 and other states as 0. This causes the calculation to be performed and the state probability of each state is stored in the algebra for each time. This final time and each state probability at that time are used to simulate transitions associated with intermittent periodic inspections, etc. For example, if it is assumed that a faulty state will be repaired by a periodic inspection, a process is performed to move the state probability of this faulty state to the normal state. The state probabilities after this movement process are used as initial values, and the ODE command is executed again using the argument for the continuous operation time and the argument file. This is repeated as many times as necessary to obtain the time and the value of each state probability at that time.

### 3.2 Using Monte Carlo Simulation (MCS) (Uncertainty analysis)

Since the analysis using the ODE solver (Section 3.1) cannot perform calculations including uncertainties, a similar simulation code is created using the Monte Carlo method to perform uncertainty analysis.

As the conditions in Section 3.1 and formulas (1) to (9), the transition rate based on the failure rate, etc. includes uncertainty, so a distribution is set. If it is a normal distribution, the “normrnd” or “lognrnd” commands are used on MATLAB. If there is another assumed distribution, uniform random numbers are generated and then used as an argument to the inverse cumulative distribution function to obtain a random number sample that approximates the distribution of the failure rate, etc.

Next, a formula is created that adds and subtracts the amount of change per unit time based on the transition rate in accordance with State transition model (Section 2.3) as same as Section 3.1. The random number samples are used and substituted into this formula the number of times equal to the quantity of samples, and the calculation is repeated until the continuous operation time is reached, with one calculation being the unit time.

The final calculation result of each state probability at that time are used to simulate transitions due to intermittent regular inspections, etc. as same as Section 3.1. The calculation is repeated again using this calculation result to add and subtract the amount of change per unit time (Figure 4).

The calculation result is generated as samples of the value of each state probability for each calculation and its quantity is same to the quantity of random samples. These values are aggregated and the median, mean, standard deviation, 5-95% value, etc. are calculated.

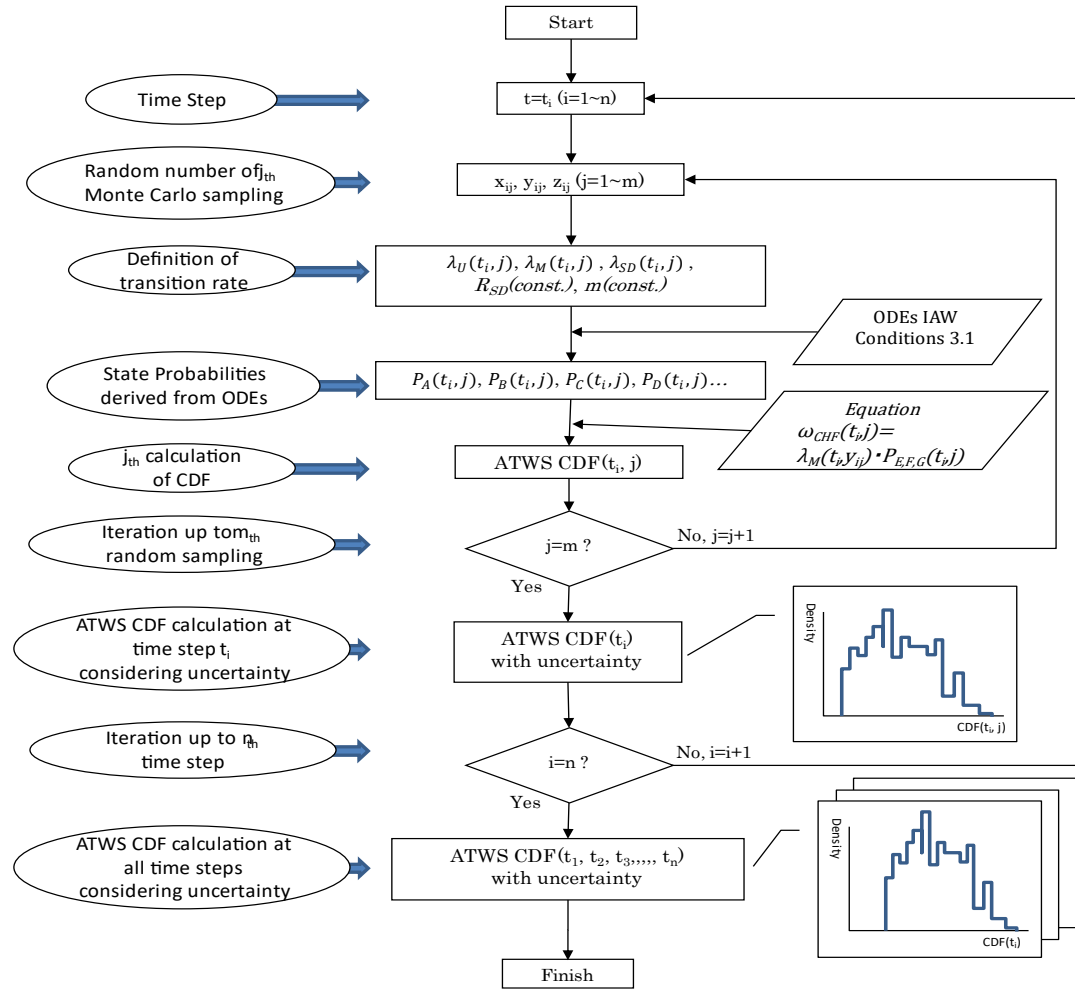


Figure 4. Flow chart: Monte Carlo simulation in State transition model

## 4. CASE STUDY

### 4.1 Point estimation by ODE solver

Conditions of State transition model (Section 2.3) for setting each state, and simultaneous differential equations (Formulas (1)-(9)), and the setting values shown in Table 1 were used to execute the code simulating the continuous operation and regular inspection exemplified in Using ODE solver (Section 3.1). Since this is point estimation, the standard deviation  $\sigma$  of the failure rate is not used. The estimation result of each state is as shown in Figure 5. Over time, the normal state decreases, the shutdown ratio and ATWS occurrence probability increase, and the other failure states are reproduced as they become 0 at each regular inspection and repair period.

Table 1 List of Parameters for Case Study

Parameters	Measurements
$\lambda_M(1/hr)$	Mean= $\ln(1.0 \times 10^{-4})=-9.2, \sigma=2.5$
$\lambda_U(1/hr)$	Mean= $\ln(5.0 \times 10^{-8})=-16.8, \sigma=2.5$
SD(1/hr)	Mean=1, $\sigma=0.1$ (After a surveillance test interval $T_S$ )
$R_{SD}(1/hr)$	1/2160 as constant
$m(1/hr)$	$\infty$ (Defined as $1.0 \times 10^{-100}$ ) as constant

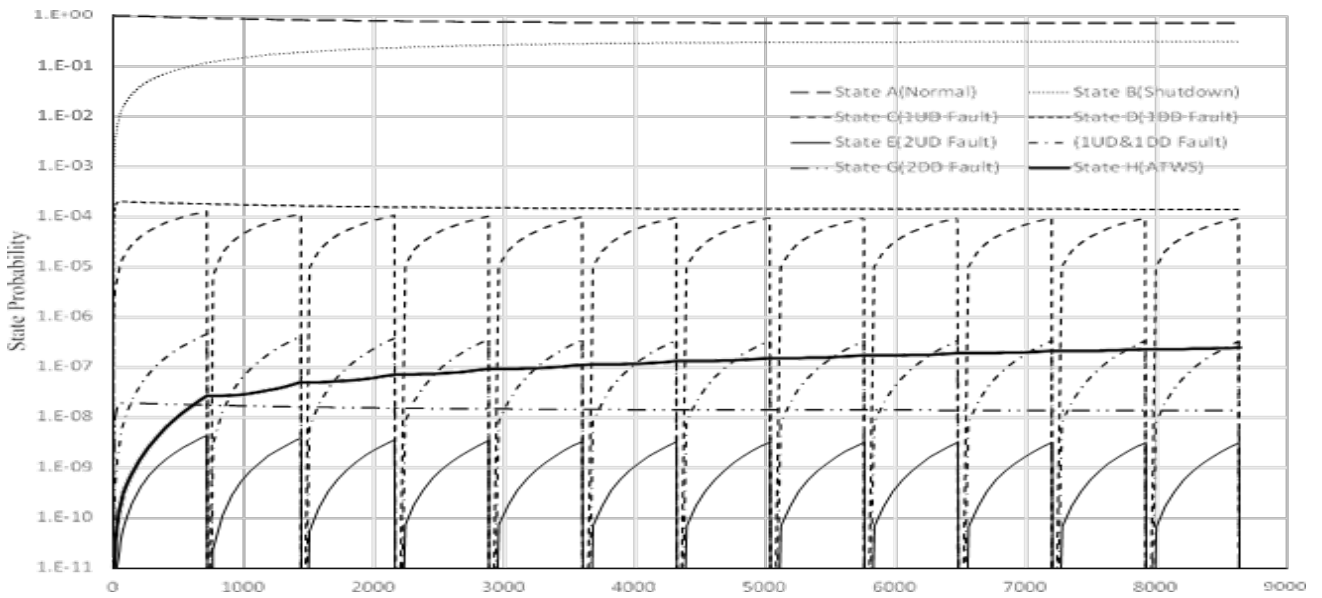
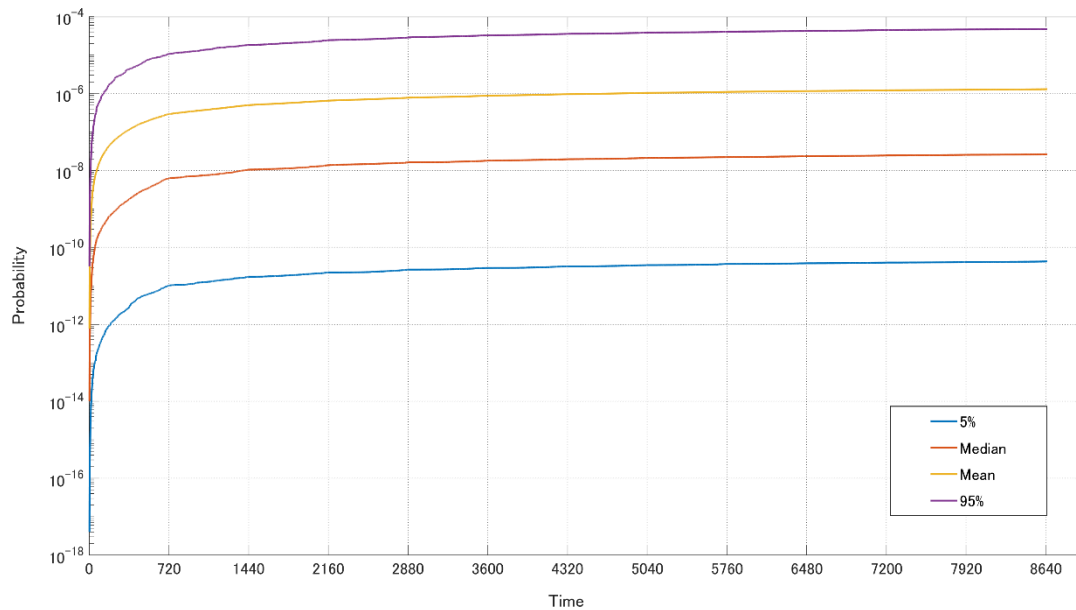


Figure 5. State probability of 8 States in State transition model

#### 4.2 Uncertainty analysis by Monte Carlo Simulation

Using Monte Carlo simulation (Section 3.2), A code was created to solve the differential equations using the Monte Carlo method, and the time-dependent changes in the probabilities of each state were analyzed using the parameters in Table 1. The 5-50-95% values and average values for each of the eight states were calculated, and samples of the state probability for each time for all eight states were calculated. The 5-50-95% values and average values for the ATWS occurrence probability were output as shown in Figure 6, and the probability was shown along with the range of uncertainty. The value used in the point estimation in Point estimation by ODE solver (Section 4.1) was used as the average value of the distribution of the transition rate, but the average value of the samples of the state probability at each time of this analysis result does not match the value in Section 4.1. In other words, the calculation result using the average value of the distribution of the transition rate does not match the average value of the calculation result using the distribution of the transition rate. Since a complex transition of eight states was simulated, the distribution of the transition rate is not preserved at each states and each time steps. And the samples of the state probability of each state are not normally distributed like the transition rate. Therefore, the samples obtained in the uncertainty analysis (4.2 Uncertainty analysis by Monte-Carlo simulation) cannot be used for point estimation.



**Figure 6. State probability results of ATWS in Monte Carlo Simulation**

## 5. CONCLUSION

In this study, based on the reliability evaluation method for Digital RPS developed by Hitoshi Muta et al., we developed a code for a solver that solves differential equations by applying the failure state of the Digital RPS hardware to the state transition model and a code for solving it using the Monte Carlo method in order to construct a dynamic reliability evaluation with uncertainty analysis. Those are reasonable quantification method that are consistent with conventional PRA and using a state transition model in a non-steady case.

In order to use it for plant risk evaluation, it is necessary to confirm the applicability to Digital RPS with a 2 out of 4 configuration that is equivalent to the actual plants. In this case, the reliability calculation will be of a more complicated configuration, and the amount of calculation is expected to be massive. Therefore, it is necessary to develop a code for a reasonable quantification method that improves the simulation accuracy and makes the calculation more efficient by using means such as importance sampling. In addition, we develop a method to evaluate the importance of each component of the Digital RPS and demonstrate its usefulness through trial calculations, and summarize the knowledge obtained from the perspective of risk by combining these reliability evaluation methods.

## 6. REFERENCES

- [1] H. Muta, K. Muramatsu, “Quantitative modeling of digital reactor protection system using Markov State-Transition model”, *Journal of Nuclear Science and Technology*, Vol.51 No.9 (2014), pp.1073-1086.
- [2] H. Muta, “Application of Markov State-Transition Model to Reliability Analysis of 2-out-of-4 Reactor Trip System”, *Transactions of the Atomic Energy Society of Japan*, Vol.14 No.1 (2015), pp.25-39.
- [3] Hitoshi MUTA, Osamu FURUYA, Ken MURAMATSU, “Proposal of PRA Methodology Considering State-Transitions and Time dependent Failure Rate of Components”, *Transactions of the Atomic Energy Society of Japan*, Vol.15 No.2 (2016), pp.70-83.



- [4] Masanobu HARUHARA, Hitoshi MUTA, Yasuki Ohtori, Shohei Yamagishi and Shota Terayama, “Proposal of quantification method of dynamic system reliability model of digital RPS using Markov state-transition model”, Journal of Nuclear Science and Technology Volume 60 (2023), pp.1154-1167.
- [5] Masanobu HARUHARA, Hitoshi MUTA, Yasuki Ohtori, Shohei Yamagishi and Shota Terayama, “Proposal of Uncertainty Analysis Methodology for LIPRA in Accordance with Markov State-transition Model”, Journal of Nuclear Science and Technology Volume 61 (2024), pp.921-934.