

# A Limit Surface Search for PWR LOCA Transients Using Adaptive Machine Learning Techniques

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**Abstract:** A limit surface is an n-dimensional surface describing a nuclear plant status as a function of selected plant parameters that identify the boundaries between failed and safe conditions for the reactor core. A limit surface can be used for a dynamic probabilistic risk assessment (PRA) analysis. In this paper, machine learning algorithms are applied to predict a limit surface efficiently for a PWR small break loss of coolant accident (LOCA). Basic ideas of the algorithms are similar to those of the RAVEN statistical analysis code. As a first step, a set of training simulations is run to construct a surrogate model for determining an approximated limit surface. The surrogate model is then used to predict where further exploration of the input space could be most informative. The new observations are used to update the surrogate model. This kind of adaptive samplings is iterated until convergence is obtained. A case study was carried out for the RELAP5 analysis of an accident management procedure of the steam generator secondary-side depressurization for a small break LOCA scenario with total failure of the high pressure injection (HPI) system in a conventional Westinghouse type PWR. The limit surface was investigated for the peak cladding temperatures as a function of the break size and the depressurization timing. Several sampling methods were compared from the viewpoint of convergence to the limit surface. It was confirmed that using adaptive sampling and machine learning techniques provided a remarkable reduction of the time required for the accurate limit surface determination.

**Keywords:** Limit Surface, Machine Learning, LOCA, RELAP5-3D

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## 1. INTRODUCTION

A limit surface is an n-dimensional surface describing a plant status as a function of selected plant parameters that identify the boundaries between failed and safe conditions for the core. A limit surface can be used for a dynamic probabilistic risk assessment (PRA) analysis. However, the evaluation of the limit surface as a function of the coordinates in the input space is computationally expensive, especially when brute force Monte Carlo methods are chosen as the sampling strategy. Therefore, applications of machine learning algorithms are desired to reduce the computational cost for the limit surface determination.

In this paper, adaptive machine learning algorithms were applied to determine a limit surface for a PWR small break loss of coolant accident (LOCA). Evaluations of the algorithms showed that they were able to calculate an n-dimensional limit surface with minimal effort.

First, the accident analyses using RELAP5-3D code [1] were carried out for a small break LOCA scenario with total failure of the high pressure injection (HPI) system in a conventional Westinghouse type PWR. In the case of total failure of the HPI system following a small break LOCA in a PWR, the steam generator (SG) secondary side depressurization is necessary for an accident management (AM) in order to allow actuation of accumulator systems and reflooding of the core. The key parameters for the AM procedure are the break size, the depressurization timing, the number of depressurizing SGs, and the depressurization rate [2]. For the sake of simplicity, the investigations in this paper were restricted to the limit surface of the first two parameters. The peak cladding temperatures (PCTs) were calculated by the RELAP5 code as a function of the LOCA break size and the SG depressurization timing. The limit surface was evaluated as the reference analysis for the machine learning applications.

Next, machine learning algorithms were applied to determine the relevant limit surface. The basic ideas of the algorithms of this paper are similar to those of the RAVEN statistical analysis code [3]. As a first step, a set of training simulations is run to construct a surrogate model for determining an approximated limit surface. The surrogate model is then used to predict where further exploration of the input space could be most informative. The new observations are used to update the surrogate model. This kind of adaptive samplings is iterated until it is converged. Verification of the guessed limit surface is then automatically performed.

Finally, the accuracy of the limit surface prediction was evaluated by comparing the prediction to the reference analysis by the RELAP5 code. The probability of success and failure was evaluated using the limit surface prediction. Several adaptive sampling methods were compared from the viewpoint of convergence to the real limit surface.

## 2. REFERENCE ANALYSIS BY RELAP5 CODE

### 2.1. Accident Scenario

In the case of total failure of the HPI system following small break LOCA in pressurized water reactors, the break size is so small that the primary system does not depressurize to the accumulator injection set point and the core is uncovered extensively. Therefore, SG secondary-side depressurization is necessary for the AM procedure in order to allow accumulator system actuation and the core reflooding. The effectiveness of SG secondary-side depressurization in small break LOCAs with HPI failure has been investigated experimentally at the Large Scale Test Facility (LSTF) (see for example, [2]). The key parameters for the AM procedure are the break size, the depressurization timing, the number of depressurizing SGs, and the depressurization rate [2]. However, the effectiveness of the AM procedure for the actual plant design and operational characteristics should be investigated.

In the author's previous study [4], the uncertainty propagation analyses were performed for a conventional Westinghouse type PWR for a small break LOCA scenario assuming conditions of the accident management (AM) procedure as based on plant emergency operation manuals.

Figure 1 shows a nodalization model of a conventional Westinghouse type PWR. The RELAP5 model consists of the reactor vessel, primary loop, pressurizer, SGs, and emergency core cooling system (ECCS). The reactor core is represented by the average power channel, the high power channel and the highest power channel, all of which are divided into six sections in the axial direction. The SG U-tubes are simulated by eight cells for the straight part and two cells for the bent part. In order to simulate countercurrent flow limitation (CCFL) in

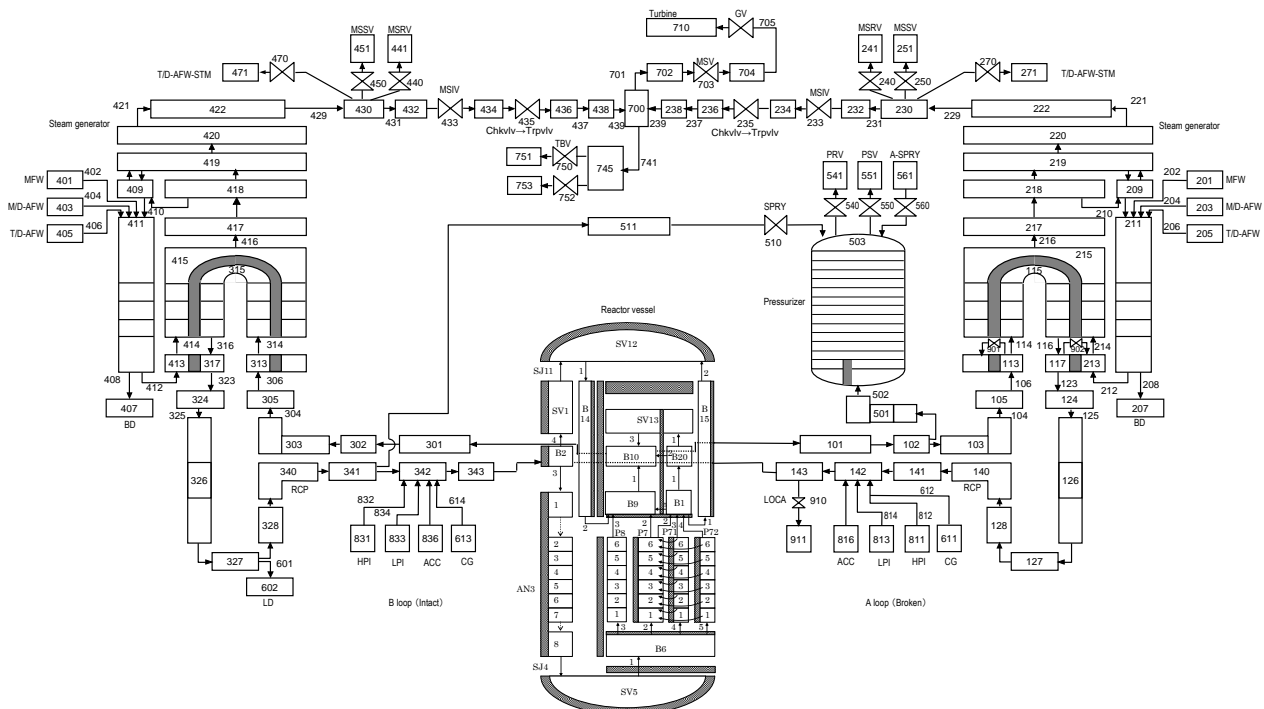


Figure 1. RELAP5 nodalization for reference PWR

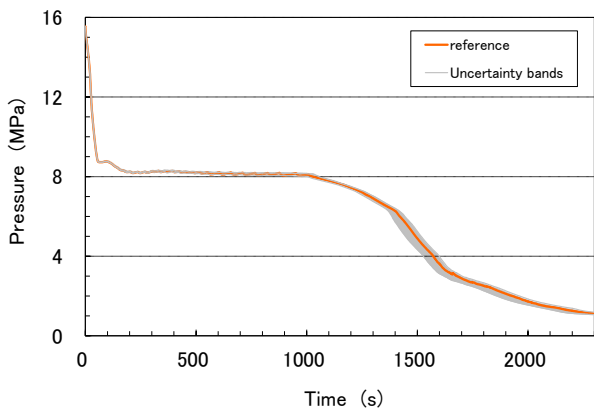
Table 1. Analysis conditions

Device operation	Analysis conditions
Initial core power	Rated power
Reactor trip	Pressurizer pressure low
Turbine trip	At the same time as reactor trip
Safety injection signal	Pressurizer pressure low
RCP coast down	At the same time as safety injection signal
Main feedwater stop	At the same time as reactor trip
Auxiliary feedwater	60s after break, All loops
Main steam relief valve	Automatic operating
Initiation of HPI system	Inoperative
Initiation of ACC injection	All loops
Initiation of LPI system	All loops
Initiation of SG secondary-side depressurization	2 min after the CET reached 350°C, Full opening of atmospheric relief valves of all loops

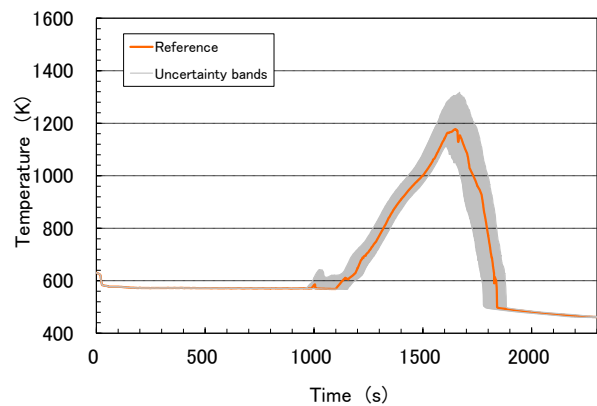
the SG U-tubes, the CCFL model is activated at the SG inlet junctions. The Wallis type correlation is used as the CCFL model. In addition to the core and SG U-tubes, heat structures in the reactor vessel, primary loop, and SG secondary-side are also simulated. The secondary system is provided with main and auxiliary feed water systems, relief and isolation valves, and the main steam line. The RELAP5 default critical flow model is activated for single-phase vapor flow at the main steam relief valves. The break is simulated with the valve component connected to the cold leg of the loop with the pressurizer.

Table 1 summarizes the analysis conditions. In accordance with the AM procedure provided as the plant emergency operations, the SG secondary-side depressurization is conducted 2 minutes after the core exit temperature (CET) reaches 350°C by opening the secondary atmospheric relief valves fully.

Figure 2 shows the analysis results for the primary system pressure and the rod surface temperature in the case of the 3 inch break. Because the HPI is assumed not to work, the core uncovering can occur when the reactor coolant system (RCS) pressure is too high for the accumulators (ACCs) and low pressure injection (LPI) system to make up for the break flow. After the initiation of the SG secondary-side depressurization, the secondary-side pressure falls below the RCS pressure, and the primary-side pressure closely follows the secondary-side pressure. After the primary-side pressure falls below the pressure set point initiating injection by the ACCs, the ACCs inject water into the cold legs and that results in the core reflooding. The effectiveness of the AM procedure should be investigated under the various break conditions and depressurization timings.



(a) Primary system pressure



(b) Rod surface temperature

Figure 2. Analysis results: 3 inch break [4]

## 2.2. Limit Surface

The limit surface is the boundary in the simulation space between system failure and system success. The key parameters for the AM procedure are the break size, the depressurization timing, the number of depressurizing SGs, and the depressurization rate. For the sake of simplicity, the investigations in this study were restricted to the limit surface defined by the first two parameters.

Figure 3 shows the PCTs calculated by RELAP5 as function of the LOCA break size and the SG depressurization timing for the reference analysis in Section 2.1. The break size was varied from 0.5 inches to 5.0 inches. The depressurization timing was varied from 600s to 18974s. The boundary between the orange and blue areas shows the discrete coarse limit surface with the threshold of 1473K.

In the case of the 5-inch break, the ACC injection was initiated early due to rapid depressurization of the primary system, the PCTs started to drop after the ACC injections, and the PCT did not reach 1473K for each depressurization timing. In the case of the 1 inch break, the accident event progressed slowly and the start time of the cladding heat up was also delayed. The PCTs started to drop after the ACC injections and they were followed by the LPI system activation. It is important to accurately evaluate the depressurization timing to ensure the success of the AM strategy for the various break sizes.

The continuous limit surface for the reference case in this study was evaluated by spline interpolation of the PCTs calculated by RELAP5.

Break size [in]	Depressurization timing [s]															
	600	755	951	1197	1507	1897	2389	3007	3786	4766	6000	7554	9509	11972	15071	18974
5.00	906.0	906.1	906.1	906.1	906.1	906.1	906.1	906.1	906.1	906.1	906.1	906.1	906.1	906.1	906.1	906.1
4.50	939.7	951.8	944.1	944.1	944.1	944.1	944.1	944.1	944.1	944.1	944.1	944.1	944.1	944.1	944.1	944.1
4.00	858.7	979.9	1037.5	1038.2	1038.2	1038.2	1038.2	1038.2	1038.2	1038.2	1038.2	1038.2	1038.2	1038.2	1038.2	1038.2
3.50	721.6	839.6	1090.7	1320.5	1334.4	1334.4	1334.4	1334.4	1334.4	1334.4	1334.4	1334.4	1334.4	1334.4	1334.4	1334.4
3.25	631.0	807.5	819.2	1252.6	1640.3	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1
3.00	631.0	648.4	709.4	966.1	2122.2	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1
2.75	631.0	631.0	631.0	697.2	1244.3	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1
2.50	631.0	631.0	631.0	719.2	656.8	1851.7	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1
2.25	631.3	631.3	631.3	631.3	631.0	735.4	2122.1	2122.2	2122.2	2122.2	2122.2	2122.2	2122.2	2122.2	2122.2	2122.2
2.00	631.0	631.0	631.0	631.0	631.0	631.0	661.2	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1
1.75	631.0	631.0	631.0	631.0	631.0	631.0	631.0	665.2	1512.3	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1
1.50	631.0	631.0	631.0	631.0	631.0	631.0	631.0	631.0	631.0	1048.1	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1
1.25	631.0	631.0	631.0	631.0	631.0	631.0	631.0	631.0	631.0	631.0	2122.1	2122.1	2122.1	2122.1	2122.1	2122.1
1.00	631.0	631.0	631.0	631.0	631.0	631.0	631.0	631.0	631.0	631.0	631.0	631.0	2122.2	2122.2	2122.2	2122.2
0.75	631.2	631.2	631.2	631.2	631.2	631.2	631.2	631.2	631.2	631.2	631.2	631.2	631.2	631.2	631.2	631.2
0.50	631.3	631.3	631.3	631.3	631.3	631.3	631.3	631.3	631.3	631.3	631.3	631.3	631.0	631.0	631.0	631.0

Figure 3. Coarse limit surface determined by the RELAP5 code

### 3. ANALYSIS METHOD USING MACHINE LEARNING

#### 3.1. Surrogate Model

In this paper, a surrogate model, also known as a reduced order model (ROM), was applied for efficient prediction of the limit surface that was evaluated in Section 2.2.

The surrogate model is a mathematical representation to predict a figure of merit of a physical system. Such a model offers a simpler and computationally faster mathematical representation which emulates the high fidelity of a computationally expensive system analysis code based on a small number of training runs. In order to predict a limit surface efficiently, a reduced order model is used to reduce the analysis computational cost by reducing the number of needed points and prioritizing the area of the input space that needs to be explored.

In this paper, the surrogate model was represented by Gaussian process regression with the squared exponential kernel. This surrogate model approximates the real response function of the system using Kriging-based interpolators.

#### 3.2. Sampling Strategy

In this paper, the following sampling strategies were used in the training process of the surrogate model, and they were compared from the viewpoints of computational cost and the accuracy of the limit surface prediction.

- (i) Monte Carlo sampling
- (ii) Latin hypercube sampling
- (iii) Adaptive sampling

In Monte Carlo sampling (MC), random values from each probability distribution are generated repeatedly at random. The random value selected for one sampling does not affect the random value for the next sampling. In Latin Hypercube sampling (LHS), each probability distribution is divided into segments of equal probability, and random values are generated from each segment, reducing the number of cases in which random value generation is biased toward a particular range of the probability distribution. Therefore, compared to Monte Carlo sampling, Latin Hypercube Sampling can achieve the same level of statistical accuracy with fewer trials.

Monte Carlo sampling and Latin hypercube sampling are forward samplers. These kinds of once-through samplers are not effective enough to train the limit surface. On the other hand, an adaptive sampling strategy is highly effective to prioritize the area of the sampling space that needs to be explored. Figure 4 shows the workflow of the algorithm for this strategy [5]. As a first step, a set of training simulations is run to sample the system responses. Those calculations are used to train a reduced order model, which is then used for determining an approximated limit surface. The reduced order model is then used to predict where further exploration of the input space could be most informative. The new observations are used to update the reduced order model. This process is iterated until convergence is obtained. This way, the most information possible is gained in a small number of carefully selected sampled points, limiting the number of expensive trials needed to understand features of the system space.

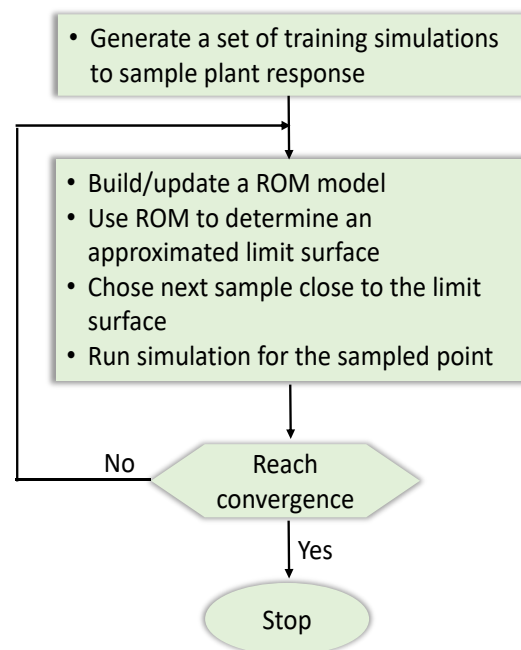


Figure 4. Adaptive sampling algorithm [5]

The procedure for the adaptive sampling in this paper is as follows. First, a small number of training data are randomly sampled, and the surrogate model is constructed using these training data. Next, the training data point is chosen randomly on the limit surface predicted in the previous step, and the surrogate model is updated using the added training data. The added training data point on the limit surface in the previous step is chosen as the point which is located the farthest from all the training data points selected in the previous step.

## 4. RESULTS AND DISCUSSION

### 4.1. Limit Surface Predictions

Figure 5 compares the limit surfaces predicted by the Gaussian process surrogate models using the sampling strategies: (a) Monte Carlo sampling, (b) Latin hypercube sampling, and (c) adaptive sampling. The number of the training data for the surrogate model was 80 for each sampling strategy. The blue points in the figure show the training data points. The red areas show the failed area of the reference analysis as obtained by the RELAP5 code, the PCTs of which were over 1473 K. PCTs over two-dimensional input space of the break area and the depressurization timing were evaluated for the 160x160 grid points by splined interpolation of the RELAP5 analysis results for the 16x16 grid points.

Figure 5(a) shows the limit surface evaluated by the Monte Carlo sampling. The crude Monte Carlo sampling could not cover the whole two-dimensional input space with 80 training data. Therefore, the predicted limit surface of 1473 K was located away from that of the reference analysis by the RELAP5 code.

Figure 5(b) shows the limit surface evaluated by the Latin hypercube sampling. Compared with the Monte Carlo sampling in Figure 5(a), the training data points in the input space were spread evenly over the whole area. However, the training data points were also located away from the 1473 K line of the reference analysis; therefore, the predicted limit surface also was located away from that of the reference analysis by the RELAP5 code.

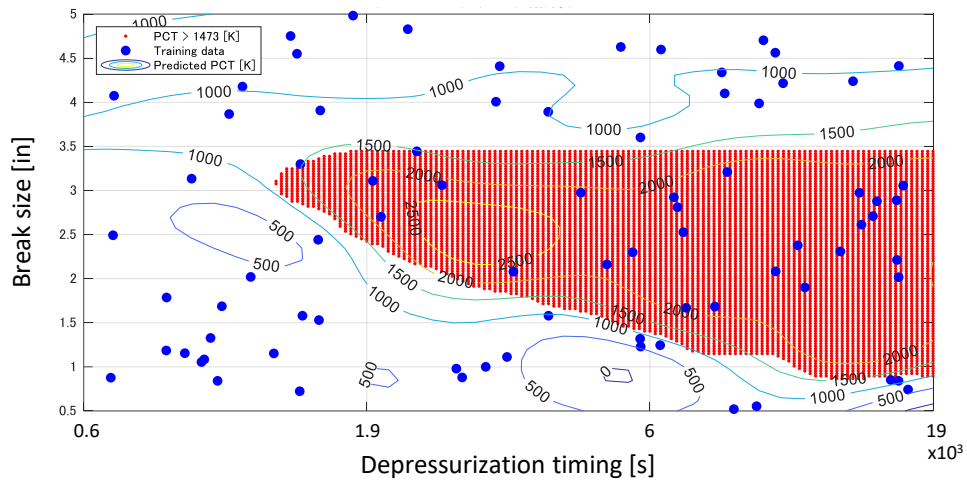
Figure 5(c) shows the limit surface evaluated by the adaptive sampling. The sampling procedure was as follows. First, 8 training data points in the input space were randomly sampled, and the surrogate model was constructed using the training data. Next, the training data point was chosen from the 1473 K line of the predicted limit surface of the previous step as the point which was the farthest from all the training data points selected in former steps. The surrogate model was updated using the RELAP5 analysis result on the added training data point. This procedure was repeated until the number of the training data was 80. As shown in the figure, the training data points were located near the 1473 K line of the reference analysis; therefore, the predicted limit surface also was located near that of the reference analysis by the RELAP5 code.

### 4.2. Accuracy Evaluation of Predicted Limit Surfaces

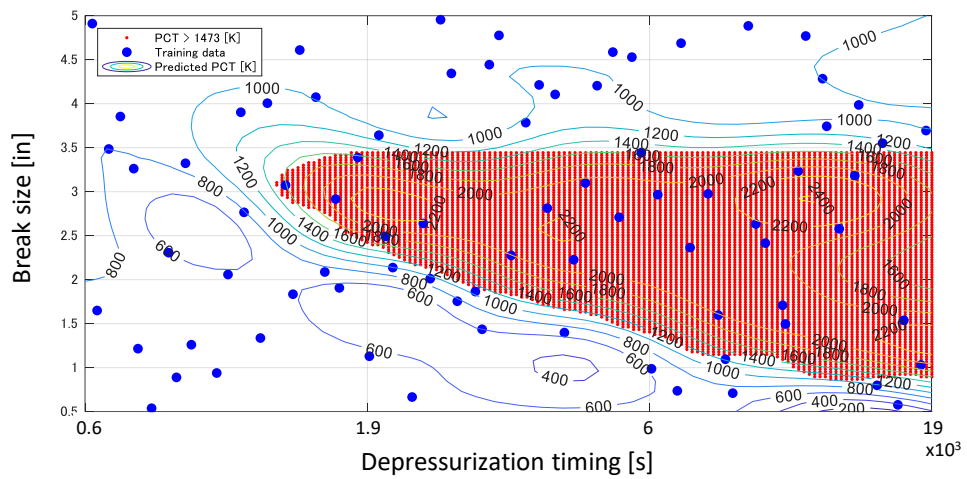
The accuracies of the predicted limit surfaces were evaluated by comparing them with the limit surface of the reference RELAP5 analysis.

Figure 6 shows the prediction accuracy of the depressurization timings on the limit surface with the increasing number of training data. The depressurization timings on the predicted limit surfaces are compared against those on the RELAP5 limit surfaces for each break size.

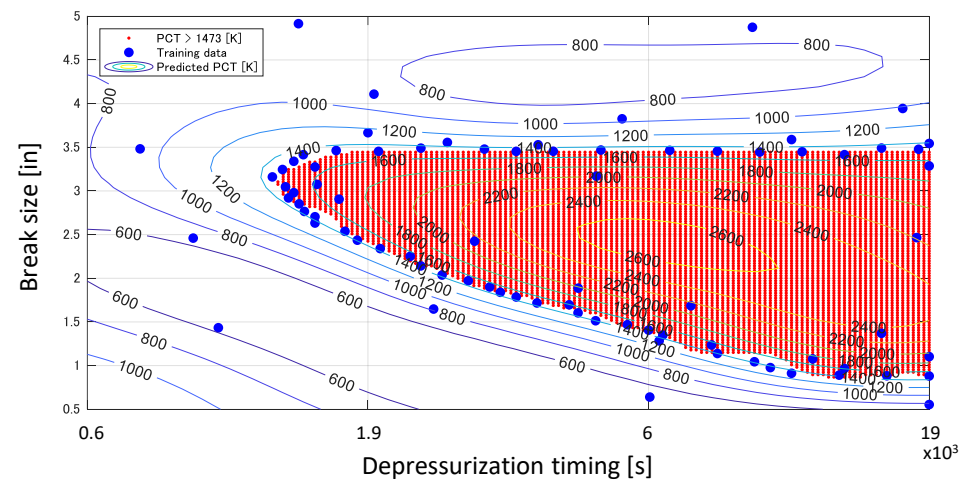
For Monte Carlo sampling, the case for 20 sample training data had poor accuracy in predicting the depressurization timings, especially for the earlier depressurization timing of break diameters around 3 inches. The case for 40 sample training data predicted late depressurization timings (i.e. non-conservative). The cases for 60 and 80 samples training data also resulted in large variations in the prediction accuracy of the depressurization timing.



(a) Monte Carlo sampling



(b) Latin hypercube sampling



(c) Adaptive sampling

Figure 5. Limit surface predictions

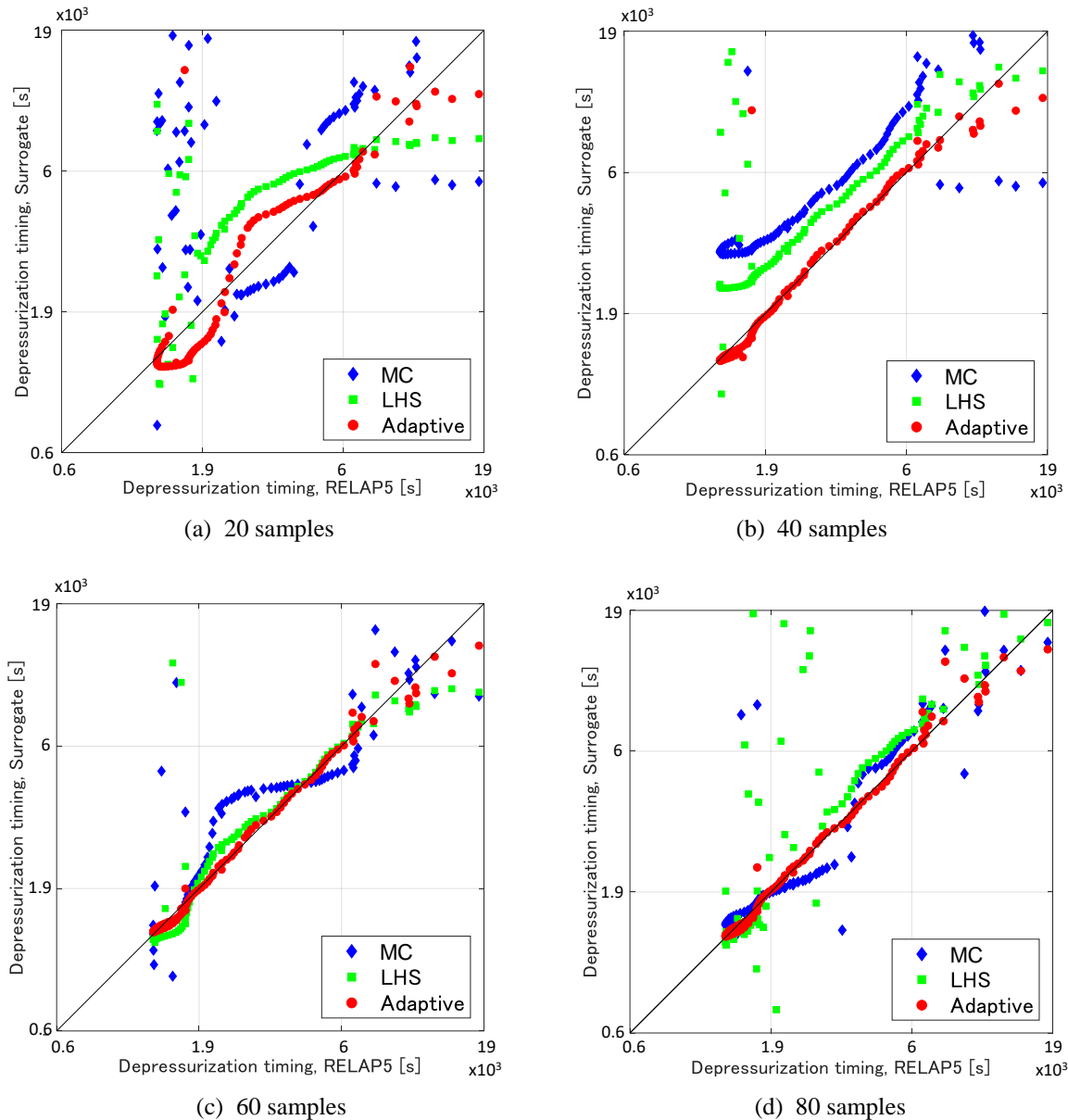


Figure 6. Prediction accuracy of the depressurization timings on the limit surface

For Latin hypercube sampling, the overall prediction accuracy was better than that of Monte Carlo sampling, but the prediction accuracy was poor and the variability was large in the case of earlier depressurization timings of break diameters around 3 inches.

For adaptive sampling, the prediction accuracy of the depressurization timing was good for the case of 40 sample training data, and the prediction accuracy improved as the number of training data increased. Although the accuracy was relatively worse for the late start of the depressurization with break diameters of around 1 inches, it is not considered to be particularly important for the accuracy evaluation when there is a late start of depressurization of the AM procedure.



### 4.3. Probability of Success and Failure

When setting the 1473 K line of the PCTs as the threshold, the probability of success and failure was evaluated for each sampling strategy.

Figure 7 shows the ratios of the numbers of points in the input space for which PCTs predicted by the surrogate models were over 1473 K versus the increasing numbers of training data. The red dashed line shows the ratio of the RELAP5 reference analysis, the value of which was 0.312. As shown in the figure, the ratios predicted by the adaptive samplings converged to the ratio of the reference analysis.

Table 2 summarizes the difference in the ratio of points in the input space for which PCTs predicted by the surrogate models were over 1473 K from that of the RELAP5 reference analysis with increasing numbers of training data. The adaptive sampling could evaluate the ratio the most accurately.

Table 2. Difference in the ratios of points in the input space for which PCTs predicted by the surrogate models were over 1473 K from that of the RELAP5 analysis with increasing numbers of training data

Numbers of Training Data	Difference from RELAP5		
	MC	LHS	Adaptive
10	-0.047	0.112	0.003
20	-0.075	-0.054	0.002
30	-0.003	-0.006	-0.013
40	-0.092	-0.053	0.006
50	0.043	-0.015	-0.003
60	-0.006	0.036	0.003
70	0.003	-0.011	0.004
80	0.007	-0.014	0.000

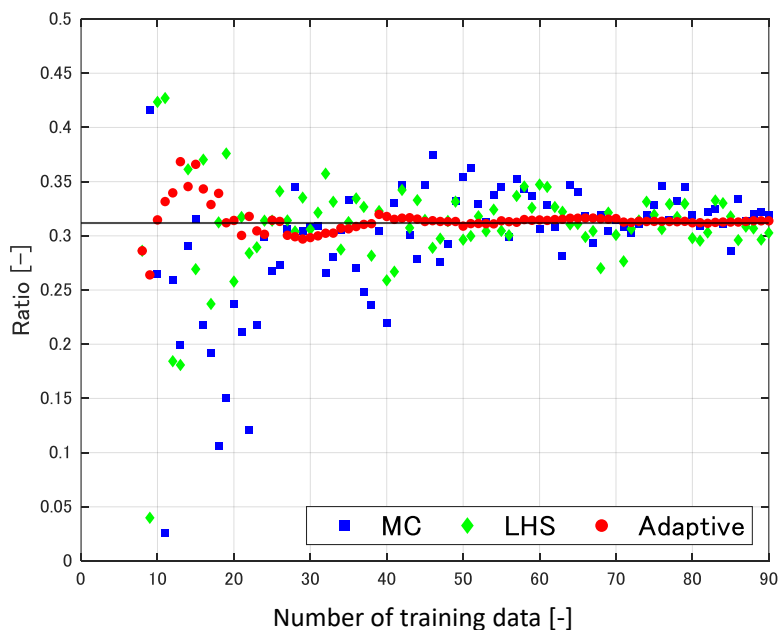


Figure 7. Ratios of numbers of points in the input space for which PCTs predicted by the surrogate models were over 1473 K with increasing numbers of training data

## 5. CONCLUSION

The limit surface is an n-dimensional surface describing the nuclear plant status as a function of selected plant parameters, and it identifies the boundaries between failed and safe conditions for the reactor core. In this paper, adaptive machine learning algorithms were applied to predict a limit surface efficiently for a PWR small break loss of coolant accident (LOCA).

As a first step, a set of training simulations was run to construct a surrogate model for determining an approximated limit surface. The surrogate model was then used to predict where further exploration of the input space could be most informative. The new observations were used to update the surrogate model. This kind of adaptive samplings was iterated until convergence was obtained.

The limit surface was investigated for the peak cladding temperatures as a function of the break size and the depressurization timing. It was confirmed that using adaptive sampling and machine learning techniques provided a significant reduction in the time required for accurate limit surface determinations.

### Acknowledgements

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