

# Development of Risk Importance Measures for Dynamic PRA Based on Risk Triplet (1) The Concept and Measures of Risk Importance

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**Abstract:** Despite the advancements in dynamic probabilistic risk assessment (PRA) methods that account for the dynamics of event progression, establishing risk importance measures for these methods remains a significant research challenge. This study proposes novel risk importance measures from the perspective of the risk triplet: Timing-Based Worth (TBW) for the timing of scenario occurrence (scenario diversity), Frequency-Based Worth (FBW) for the frequency (probability) of scenarios, and Consequence-Based Worth (CBW) for the consequences of scenarios. To assess the effectiveness of these measures, a static PRA using the event tree method and a dynamic PRA using the continuous Markov chain Monte Carlo (CMMC) method are performed on a simplified reliability model. The results indicate that the proposed measures facilitate a comprehensive risk importance evaluation, incorporating resilience effects (the time margin) and consequence mitigation, alongside traditional frequency-based evaluations. This advancement is anticipated to improve the utilization of risk information derived from dynamic PRA.

**Keywords:** Dynamic PRA, Risk Importance Measure, Risk Triplet, Resilience.

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## 1. INTRODUCTION

Despite the ongoing development of dynamic probabilistic risk assessment (PRA) methods that consider the dynamics of event progression, risk importance evaluation methods including risk importance measures for these methods have not been established. This is one of the research challenges in utilizing risk information obtained from dynamic PRA.

Characteristics of dynamic PRA include (1) a comprehensive scenario coverage, including changes in the sequence and timing of events, (2) the dynamic analysis without relying on predefined success criteria, and (3) the direct quantification of consequences using deterministic accident simulation codes, to name a few. These features enable the straightforward assessment of the risk triplet [1,2], defined by the following equation:

$$Risk = \left\{ \langle S_i, L_i, X_i \rangle \right\}_c, \quad i = 1, 2, \dots, N \quad (1)$$

where *Risk* is the risk associated with the system or activity,  $S_i$  is the  $i$ th risk scenario,  $L_i$  is the likelihood of the scenario occurring, and  $X_i$  is the consequence if the scenario actually occurs. The subscript  $c$  under the curly bracket signifies “complete,” meaning all significant scenarios, or at least those that should be included as scenarios, are considered.

In this study, we propose risk importance measures for dynamic PRA based on the risk triplet. Furthermore, we conduct a dynamic PRA on a simplified reliability model to evaluate the functionality of the proposed risk importance measures.

## 2. RISK IMPORTANCE MEASURES BASED ON RISK TRIPLET

This study proposes risk importance measures for dynamic PRA from the perspectives of the risk triplet, assessing time dependency, evaluating resilience including accident management (AM), and aligning with existing risk importance measures. The proposed risk importance measures are Timing-Based Worth (TBW) related to the timing of scenario occurrence (scenario diversity), Frequency-Based Worth (FBW) related to the

frequency (probability) of scenario occurrence, and Consequence-Based Worth (CBW) related to the consequence of scenarios. Each of these measures is defined as follows.

a) TBW

$$TBW^{SorF} = \log_{10} \left( \frac{E_{I=SorF}[T(R)]}{E[T(R)]} \right) \text{ for } E[T(I = S or F)] \leq E[T(R)] \quad (2)$$

where  $R$  is the evaluation event, such as core damage or containment failure,  $I$  is the basic event, such as component operation or AM,  $E[T(R)]$  is the expected time of occurrence of  $R$  over the entire analysis,  $E[T(I = S or F)]$  is the expected time of occurrence of  $I$ , where  $I$  is either success ( $S$ ) or failure ( $F$ ), and  $E_{I=S or F}[T(R)]$  is the expected time of occurrence of  $R$ , conditional on  $I$ .

Figure 1 shows the relationship between  $E[T(R)]$ ,  $E[T(I = S or F)]$ , and  $E_{I=S or F}[T(R)]$ .

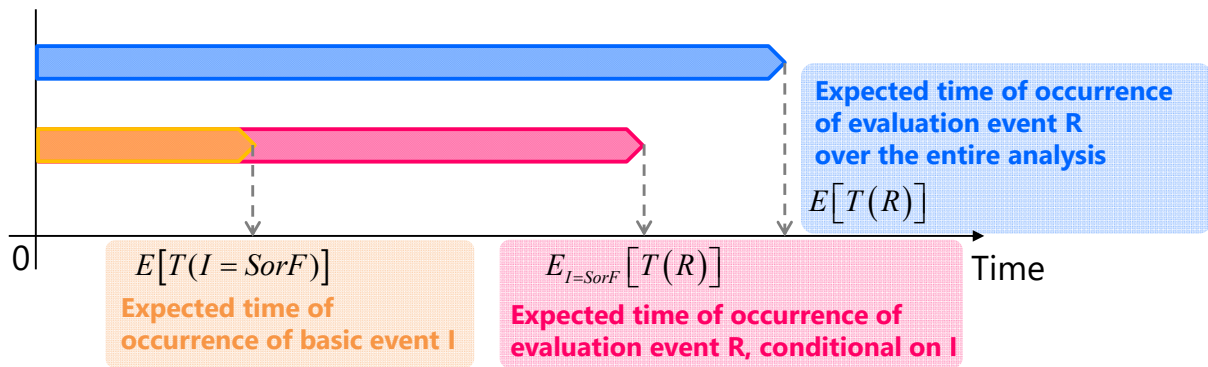


Figure 1. Relationship between  $E[T(R)]$ ,  $E[T(I = S or F)]$ , and  $E_{I=S or F}[T(R)]$

b) FBW

$$FBW^{SorF} = -\log_{10} \left( \frac{E_{I=S or F}[F(R)]}{E[F(R)]} \right) \quad (3)$$

where  $E[F(R)]$  is the expected frequency of  $R$  over the entire analysis, and  $E_{I=S or F}[F(R)]$  is the expected frequency or probability of  $R$ , conditional on  $I$ .

FBW has a relationship with the risk importance measures Fussell-Vesely (FV), Risk Achievement Worth (RAW), and Risk Reduction Worth (RRW) [3] as shown in the following equations, demonstrating its compatibility with traditional importance measures.

$$FBW^S = -\log_{10} (1 - FV) \quad (4)$$

$$FV = \frac{E[F(R)] - E_{I=S}[F(R)]}{E[F(R)]} \quad (5)$$

$$FBW^F = -\log_{10} RAW \quad (6)$$

$$RAW = \frac{E_{I=F}[F(R)]}{E[F(R)]} \quad (7)$$

$$FBW^S = \log_{10} RRW \quad (8)$$

$$RRW = \frac{E[F(R)]}{E_{I=S}[F(R)]} \quad (9)$$

c) CBW

$$CBW^{SorF} = -\log_{10} \left( \frac{E_{I=SorF}[C(R)]}{E[C(R)]} \right) \quad (10)$$

where  $E[C(R)]$  is the expected consequence of  $R$  over the entire analysis, and  $E_{I=SorF}[C(R)]$  is the expected frequency or probability of  $R$ , conditional on  $I$ .

The concept of risk importance evaluation using these measures is illustrated in Figure 2. All the measures indicate an improvement in safety with positive values. The key differences from existing risk importance measures lie in TBW and CBW. TBW indicates the time margin until the occurrence of an evaluation event, making it an important measure for assessing the feasibility and success probability of AM, thus contributing to resilience evaluation. CBW represents the consequence, such as the release of fission products, serving as a measure to evaluate the consequence mitigation effect of AM, etc.

Thus, by using the three measures defined in this study, it becomes possible to measure risk importance considering resilience effects and consequence mitigation effects, which was challenging with existing importance measures focused solely on scenario frequency.

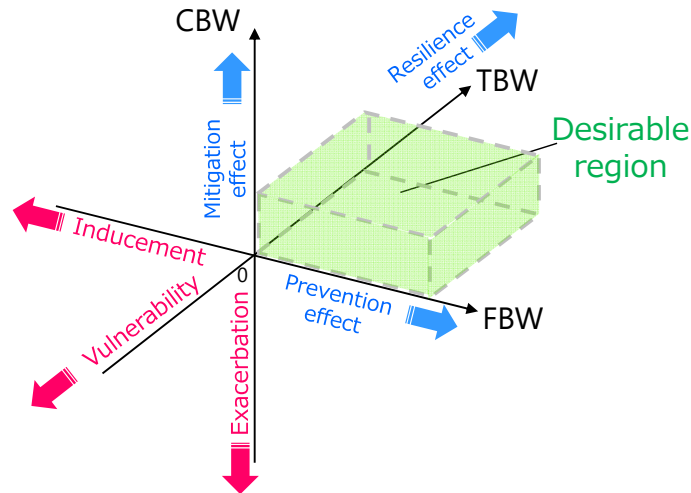


Figure 2. Concept of Risk Importance Evaluation Based on Risk Triplets

### 3. APPLICATION TO DYNAMIC PRA

A dynamic PRA was conducted on the Holdup Tank problem [4] to evaluate the functionality of the proposed risk importance measures. Additionally, a static PRA was also performed to verify the dynamic PRA.

#### 3.1. Holdup Tank Problem

Figure 3 shows a schematic of the Holdup Tank model. This model consists of a water storage tank with one valve and two pumps (pump 1 and pump 2). When the valve opens, water is drained from the tank, and when the pumps operate, water is injected into the tank.

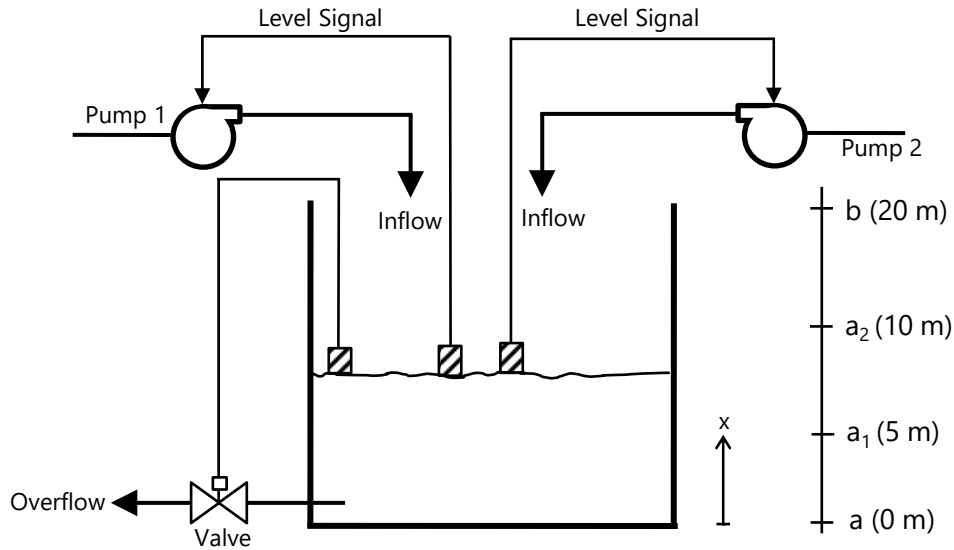


Figure 3. Schematic of Holdup Tank Model

The operational states of the valve and pumps depend on the tank water level. Table 1 shows the relationship between the tank water level and the operational states of the components. When the water level is near the middle, the valve is open, pump 1 is on, and pump 2 is off. If the water level increases from this state, pump 1 turns off, changing the operational state to lower the water level. Conversely, if the water level decreases, the valve closes, and both pump 1 and pump 2 operate, changing the operational state to raise the water level.

Table 1. Operational States of Valve and Pumps

Water level ( $x$ )	Valve	Pump1	Pump 2
$a_1 < x < a_2$	Open	On	Off
$a_2 < x$	Open	Off	Off
$x < a_1$	Closed	On	On

Table 2 shows the flow rate, failure modes, and failure rates of the valve and pumps. The failure modes are categorized into demand failures and operational failures. Poisson processes are assumed for the latter failures. For the failure states, both open and closed conditions are considered for the valve, and both on and off conditions are considered for the pumps. For simplicity, the impact of water level changes on the flow rate is not considered.

Table 2. Flow Rate, Failure Modes, and Failure Rates of the Valve and Pumps

Component	Flow rate (m <sup>3</sup> /h)	Failure mode	Failure rate
Valve	1	Demand failure (open/closed)	0.05 (/demand)
		Operational failure (open/closed)	0.001 (/h)
Pump 1	1	Demand failure (on/off)	0.05 (/demand)
		Operational failure (on/off)	0.001 (/h)
Pump 2	0.5	Demand failure (on/off)	0.05 (/demand)
		Operational failure (on/off)	0.001 (/h)

The case where the tank water level reaches  $a$  (0 m) is defined as “dryout,” and the case where the water level exceeds the allowable level of  $b$  (20 m) is defined as “overflow.” These conditions are considered as evaluation

events. For this Holdup Tank model, both static and dynamic PRAs were conducted focusing on the scenario where pump 1 fails to turn on (failed off) as the initiating event.

### 3.2. PRA Methods

#### 3.2.1. Static PRA Method

The static PRA was conducted using an event tree based on the preceding study [4]. Figure 4 illustrates a simplified hardware-oriented event tree created for the Holdup Tank model. Using this event tree, the frequencies of dryout and overflow were analytically calculated.

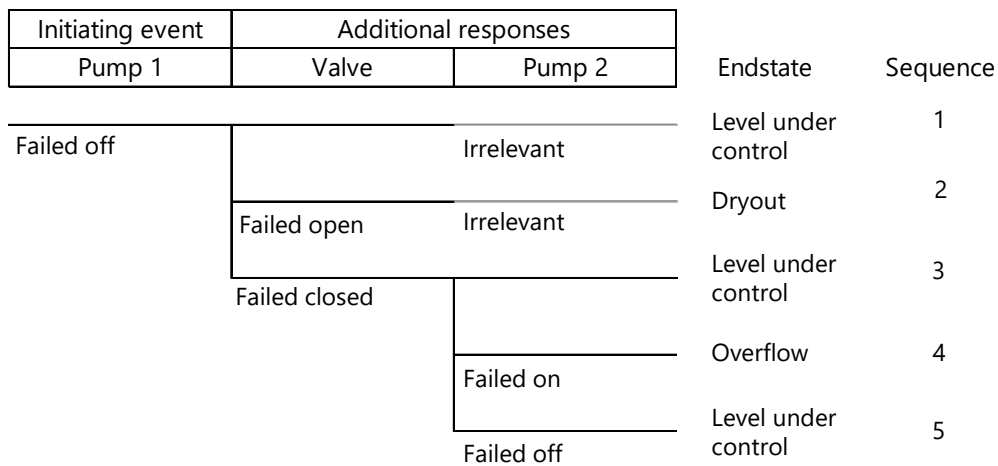


Figure 4. Hardware-Oriented Event Tree

#### 3.2.2. Dynamic PRA Method

The continuous Markov chain Monte Carlo (CMMC) method [5] was used for the dynamic PRA. The CMMC method assumes a Markov chain for the state transitions of the system and uses Monte Carlo simulation to evaluate the dynamic behavior of the system. The calculation procedure of the CMMC method is shown in Figure 5, and the analysis conditions are presented in Table 3.

Table 3. Analytical Condition of the CMMC Method

Item	Set value
Time step ( $\Delta t$ )	0.1 (h)
End time ( $t_{end}$ )	50 (h)
The number of simulation ( $n_{all}$ )	10,000 (-)

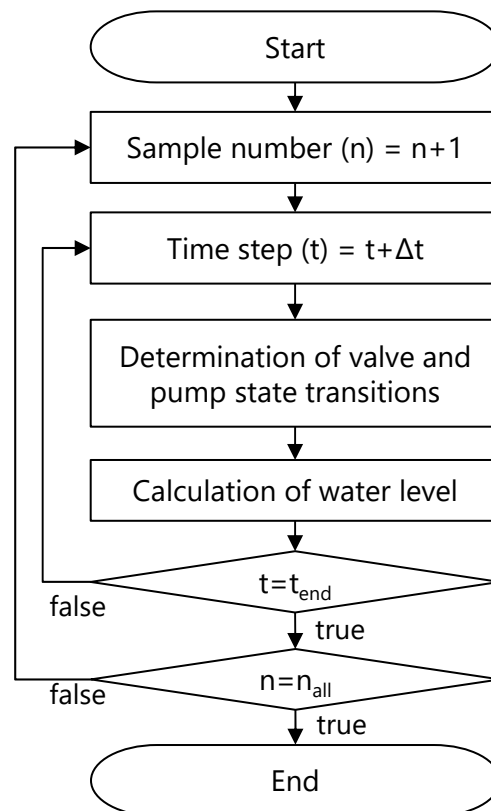


Figure 5. Calculation Procedure of the CMMC Method

### 3.3. Results and Discussion

#### 3.3.1. Time History of Water Level and Components Status

Figure 6 shows the time history of the tank water level and the operational states of the components as estimated by the CMMC method. When the initiating event of pump 1 failing off occurred, the water level dropped. The operational states of the valve and pumps were observed to change when the water level reached  $a_1$  (5 m). Subsequently, numerous demands occurred early in the event as the tank's water level fluctuates.

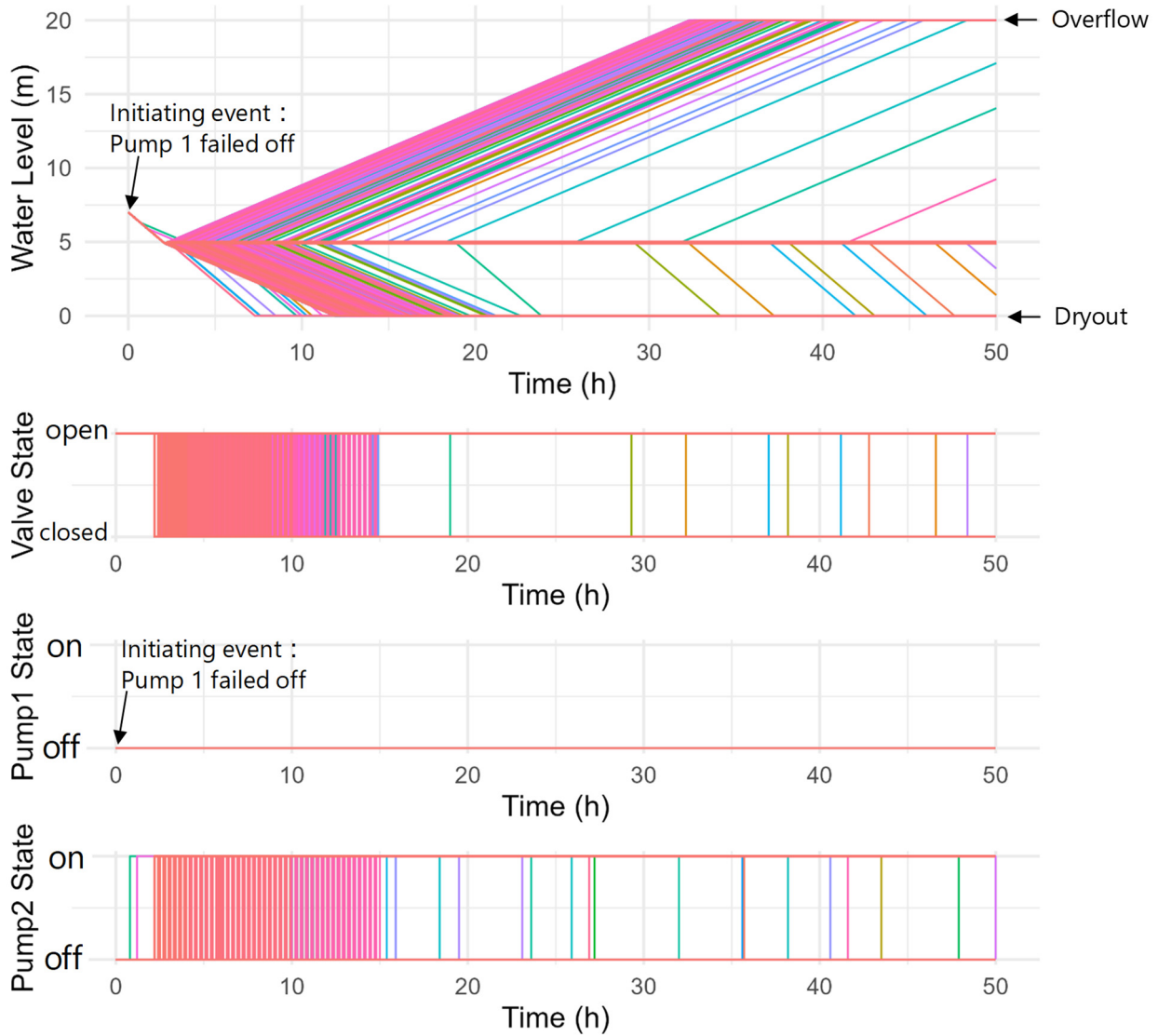


Figure 6. Time History of Tank Water Level and Status of Valve and Pumps  
(The first 1,000 samples are presented.)

### 3.3.2. Comparison of Existing Risk Importance Measures with Static PRA

The dynamic PRA results were verified by comparing them with the static PRA results. The existing importance measures, RAW and FV, were used for this comparison.

Table 4 shows the calculation results of FV and RAW, highlighting the maximum value for each measure. For both overflow and dryout, the highest importance measures are highlighted in the table. The low impact of pump 2 on dryout was consistent in both the static and dynamic PRAs.

In contrast, for both dryout and overflow, the static PRA showed high RAW values for both demand and operational failures, while the dynamic PRA showed high FV values for demand failures. This result can be attributed to 1) the dynamic PRA having a relatively high probability of demand failures due to the numerous demands occurring early in the event, as shown in Figure 6, and 2) the static PRA calculating the probability of operational failures as the product of the failure rate and the end time ( $t_{end}$ ), resulting in a relatively higher probability of operational failures.

Based on these results, although there are differences attributable to the inherent characteristics of dynamic PRA, such as the consideration of system state transitions, the importance measures obtained from the dynamic PRA are generally consistent with those from the static PRA.

Table 4. FV and RAW Calculated by Static and Dynamic PRAs  
(The maximum value is highlighted.)

Evaluation event	Basic event	FV		RAW	
		Static	Dynamic	Static	Dynamic
Overflow	Valve demand failure	0.49	0.99	10.38	1.35
	Valve operational failure	0.51	-0.01	10.37	0.10
	Pump 2 demand failure	0.49	0.91	10.38	1.87
	Pump 2 operational failure	0.51	0.02	10.37	1.67
Dryout	Valve demand failure	0.49	0.88	10.38	1.31
	Valve operational failure	0.51	0.02	10.37	2.03
	Pump 2 demand failure	0.00	-0.30	1.00	0.70
	Pump 2 operational failure	0.00	0.01	1.00	1.23

### 3.3.3. Measurement of Risk Importance Based on Risk Triplet

Figure 7 shows the results of the proposed risk importance measures FBW and TBW calculated using dynamic PRA. Figure 8 illustrates the concept of risk importance evaluation using FBW and TBW.

Among the four basic events (demand and operational failures of pump 2 and valve), the conditional dryout for operational failures of pump2 and valve did not meet the TBW criteria, as the expected occurrence times of these basic events were later than the expected occurrence time of dryout.

For the conditional dryout of pump 2's demand failure, both TBW and FBW had positive values, indicating both preventive and resilience effects. Pump 2 failing to turn on occurs when the water level is low. In this situation, the valve is closed, and pump 1 has failed off, preventing further changes in the tank's water level. As a result, the demand failure rate of the valve becomes zero, leading to a decrease in the probability of dryout and a delay in its occurrence time.

On the other hand, the conditional overflow for pump 2's demand failure, as well as the conditional dryout and overflow for the valve's demand failure, had negative values for both TBW and FBW, indicating no preventive or resilience effects. This result can be interpreted as shown in Figure 6: the numerous demands occurring early in the event make the demand failures of the valve and pump 2 dominant factors for dryout and overflow, thereby reducing the time margin.

These results confirm that the proposed risk importance measures allow for a comprehensive risk importance evaluation including the consideration of resilience effects, which were challenging with traditional measures focused solely on scenario frequency.

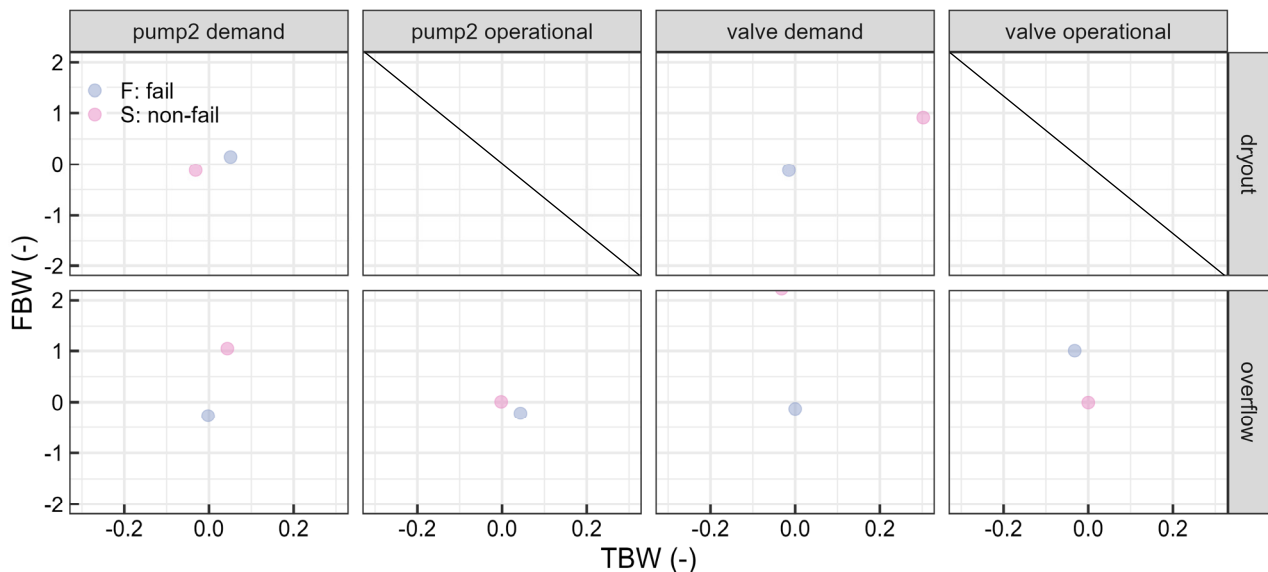


Figure 7. Calculation Results of FBW and TBW



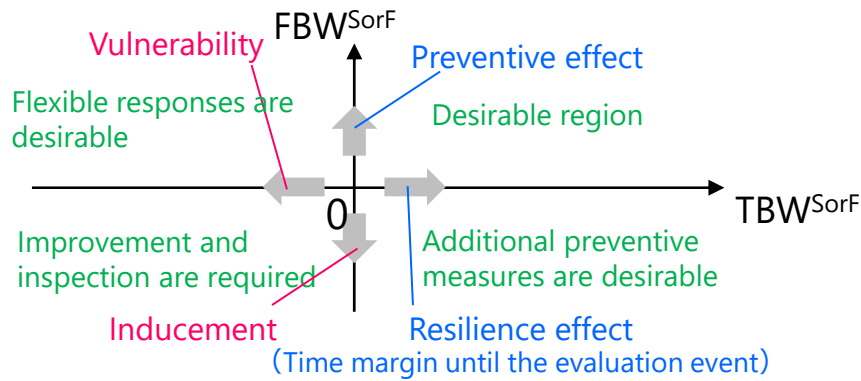


Figure 8. Concept of Importance Evaluation using FBW and TBW

#### 4. CONCLUSION

Although dynamic PRA methods that account for the dynamics of event progression have been developing, risk importance measures for these methods have not yet been established, remaining a research challenge in effectively utilizing the risk information obtained from dynamic PRA. This study proposed risk importance measures from the perspective of the risk triplet: Timing-Based Worth (TBW) for the timing of scenario occurrence (scenario diversity), Frequency-Based Worth (FBW) for the frequency (probability) of scenarios, and Consequence-Based Worth (CBW) for the consequence of scenarios.

To evaluate the functionality of the proposed measures, static and dynamic PRAs were conducted on a simplified reliability model. The results demonstrate that the proposed risk importance measures enable a comprehensive importance evaluation, considering resilience effects (time margin) and consequence mitigation effects, in addition to the traditional frequency-based evaluation. This advancement is expected to enhance the utilization of risk information obtained from dynamic PRA.

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