Comparing Environmental Contours and a Sequential Sampling Method for Probabilistic Structural Reliability Assessment: A Case Study on Offshore Wind Turbines

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Abstract: Probabilistic design of offshore wind turbines aims to ensure structural safety in a cost-effective way. This involves considering various structural responses to different environmental loads and conducting structural reliability assessments for different design options. In particular, ultimate limit state (ULS) assessment considers extreme structural responses due to extreme loading that the structure is expected to withstand without failing. There are several probabilistic structural reliability methods addressing ULS, which accounts for both the long-term variability of environmental conditions and the short-term conditional structural response. In principle, a full long-term analysis is recommended, but this is often unfeasible due to time-consuming and computationally costly response calculations. Hence, simplified approximate approaches are useful, particularly in early phases of design. This paper compares two such approaches for assessment of extreme structural responses, i.e., the environmental contour method – a well-known method often used in ocean engineering applications - and a more novel approach based on sequential sampling and Gaussian processes regression. Both methods utilize the same probabilistic model to describe the environmental conditions and a computationally efficient surrogate model to approximate the conditional structural response. The approaches are generic and can be applied to any structure exposed to environmental loading, but this study considers a simplified use case on an offshore wind turbine. Estimates of the 50-year extreme response are obtained with the two methods and compared, for a selected structural response. The results demonstrate that the two methods can give quite different results, and possible explanations for this are discussed. Probably, this can be explained by the fact that some of the assumptions made with one of the approaches are not fulfilled. The two methods are also compared in terms of computational efforts, which are found to be comparable. Results from this simple case study suggests that the sequential sampling method can be a robust and computationally effective approach for probabilistic structural reliability assessment.

Keywords: Probabilistic Risk Assessment, Structural Safety, Ultimate Limit State, Renewable Energy.

1. INTRODUCTION

Structural reliability assessment is required to ensure that offshore structures can withstand the environmental conditions they are expected to encounter throughout their lifetimes. Different structural responses are associated with certain limit states that describe under what conditions the structure is expected to fail, and the ultimate limit state (ULS) describes failure when subjected to extreme loads. In probabilistic design, one will typically design against a target reliability level, corresponding to a maximum probability of failure. There are several ways to estimate such small failure probabilities, or conversely, estimate the extreme structural loads and responses associated with a prescribed return period. In this paper, two approaches to structural reliability will be compared by way of a simple offshore wind case study. The traditional environmental contour approach [1], [2] will be compared to a more novel approach based on sequential sampling and Gaussian processes (GP) regression [3].

Structural reliability analysis is typically performed to determine the reliability R, or conversely the failure probability P_f, of a given design. The limit state function is a performance function that defines when a structure will fail. Generally, the limit state is on the form $g(\boldsymbol{X}) = y_{capacity} - Y(\boldsymbol{X})$, where $y_{capacity}$ is the structural strength, $Y(\boldsymbol{X})$ is the actual load on the structure and $\boldsymbol{X}$ is a vector of relevant input variables. If the loads are greater than the structural capacity, i.e. when $g(\boldsymbol{X}) < 0$, the structure will fail. Hence, the reliability may be determined by integrating the probability density function of the input variables, $\boldsymbol{X}$, over the safe region, i.e.

the set of input variables where the structure survives. Denoting the structural performance function $g(\boldsymbol{X})$, and the joint density function of the input variables $f_{\boldsymbol{X}}(\boldsymbol{x})$, the reliability of a structure may be defined as

$$R = 1 - P_f = P[g(\boldsymbol{X}) > 0] = \int_{g(\boldsymbol{X})>0} f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x}. \tag{1}$$

Such integrals are often difficult to solve exactly since the joint density function of the input (environmental) parameters and the performance function may be complicated functions. Two commonly used methods to approximate these integrals are the First Order Reliability Method (FORM) and the Second Order Reliability Method (SORM), where the failure boundary is approximated by a first- or second order Taylor expansion, respectively, at the design point.

Often, the environmental input conditions are modelled as piece-wise stationary processes, and the description of the structural response will be a combination of the long-term environmental conditions and conditional short-term responses. Full long-term extreme response analysis would then involve integrating the conditional short-term structural response over all long-term environmental conditions, see e.g. [4], [5]. In principle, this may be done by brute force Monte Carlo simulations. However, in practice, the evaluation of the short-term response in a given environmental condition will be very computationally demanding, making this infeasible in most realistic applications. Two fundamental approaches can be taken to alleviate this: 1) more efficient short-term response analysis and 2) the need for fewer short-term response calculations to determine the long-term extreme response. In this study, two approximative approaches for long-term extreme response estimation will be compared, i.e. environmental contours and sequential sampling, which both aim at solving the ULS problem with fewer short term response calculations. However, with both approaches a surrogate model is utilized for making the short-term response analysis more efficient.

In this study the long-term environment is described in terms of $\boldsymbol{x} = (U, \sigma_U)$, where U is the average horizontal wind speed at hub height and σ_U is the turbulence intensity defined as the temporal standard deviation of the wind speed. The long-term parameters $\boldsymbol{x}$ are assumed stationary over a period of 1 hour and are described by a joint probability distribution $f_{\boldsymbol{X}}(\boldsymbol{x})$, as presented in [6].

The short-term response of interest is the maximum flapwise blade root bending moment and is for a given long-term condition $\boldsymbol{x}$ a stochastic process $S_t(t)$ for $t \in [0, 10]$ min. In this study the maximum response during each 10-minute time-period is considered, denoted $Y = max_{t\in[0,10]} S_t(t)$. Hence, for a given long-term input, Y is a stochastic random variable from an (unknown) probability distribution $g_{Y|X}(y|x)$.

The marginal long-term distribution of the maximum blade root bending moment can then in principle be found by integrating over the long-term environment, i.e.

$$g_Y(y) = \int g_{Y|\boldsymbol{X}}(y\,|\boldsymbol{x}) f_X(\boldsymbol{x}) d\boldsymbol{x} \ . \tag{2}$$

In the present case the distribution $g_{Y|\boldsymbol{X}}(y\,|\boldsymbol{x})$ is not known, but samples from it can be obtained by running simulations of the structural response, which in this case is provided by an mNARX surrogate model [7]. The response model is only applicable between the cut-in wind speed (3 m/s) and cut-out wind speed (25 m/s), but wind conditions above the cut-out wind speed are not assumed to contribute to the long-term extreme blade root responses for these turbines, so this is deemed appropriate. In principle, the integral (2) can then be estimated through a brute-force Monte-Carlo approach. However, this is still a computational demanding task, and may in many cases be infeasible if simulating the structural response is computational demanding.

Even though the case study presented in this paper is a simplified 2-dimensional case, both the environmental contour method and the sequential sampling method may be generalized to higher dimensional problems, see e.g. [8], [9].

1.1. Environmental Contours

The environmental contour method for structural reliability is a well-adopted method, in particular in ocean engineering applications, and it is recommended in DNV's recommended practice on environmental conditions and environmental loads [10]. There are different methods to construct environmental contours, including

IFORM contours [1], [2] and direct sampling contours [11], [12], see e.g. [13], [14], and this study uses IFORM contours to obtain estimates of 1- and 50-year extreme responses.

Environmental contours are associated with a target exceedance probability of the input variables. Assuming that the largest response occurs in the most severe conditions, one may define a limit state function as a function of long-term variables only and construct environmental contours in the long-term variable space. One may then assume that any design with a limit state function fully outside of the environmental contour has a failure probability smaller than the exceedance probability associated with that contour. Alternatively, one may assume that the long-term extreme response with a return period corresponding to the exceedance probability of the environmental contour is the maximum response evaluated at points along the contours. Hence, environmental contours can be used to estimate extreme long-term responses for desired return periods with only a limited set of short-term response analyses; $g_{Y|\boldsymbol{X}}(y\,|\boldsymbol{x})$ need only be evaluated at selected points along the contours. This approach ignores the effect of the short-term variability of the response, but this may be accounted for by using a higher quantile of the conditional short-term extreme response.

1.2. Sequential Sampling with Gaussian Process Regression

The sequential sampling approach is an alternative approach that accounts for the effect of both long-term variability of the environment and the short-term variability on the extreme response while minimizing the number of short-term response evaluations. The methodology applied to estimate the integral (2) is described in [3], see also [9], and can be summarized by the following main steps:

- Introduce a parametric distribution $\hat{g}_{Y|\boldsymbol{X}}(y\,|\boldsymbol{x},\boldsymbol{\theta}(\boldsymbol{x}))$ for the short-term response (i.e. $\hat{g}_{Y|\boldsymbol{X}}(y\,|\boldsymbol{x},\boldsymbol{\theta}(\boldsymbol{x}))$ represents an approximation of the "true" distribution $g_{Y|\boldsymbol{X}}(y\,|\boldsymbol{x})$).
- A Gaussian process (GP) regression model is used to represent the distribution parameters , $\boldsymbol{\theta}(\boldsymbol{x})$, which are fitted based on a limited number of short-term response simulations.
- The estimated long-term response distribution is obtained from (2) by replacing $g_{Y|\boldsymbol{X}}(y\,|\boldsymbol{x})$ with $\hat{g}_{Y|\boldsymbol{X}}(y\,|\boldsymbol{x},\boldsymbol{\theta}(\boldsymbol{x}))$.

2. ANALYSIS AND RESULTS

2.1. Extreme Responses Estimated by the Environmental Contour Method

To estimate the extreme response of the wind turbine at South Brittany, 2-dimensional environmental contours based on IFORM and direct sampling are considered for the mean wind speed and turbulence variables. The predefined surrogate model described in [7] is used to calculate the short-term extreme response (i.e. maximum flapwise blade root bending moment M_y^{Bld}) in selected wind conditions. Environmental contours corresponding to n-year extreme of 1-hour conditions are calculated, i.e. corresponding to an exceedance probability of

$$p_e = \frac{1}{365.25 \times 24 \times n}. \tag{3}$$

The resulting contours based on the fitted omnidirectional joint distribution are shown in Figure 1 for 1- , 20- 50- and 100-year return periods. The differences between contour methods are negligible (the contour lines are nearly indistinguishable in the plot) and the points along the 1- and 50-year IFORM contours are taken as input for the surrogate model. There are 94 input points from the 1-year contour, and 74 input points from the 50-year contour. In total 100 seeds are run with the surrogate for each input point. The long-term extreme responses of maximum flapwise blade root bending moment are estimated from the 50%, 90% and 99% fractiles of the resulting conditional response distributions as presented in Table 1. Note that the 99% fractile should be used with caution since these results are based on only 100 seeds. The estimates corresponding to the 90% fractile are presented in bold font, since this seems to be the most reasonable choice. The long-term extreme estimates of maximum flapwise blade root bending moment from the 50-year contour is slightly higher than from the 1-year contour.

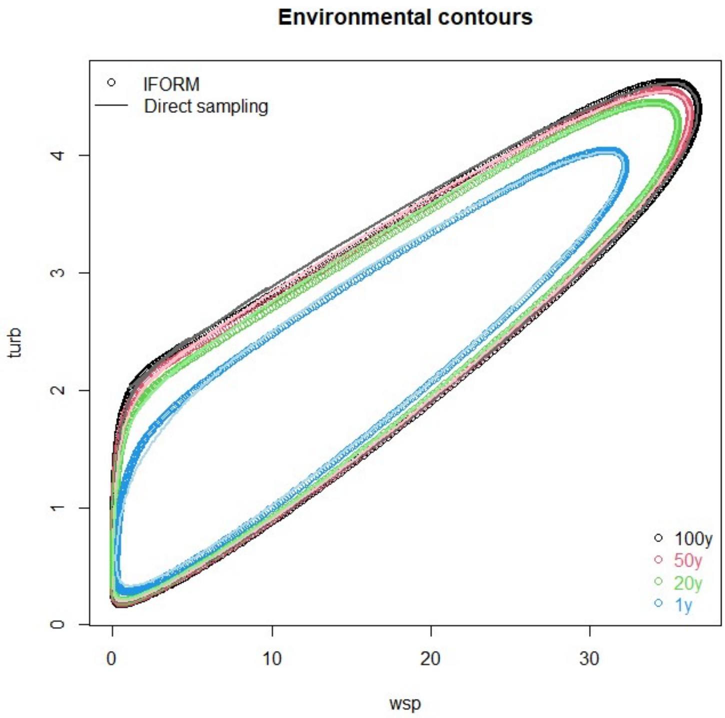

Figure 1. Environmental contours

Table 1. Long-term extreme responses based on environmental contours.

IFORM – 1 year		
U [m/s]	σ_U [m/s]	M_y^{Bld} [MNm]
11.11	2.57	14.95 (50% fractile)
11.11	**2.57**	**15.87 (90% fractile)**
10.63	2.53	16.88 (99% fractile)
IFORM – 50 year		
U [m/s]	σ_U [m/s]	M_y^{Bld} [MNm]
13.31	3.04	15.53 (50% fractile)
12.69	**2.99**	**16.54 (90% fractile)**
13.31	3.04	17.54 (99% fractile)

Figures 2 shows the 50-year IFORM contours and the corresponding long-term extreme responses (90% fractile). The rainbow colors denote the range of the maximum value, and the blue cross denotes the combination of the wind speed and turbulence leading to the maximum flapwise blade root bending moment along the contour. Results are only shown for wind conditions between the cut-in and cut out wind speeds.

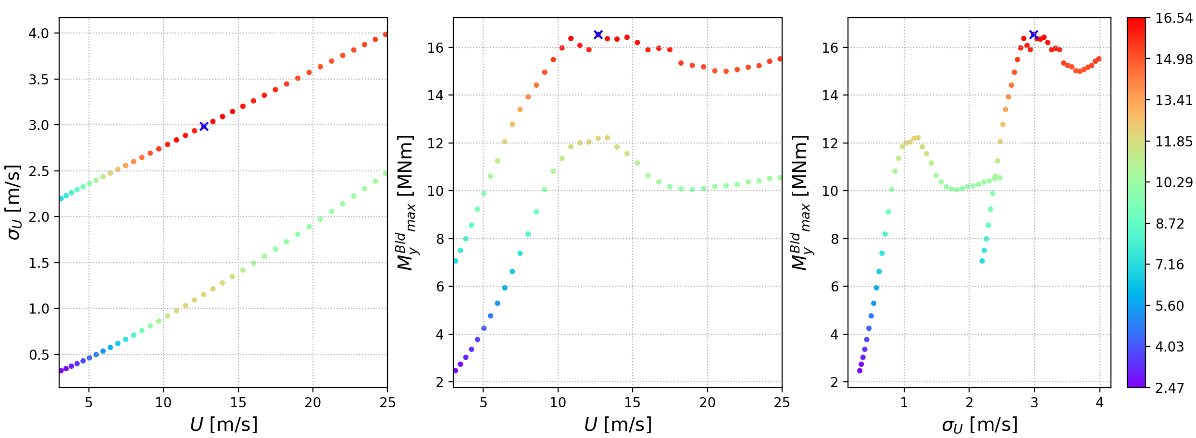

Figure 2. 50-year contour and corresponding extreme maximum flapwise blade root bending moment

2.2. Extreme Responses Estimated by Sequential Sampling and Gaussian Processes

Two different models $\hat{g}_{Y|\boldsymbol{X}}(y\,|\boldsymbol{x},\boldsymbol{\theta}(\boldsymbol{x}))$ for the short-term extreme response have been considered in this study: the Gumbel-distribution and the generalized extreme value (GEV) distribution. The Gumbel distribution has two parameters $\boldsymbol{\theta} = (\alpha, \beta)$ (location and scale) and the GEV distribution has three parameters $\boldsymbol{\theta} = (\alpha, \beta, \gamma)$ (location, scale and shape). In this paper, only results with the Gumbel alternative are presented. Results for the GEV model tend to be similar but takes a bit more time to converge. This is probably due to the extra shape parameter that introduces additional variability (see [15] for more details).

From n_{seeds} random response simulations for a given long-term input $\boldsymbol{x} = (U, \sigma_U)$, the best fit parameters $\boldsymbol{\theta}$ are found as the maximum likelihood estimate (MLE) for the given observations $y = (y_1, \cdots, y_{n_{seeds}})$. The uncertainty in the distribution parameters is accounted for throughout the analysis by considering the likelihood of the distribution parameters under the given observations, i.e. $p(\boldsymbol{y}|\boldsymbol{\theta})$. These uncertainties are incorporated into the GP-model, by approximating the Gumbel and GEV-parameters' likelihoods by 2- and 3-dimensional Gaussian likelihoods, respectively. The best-fit Gaussian likelihoods are found by drawing samples from the distribution proportional to $p(\boldsymbol{y}|\boldsymbol{\theta})$ using Markov Chain Monte Carlo (MCMC). From the MCMC-samples, the means and covariance matrix of the distribution parameters are estimated. The set of MCMC samples is increased in batches until three consecutive estimates of the means and covariances are within 1% of each other.

The fitted Gaussian likelihood for the distribution parameters (i.e. the mean vector and covariance matrix) is then used to fit a GP model. In the general case that the Gaussian process have *m*-dimensional output (i.e. models *m* distribution parameters $\theta = (\theta^{(1)}, \cdots, \theta^{(m)}\}) \in \mathbb{R}^m$) and *d*-dimensional input (i.e. is a function of *d* long-term parameters $x \in \mathbb{R}^d$), consider a Gaussian process given as a prior over functions $\boldsymbol{\theta}$: $\mathbb{R}^d \rightarrow \mathbb{R}^m$:

$$\theta(x) \sim GP(\boldsymbol{\mu}(x), K(x, x')). \tag{4}$$

where the prior mean $\boldsymbol{\mu}(x) = [\mu_1(x), \cdots, \mu_m(x)]$ is assumed zero, and where *K* is the diagonal matrix

$$K(x, x') = \begin{bmatrix} K_1(x, x') & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & K_m(x, x') \end{bmatrix}, \tag{5}$$

where each K_j is of the Matérn 3/2 type as in [3]. In the present case $m = 2$ or $m = 3$ for the Gumbel and GEV distributions, respectively, and $d = 2$ for $\boldsymbol{x} = (U, \sigma_U)$.

Given some training data, i.e. observed distribution parameters $\boldsymbol{\zeta}_j = (\zeta_j^{(1)}, \cdots, \zeta_j^{(m)})$ for points $\boldsymbol{x}_j$: $D = \{x_j, \zeta_j\}_{j=1}^N$ one can derive the posterior predictive distribution for unobserved points under the observed training data. The parameters are assumed to come with Gaussian noise, so that $\boldsymbol{\zeta}_j = \boldsymbol{\theta}(\boldsymbol{x}_j) + N(0, \Sigma_j)$, where Σ_j is the covariance matrix of each set of the *m* distribution parameters, as estimated using the procedure described above.

Given the GP-model that enables drawing random samples of the distribution parameters for any long-term parameter $\boldsymbol{x}$, a Monte-Carlo estimate of the response distribution is obtained based on simulations of 10000 years of long-term conditions. First, distribution parameters $\boldsymbol{\theta}_j$ are sampled from the GP-model for each long-term condition x_j, $j = 1, \cdots, 10000 \cdot 365.25 \cdot 24 \cdot 6$. Then, for each $\boldsymbol{\theta}_j$ a short-term response y_j is sampled from the Gumbel- or GEV distribution $\hat{g}_{Y|\boldsymbol{X}}(y\,|\boldsymbol{x},\boldsymbol{\theta}(\boldsymbol{x}))$. From the 10000 years of responses, the relevant return values are estimated.

A sequential update of the GP-model is applied, as described in [3], where a new point $\boldsymbol{x}_{new}$ for which to run new short-term response simulations is selected based on a trade-off between increasing accuracy in the areas of the long-term input space that contributes to the extreme response (here responses above the estimated 100-year level) and areas where the uncertainty is large. More specifically, the following acquisition function is applied

$$\boldsymbol{x}_{new} = \underset{\boldsymbol{x}}{\operatorname{argmax}}\, s(\boldsymbol{x})|\boldsymbol{\sigma}_\theta(\boldsymbol{x})|, \tag{6}$$

where $\boldsymbol{\sigma}_\theta(\boldsymbol{x}) = (\sigma_1(\boldsymbol{x}), \cdots, \sigma_m(\boldsymbol{x}))$ are the standard deviations of each of the distribution parameters as function of the long-term variables $\boldsymbol{x}$, and $s(\boldsymbol{x})$ is the probability density function of responses above the 100-year return value, which is estimated using kernel density estimation.

Results of the sequential sampling assuming the Gumbel distribution are shown in Figure 3, as a function of the number of short-term simulations used to train the GP. The estimate obtained from the environmental contour method is indicated in the figure. It is interesting to note that the contour approach underestimates the 50-year return value significantly compared to the sequential sampling approach. It is likely that this because the main contribution to the 50-year response is coming from long-term parameters well inside the contour. Hence, this represents a situation where extreme short-term responses in relatively common long-term conditions dominates the extreme response. This is illustrated in Figure 4, which shows the area in the long-term space that has responses exceeding the 50-year return level. Most of the simulated extreme responses lie well within the contours, corresponding to relatively benign wind conditions. Both IFORM and direct sampling (DS) contours are shown in the figure.

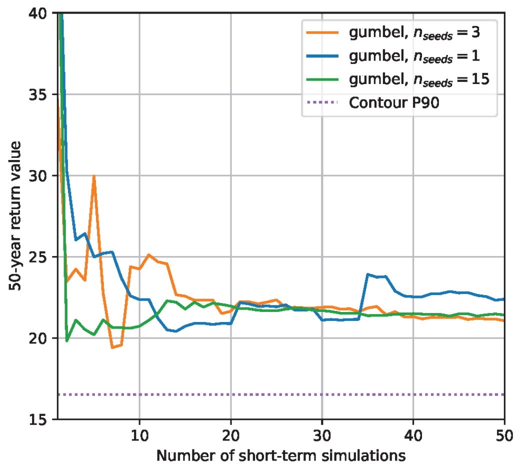

Figure 3. Estimated 50-year return values from the sequential sampling method

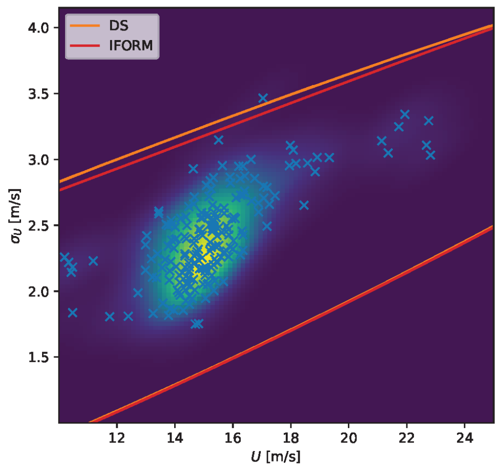

Figure 4. Contribution to the 50-year return value as a function of long-term parameters

3. DISCUSSION

In this study, different approaches to ULS reliability assessment have been applied to an offshore wind case and compared. In both cases, the extreme maximum flapwise blade root bending moment has been estimated by environmental contours and by a Gaussian processes regression model with a sequential sampling approach. These results may be used as a reference and for comparison with other approaches to ULS of the same problem.

The main reason for these approximate methods in probabilistic structural reliability assessment for ULS is that response calculations are too time consuming and expensive to allow for full long-term assessments. Hence, both environmental contours and the sequential sampling approach enables long-term extreme response estimation from a limited number of short-term calculations. In this study, both methods achieved their results with a reasonable number of response calculations. With the environmental contour method, a total of 37 points along the upper contour were used. Results from the sequential sampling approach for this case study indicate that this method converges within a similar number of simulations. Thus, in terms of computational efficiency the two approaches are comparable.

The case study presented in this paper illustrates the uncertainties in estimate extreme structural responses in probabilistic design and risk assessment. Two different approaches have been compared, that are both believed to be valid, but estimated 50-year extreme responses are deviated notably. Obviously, the true 50-year return value is not known, and response calculations are to computationally costly to be able to perform brute force Monte Carlo. However, it is believed that the results from the sequential sampling approach are more reliable since this method takes the short-term variability of the conditional response into account. Hence, this study highlights potential difficulties and shortcomings with the environmental contour method when the short-term variability is large.

The contour estimates of the 50-year response are significantly lower than the estimates from the sequential sampling approach. Most likely, this is because the short-term variability is dominating, violating the implicit assumptions of the environmental contour method. This is confirmed by Figure 4, which shows that the wind conditions most likely to be responsible the long-term extreme response lie well within the environmental contour lines. Hence, the assumption that the largest response will occur in the most severe wind conditions is not true. Indeed, due to large short-term variability, significant contributions to the long-term extreme response come from frequently occurring non-extreme wind conditions. This observation is also substantiated by the observation that there are relatively small differences between the 1-year and the 50-year extreme response estimates. One remedy for the environmental contour approach is to increase the quantile level of the short-term response distribution, but the exact level to choose can be difficult to determine. Alternatively, one may inflate the contours [10], but again, it may be difficult to determine an appropriate inflation factor.

4. SUMMARY AND CONCLUSION

Two approximate approaches for probabilistic structural reliability assessment have been compared in this paper, i.e., the environmental contour method and a sequential sampling method using Gaussian process regression. A case study considering the extreme response of an offshore wind turbine has been considered, and the 50-year return value of a selected response is estimated by the two approaches. The computational efforts needed for both methods are comparable. However, results from this case study suggests that the sequential sampling approach should be favored, since the environmental contour results are considerably lower. Although the true extreme response is not known, the fact that large contributions to the extreme response comes from environmental conditions well inside the contour lines suggests that the contour may not be accurate. However, without knowledge of the true extreme response, final recommendations cannot be made with high confidence.

Further case studies for other structural problems are recommended in order to validate the sequential sampling approach to ULS reliability assessment in general. It will be of interest to investigate how this method performs for higher-dimensional problems where the structural response depends on more environmental variables. This will be the focus of further work, where the simplified 2-dimensional case study will be extended to higher-dimensional cases.

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