Application of Bayes' Theorem for Risk-Informed Decision-Making at the Decommissioning of Fukushima Daiichi Nuclear Power Plant

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Abstract: This paper proposes an easily updated and rational framework for the treatment of measurement data using Bayesian updating for fast and proper quantification of uncertainty. The approach is demonstrated to be robust and responsive in detecting anomalies. The use of Bayesian Networks is proposed as a more rigorous way of combining multiple sources of information, and is demonstrated to be useful in resolving the presence of conflicting evidence.

Keywords: Bayesian updating, Fukushima Daiichi NPP Decommissioning, uncertainty, risk management

1. INTRODUCTION

1.1. Background

In 2011, a magnitude 9.0 earthquake and tsunami struck the Fukushima Daiichi Nuclear Power Plant, leading to the now infamous nuclear disaster. Cleanup and decommissioning schedules have been implemented [1] [2], but numerous problems exist to make the decommissioning works a challenging task. One of the main issues is the lack of data and knowledge [3]. This leads to three requirements when it comes to rational decision-making during the decommissioning:

1. We must make full use of all available information by considering all information sources under a unifying framework
2. A framework is required to allow for rapid updating when new information is obtained while the decommissioning work is ongoing
3. Where possible, our confidence and uncertainty in estimated parameters should be quantified

1.2. Overview of the Bayesian Approach

The Bayesian approach allows us to combine all the different information, whether objective or subjective, and obtain a quantitative result. By using this quantitative result instead of intuition, the Bayesian approach can aid the decision-maker in making a rational decision when using limited and possibly conflicting information from different sources.

The key mathematical idea behind Bayesian Statistics is the Bayes Theorem [4] [5], which in its simplest form is given by Equation (1).

$$P(H_n|E) = \frac{P(H_n) \times P(E|H_n)}{\sum_m P(H_m) \times P(E|H_m)} \tag{1}$$

In essence, it answers the question: "Given that evidence E is observed, how likely is hypothesis Hn to be true?". This "updated" belief P(Hn|E) is known as the posterior, and the "old" belief P(Hn) is known as the prior. In updating from the prior to the posterior, the likelihood of observing this evidence if the hypothesis is true is weighted against the total likelihood of observing this evidence. The likelihood function depends on the problem and hypothesis, and can be anything from a Heaviside Step function to a Normal probability density function.

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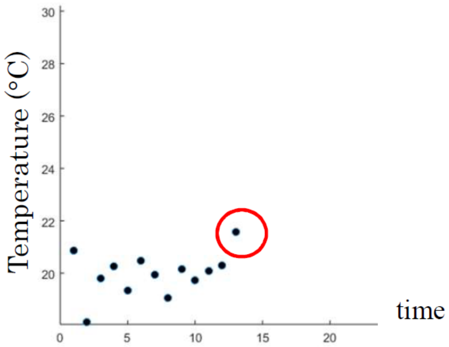
Simply put, this is a more mathematically rigorous way of combining our prior belief with new evidence to provide an updated belief.

2. DATA TREATMENT AND ANOMALY DETECTION USING BAYESIAN UPDATING

2.1. Problem Definition

When we measure temperature using any detector, we can assume that measurement uncertainty will cause the readings to follow a Normal distribution Normal(μ_T,σ_T), where μ_T is the "true" temperature and σ_T is the aleatoric uncertainty. As temperature measurements come in, we want to estimate the values for μ_T and σ_T, together with our confidence in those estimates. Then, if an increase in temperature is measured, we can estimate the probability of it being due to measurement noise or due to a real increase in temperature (Figure 1).

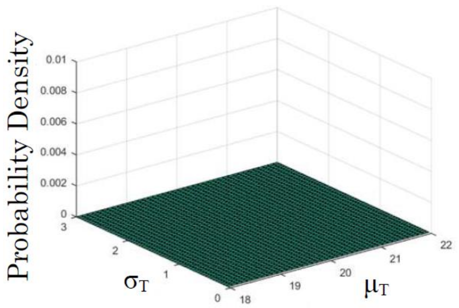
Figure 1: Real increase or due to aleatory uncertainty?



2.2. Theory and Equations

The conjugate prior for the Normal distribution with unknown μ and unknown σ is the Normal-Gamma Distribution [6], with unknown parameters μ, k, α, β. A judicious choice of prior values for these parameters will be useful, but in the absence of any knowledge we can use a non-informative prior where μ=0, k=0, α=-0.5, β=100, as shown in Figure 2.

Figure 2: Non-informative prior Normal-Gamma distribution with μ=0, k=0, α=-0.5, β=100



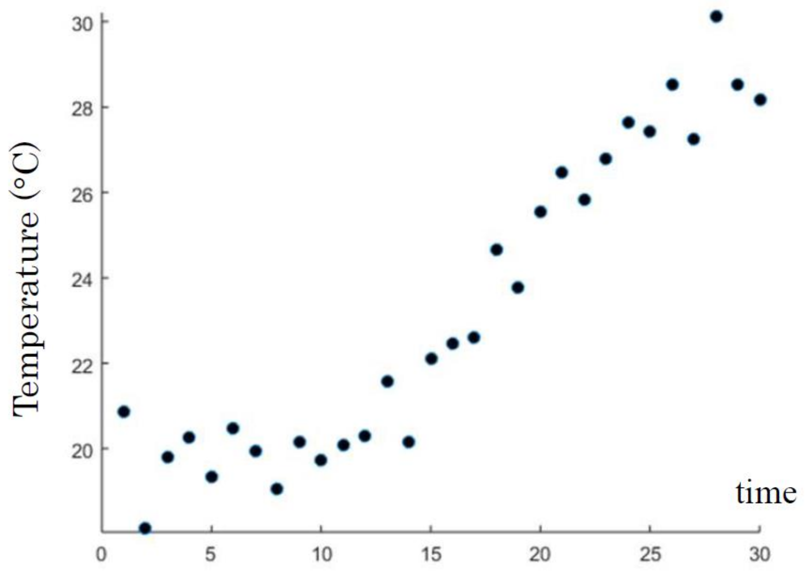
When n number of temperature measurements $x_1, \ldots, x_n$ is taken with average $\bar{x}$, the parameters can be updated as follows in Equation Set (2) [7]:

$$\mu^* = \frac{k\mu + n\bar{x}}{k+n}$$
$$k^* = k + n$$
$$\alpha^* = \alpha + \frac{1}{2}n$$
$$\beta^* = \beta \quad + \frac{1}{2}\sum\nolimits_i (x_i - \bar{x})^2 \; + \; \frac{kn(\bar{x}-\mu)^2}{2(k+n)} \tag{2}$$

2.3. Simulated data and Methodology

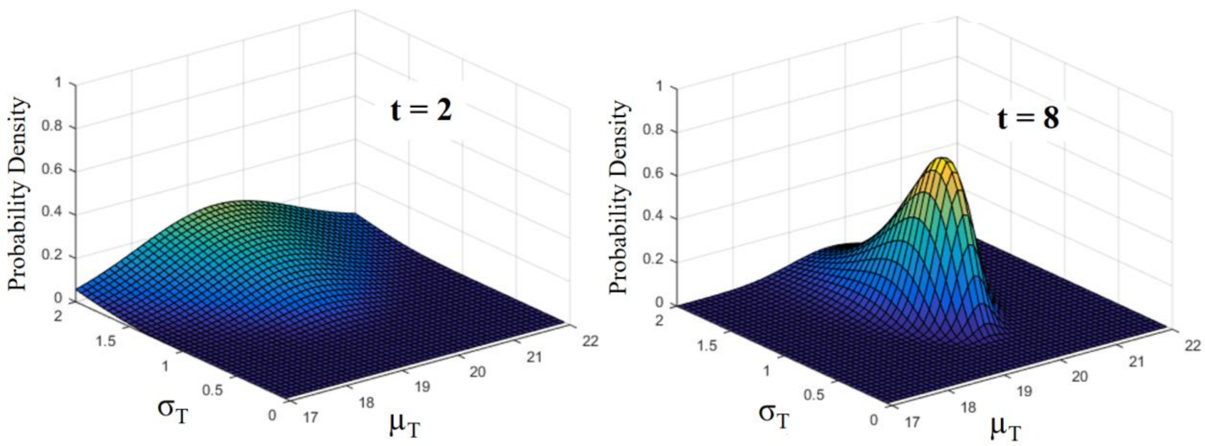
To test and demonstrate the robustness of the approach, a simulation was run using Matlab. A dummy set of 10 Temperature data was first generated using Normal(μ_T=20°C, σ_T =1°C), followed by a steady increase of 0.5°C per timestep from t=11 to t=30, as shown in Figure 3.

Figure 3: Simulated Temperature Data



Beginning with the non-informative prior and updating sequentially at every time step using Equation Set (2), a Normal-Gamma distribution is obtained at every time step. An example at t=2 and t=8 is shown in Figure 4.

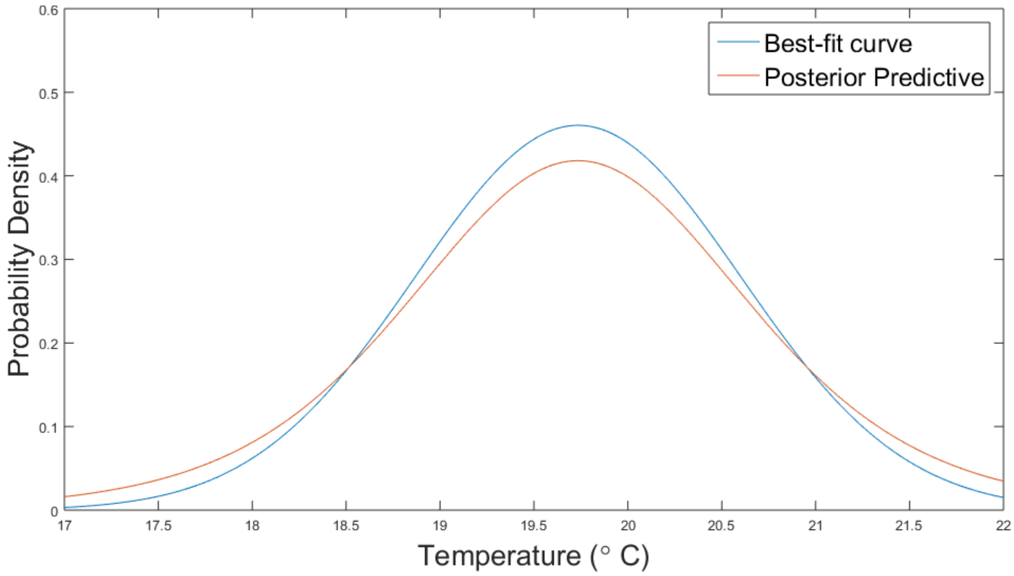
Figure 4: Updated Normal-Gamma Distribution of parameters at t=2 and t=8



Compared with traditional curve fitting methods, it is evident that this approach better encapsulates the uncertainty, since aleatoric uncertainty (σ_T) and epistemic uncertainty (introduced during model fitting and shown as the probability density) are treated separately.

The posterior probability density at each time step is then used to produce a posterior predictive as shown for t=8 in Figure 5. The posterior predictive ($P(D_{new}|D)$) effectively answers the following question: Given the set of observations so far (D), what is the likelihood of a new observation (D_{new}) being equal to *x*?

Figure 5: Posterior Predictive (Constant Temperature) vs Best-fit curve at t=8



In this case, it takes the form of a non-standard Student's t-distribution with center μ, precision Λ, and degree of freedom 2α (Equation (3), with parameters related to μ, k, α, β from the Normal-Gamma distribution) [6].

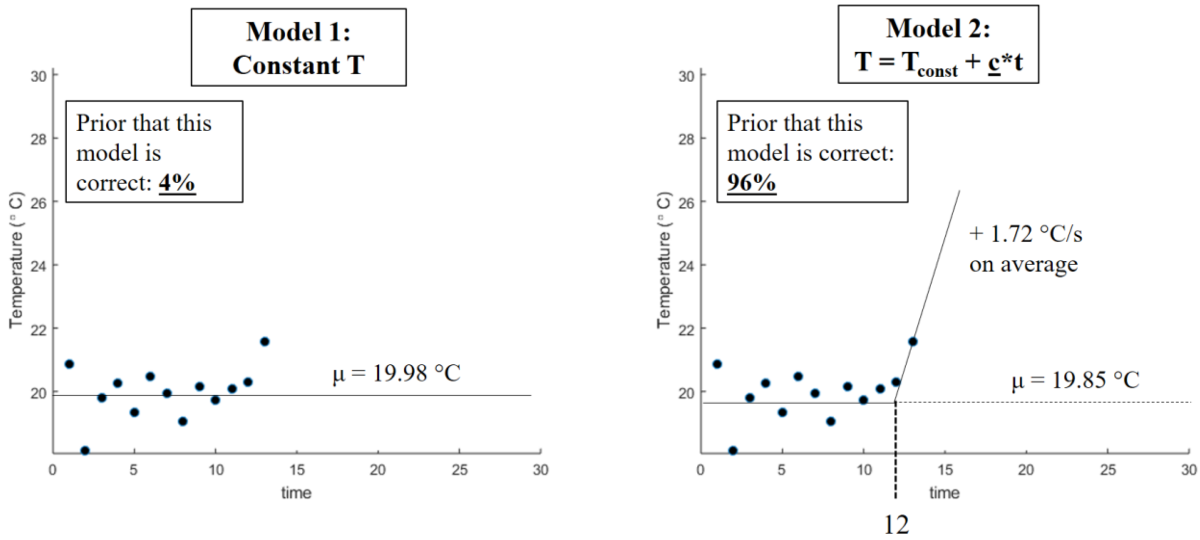
$$P(D_{new}|D) = \pi^{-\frac{1}{2}} \frac{\Gamma\left(\frac{2\alpha+1}{2}\right)}{\Gamma\left(\frac{2\alpha}{2}\right)} \left(\frac{\Lambda}{2\alpha}\right)^{\frac{1}{2}} \left(1 + \frac{\Lambda(x-\mu)^2}{2\alpha}\right)^{-(2\alpha+1)/2}, where\ \Lambda = \frac{\alpha\kappa}{\beta(\kappa+1)} \quad (3)$$

While similar results may be obtained by simply curve-fitting the data and using the best-fit parameters, epistemic uncertainty is ignored in such an approach. The Bayesian approach, on the other hand, considers all possible μ_T and σ_T estimates weighted as shown in Figure 4. The difference is obvious and important especially when data is sparse and uncertainties are large.

When a new measurement is made, for example at t=9, the value is checked against the posterior predictive to see if it lies within the 50% confidence range. If it does, the Normal-Gamma Distribution for the parameters are continued to be updated, producing a new posterior predictive to be used for the next time step, and so on. If it does not, a noise-vs-anomaly subroutine is started to calculate the relative probabilities of each case. For this data set (Figure 3), the subroutine was automatically triggered at t=13 due to the low likelihood of observation (4%) based on the posterior predictive at t=12.

A simple linear approximation is used for the non-constant model, with the gradient being the average change in value from the last known stable point (in this case, t=12). For the constant-temperature model, updating continues as per usual. The above is illustrated in Figure 6.

Figure 6: Models used for the anomaly characterization sub-routine



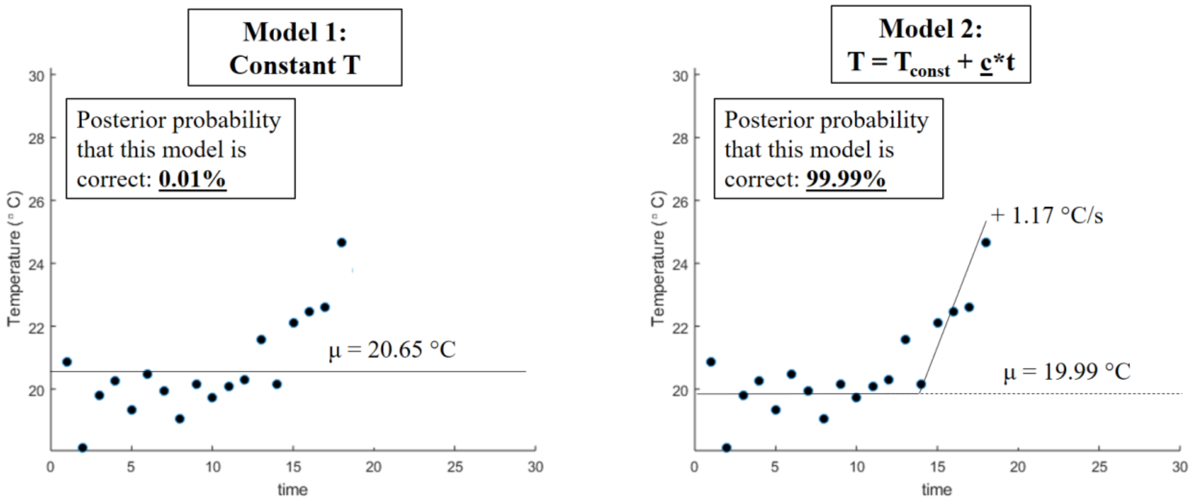
At the next time step (t=14), the measured value is compared with the 2 models to obtain the likelihood of observation, which is then used to update the posterior probability of each model via Equation (1).

All of the above continues until the constant-temperature model reaches posterior probability greater than 50%, at which point the sub-routine ends and updating continues as before, or until Model 2 reaches a posterior probability greater 99.99%, at which point the entire algorithm stops and concludes that the temperature is no longer constant.

2.4. Results and comments

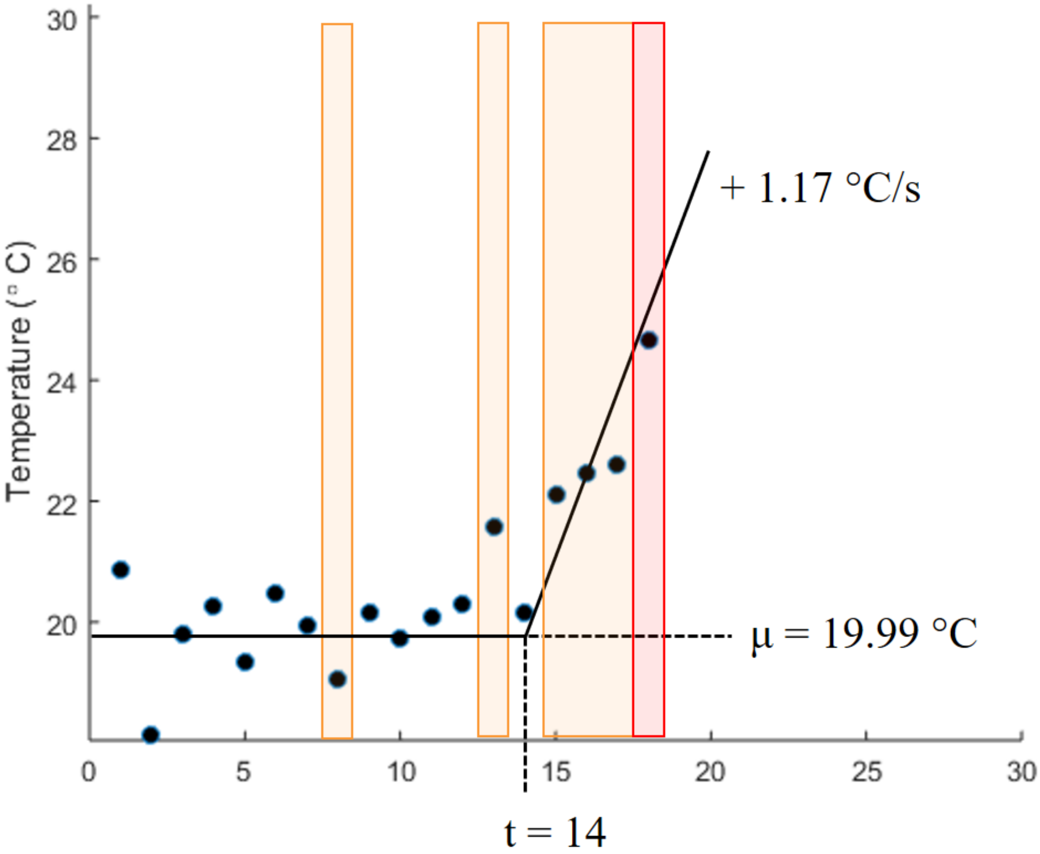
At t=18, the algorithm concludes with >99.99% confidence that the temperature is no longer constant (Figure 7).

Figure 7: Temperature confirmed (>99.99% confidence) to be changing at t=18.



The full summary of the results produced by the algorithm is shown in Figure 8, where yellow highlighted portions denote suspected anomalies (the first two of which were concluded to be false positives), and the red highlighted portion denotes the point where the temperature was confirmed to have been increasing.

Figure 8: Summary of results from simulation



Thus, the Bayesian approach has been demonstrated to be able to provide clear information on not just the estimates themselves, but also our confidence in those estimates. It is also easily updated, and can be used to produce quantitative probability estimates that will be useful as an aid for decision-making.

3. BAYESIAN NETWORKS FOR RATIONAL RISK-INFORMED DECISION-MAKING

3.1. Problem Definition

The previous chapter considered only information from a single detector. In the real world, we may have multiple detectors for each plant parameter, and several plant parameters to consider. Often times, such fusion of information from multiple disparate sources is performed based on the decision-maker's intuition and "experience". A more rigorous approach is needed such that even when conflicting information is obtained, decision-making can proceed rationally, based on quantitative results.

3.2. Theory and overview of Bayesian Networks

Bayesian networks are based on Directed Acyclic Graphs and can represent real-world processes rather than information flow [8]. Thus, it is possible to model highly complex systems where the information flow is probabilistic. This is in contrast with traditional methods where all the information must be explicitly modeled as input variables in a deterministic function. In essence, traditional methods can be thought of as "all-or-nothing" models, while Bayesian Networks allow a model to be built based on limited information and then updated as more information becomes available.

The underlying mathematics are the same as that given in Equation (1), as explained in Section 1.2.

3.3. Methodology and results

A model of the situation during fuel debris retrieval operations at Fukushima Daiichi is created in the form of a Bayesian network in GeNIe 2.1 (32-bit Academic Edition). Conditional Probability Tables (CPT) are then defined as such:

- Re-criticality can happen with a 10% probability, while a failure in decay heat removal system can happen with a 20% probability.
- The first scenario will lead to an increase in temperature, neutron flux, and amount of short-lived radioisotopes, while the second will only lead to an increase in temperature.
- Even if neither happens, other factors such as the weather or the statistical nature of radioactive decay may cause an increase in temperature or neutron flux with a 5% probability.
- Detectors are assumed to be 95% reliable, except for the radioisotope detector which is assumed to be more prone to giving false readings and thus assigned only a 70% reliability rating.

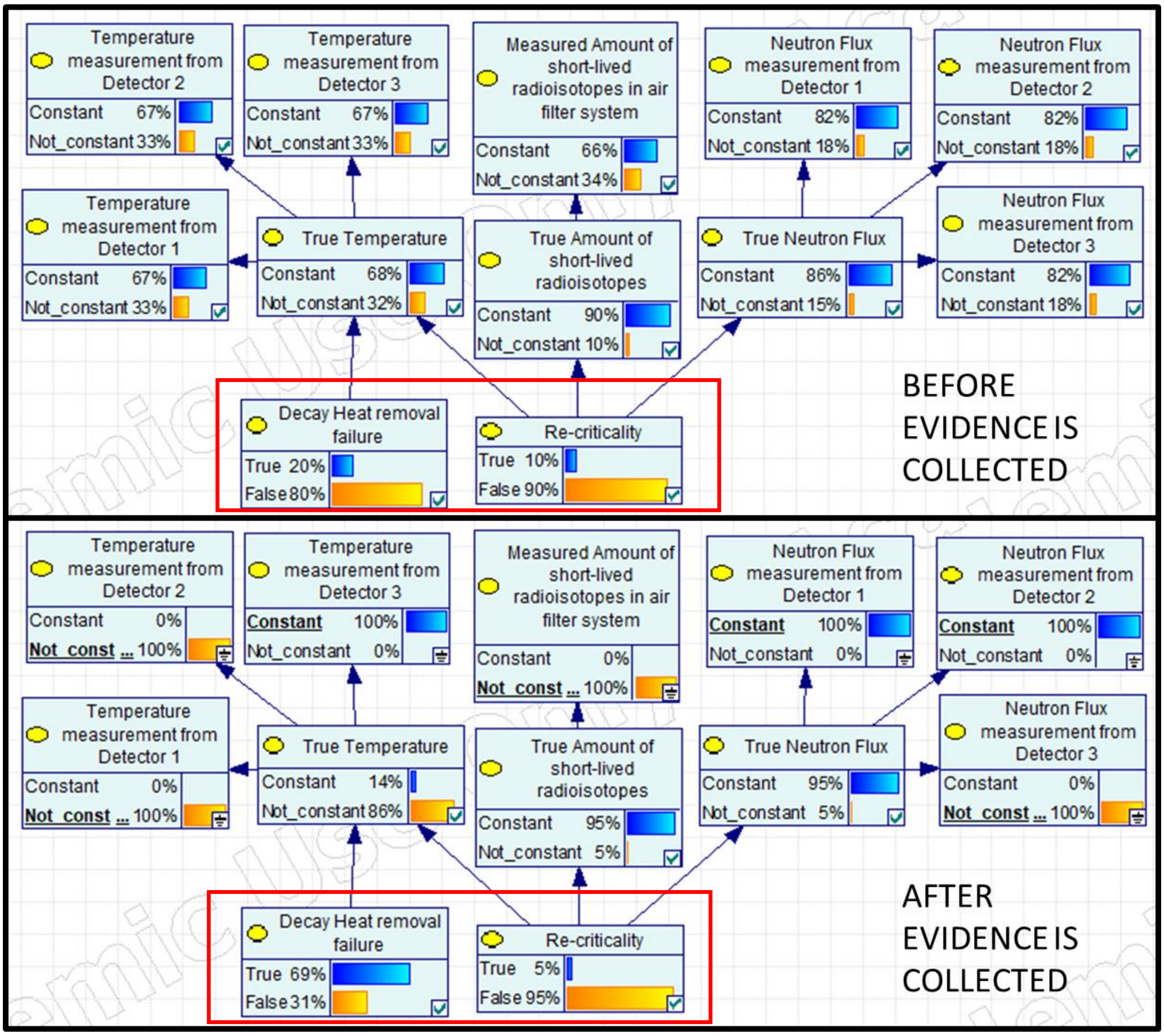
Several scenarios are then considered to demonstrate the flexibility of Bayesian Networks and its usefulness as a real-time decision support system. One example that highlights the utility of such a system in dealing with conflicting evidence is shown in Figure 9. Before any evidence is available, the network as defined by the CPTs is shown in the top of Figure 9, where it can be seen that the two scenarios "Re-criticality" and "Decay heat removal failure" have 10% and 20% probabilities, as we have defined in the CPTs based on prior knowledge.

Now, we assume that evidence becomes available as follows:

- Two of the temperature detectors measure an increase, while the third registers no change
- One of the Neutron detectors measure an increase in Neutron flux, while the other two register no change
- An increase in short-lived radioisotopes is measured in the air filter system

Given such a combination of conflicting evidence, and bearing in mind the complex combination of factors as defined in the CPTs, human intuition is likely to fail. However, with the Bayesian network, quantitative results can immediately be obtained as shown in the bottom half of Figure 9, which is useful as an aid for decision-making.

Figure 9: Using a Bayesian Network to update scenario probabilities with new but conflicting evidence



4. CONCLUSION

The simplicity and intuitiveness of Bayes' theorem mean that it can extremely powerful when applied correctly. This thesis first applies it in a dynamic form as a more robust way of quantifying uncertainty and to detect anomalous data points in real-time. The obtained results from multiple detectors and for different plant parameters are then combined in a Bayesian network, which can be used as a real-time decision support system. Work is ongoing to perform more simulations to test the robustness of the approach, especially when uncertainty is large. Work is also ongoing to integrate the use of Bayesian Networks into the anomaly detection sub-routine directly.

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