

# A Collision Risk-Based Ship Domain Method Approach to Model the Virtual Force Field

Tengfei Wang<sup>a,b,c</sup>, Xiping Yan<sup>a,b\*</sup>, Yang Wang<sup>a,b</sup>, and Qing Wu<sup>a,b,c</sup>

<sup>a</sup> Intelligent Transportation System Research Center, Wuhan University of Technology, Wuhan Hubei 430063, China

<sup>b</sup> National Engineering Research Center for Water Transport Safety, Wuhan Hubei 430063, China

<sup>c</sup> School of Logistics Engineering, Wuhan University of Technology, Wuhan Hubei 430063, China

---

**Abstract:** Ship domain is a concept which is widely used in research on collision avoidance and traffic engineering among others. The paper offers a systematic and critical review of the newer ship domain models and related research. It discusses multiple differences in approach to ship domain concept: from definitions and safety criteria, through research methodologies and factors taken into account, to sometimes mostly different results obtained by various authors. Then, an attempt of applying the risk based ship domain is made to build a new VFF in this paper. The aim of this approach is to establish a Virtual Field Force method(VFF) model with clear scientific meaning of risk and the ship domain model is used as a bridge during this process. The VFF method presented in this paper can be detailed and quantitative interpretation the collision risk of multi-ship encounter scenario. It can provide an essential basis for decision-making in multi-ship collision avoidance.

**Keywords:** Maritime Safety, Ship Domain, Collision risk, VFF.

---

## 1. INTRODUCTION

The concept of ship domain was widely used in the field of ship collision risk assessment and collision avoidance decision-making since the 1970s[1, 2]. From then on, it becomes an important tool to estimate the safe distance or safe area around the own ship in an encounter situation. Although there is never a complete and precise definition of ship domain especially when it comes to the details such as the terms of size and shape, the general principles of this concept are never change: ship domain is an area that it can be used to evaluated and visualize the collision risk with target ships in current scenario. It is more intuitive and taking the speed and course change into account in some terms compared with CPA, another geometric method widely used in ship collision avoidance.

Although there is not a clear classification of ship domains, it can be divided into three main methods: developed by theoretical analyses, based on experts' knowledge and depended on work experience. These three methods are not mutually exclusive, and a ship domain model is usually constructed by a combination of various methods in actual use., e.g. Dinh and Im [3]. What all models have in common is that they are influenced by water regions, albeit at different levels. Under the condition of capacity or waterway risk, the particular water region which is of interest is mainly determined by its shape, traffic density and traffic patterns. For instance, the water can be divided into three types in collision avoidance research: open waters, restricted (but considerable) waters or narrow waterways.

The methods of determining and calculating ship domains have developed over time. Early models were often based on statistically processed marine radar data[1, 2, 4]. This kind of empirical approach has become the mainstream method, with continuous development, the data source changed from radar to AIS and more advanced statistical methods have been applied to data processing [5, 6]. Analytical approach [7, 8], expert navigators' knowledge[9], or a combination of both[3] is widely used especially it comes to collision risk analysis, near miss detection or collision avoidance decision-making supporting systems.

The Virtual Field Force method(VFF) model[10] is a classical deterministic approach method for robot path planning, in which the search area of robot is changed into a virtual risk field. In VFF method, the safe path can be found along the virtual force. While there are a lot of similarities in the

connotations and application between the concept of influence range of obstacles in VFF and ship domain, VFF is used to analysis ship domain in more details[11], and the ship domain is used to make the meaning of the risk value in VFF clearer[12]. During the development of unmanned surface vessels (USV) and unmanned cargos, the application of VFF in decision-making of unmanned ships is getting more and more attention. Bucknall et.[13, 14] developed a path planning method for USV based on both VFF and ship domain, which can be used in a time-variant maritime environment. Most of the approach in unmanned ships are mainly based on VFF as a continuation of robotics in the field of navigation, and the background of the risk of ship domain has gradually been neglected. To reduce this gap, an attempt of applying the risk based ship domain is made to build a new VFF in this paper. The aim of this approach is to establish a VFF model with clear scientific meaning of risk and the ship domain model is used as a bridge during this process.

The paper is organized as follows: different kinds of definitions of ship domain are presented in Section 2, followed by the analysis and discussion. Then in Section 3, several classic ship domain models are mainly discussed in details. Based on these analysis, a collision risk-based ship domain method which can approach to collision avoidance decision-making systems for unmanned cargo ship is presented in Section 4. a case study is presented in Section 5, followed by discussion and conclusions in Section 6.

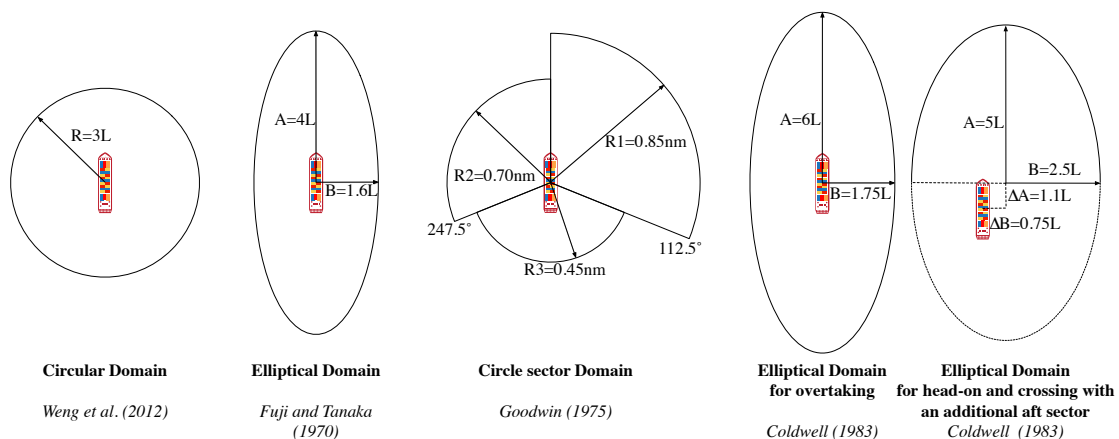
## 2. SHIP DOMAINS DEFINITIONS, METHODS AND FACTORS TAKEN INTO ACCOUNT

### 2.1. Definitions of Ship Domains

Although all researchers propose various dimensions of the ship domain models to fit their approaches, it must be pointed out that these dimensions may lead to different vessel spacing, depending on the specific definition, application scenarios and relevant safety standards. Therefore, four typical definitions are reviewed and discussed here, and the typical methods and factors according to different kinds of ship domains are discussed later in this section.

These four main definitions of ship domain are the circular domain [15, 16], Fujii [1]'s and Coldwell [4]'s elliptical domain and M. Goodwin [2]'s circle sector domain, the details of shape and size of each domain are depicted in Fig. 1. Except of the difference of the shape and size between these models, the risk criteria which often combined with ship domain in practice is another major cause of differences. Fig. 2 illustrates the mainly used four kinds of risk criteria, which are:

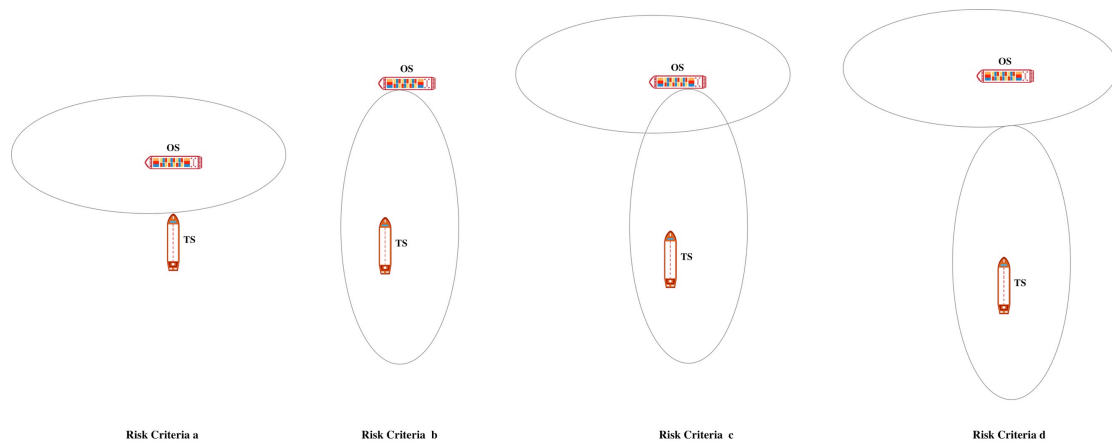
- A) own ship's (OS) domain is only concerned and should keep a target ship (TS) out of this rang;
- B) any TS's domain is only concerned and OS should keep out of this rang;
- C) OS's and TS's domains can be overlapped but every domain should keep clear;
- D) neither OS's and TS's domains should be overlapped.



**Figure 1: Definition of four typical ship domains**

At beginning, these criteria are always represented with ship domain together by researchers. In Fujii [1] ship domain definition, the give way ship have the duty to avoid to violate the stand-on ship's domain, so the risk criteria is one of A and B. While in Coldwell [4] ship domain, every ship should only be responsible for her own security, that's obviously the case of A. As for M. Goodwin [2], the ship domain is defined and analyzed under the premise that neither OS or TS's domains should be violated during an encounter situation, that is the same meaning like case C. While in the latest research like Andrew, et al. [15] and Wang and Hoong-Chor [16], the ship domain is defined that should not be overlapped any time, which is the case D. With the promotion and application of the concept of ship domain, the ship domain and risk criteria Are combined in different ways, which also makes the model of ship domain become wider variety.

As one of the main factors of collision risk analysis, the ship domain and its companied risk criteria represent the understanding and attitude towards collision risk. Because different ship domain model could lead to a different safety area in shape and size between encountered ships, and largely affect the collision risk identification and conformation as well as the process of collision avoidance decision-making during a ship's navigation. As the effective spacing between the encountered ships is an important index for collision safety – the larger effective spacing there is, the safer the encountered ships will be. The calculation formula of effective spacing of the risk criteria A and risk criteria B for four typical ship domains and seven encounter situations are given in Table 1. As is depicted in Fig. 1, the effective spacing in risk criteria C is the larger one among risk criteria A and B, and the risk criteria D is the sum of A and B.



**Figure 2: Different domain-based risk criteria**

The encounter situations are divided into seven scenarios rather than the typical three (head-on, crossing and overtake), as the effective spacing is different when different sides of OS face to the TS. The seven types of encounters are presented in details in [17]. Then the values are calculated by setting the ships' dimensions of OS and TS as 160m and 200m in length, and 20m and 30m in beam. Fig. 3 illustrates these calculation results. By this way the effective spacing of ship domains under different risk criteria can be compared intuitively.

The presented results in Table1 and Fig.3. yield the conclusions as follows:

- 1) The risk criteria A (first line in fig.3.) may lead to a relatively small spacing for a crossing encounter situation, no matter from astern or ahead of a TS. Specifically, when OS is a Stand-on ship, which represents the TS appearing on starboard, the effective space is large enough to meet the requirements of COLREGS. When OS is a Give-way ship, which represents the TS appearing on port, the effective space is quite small and OS should take measures earlier to keep a safe situation.
- 2) The risk criteria B (second line in fig.3.) is similar with risk criteria A. The only difference is that the ship domain is set on the base of TS rather than OS. Just like it in risk criteria A, when OS is a Give-way ship, the ship domain is not enough to keep safe.

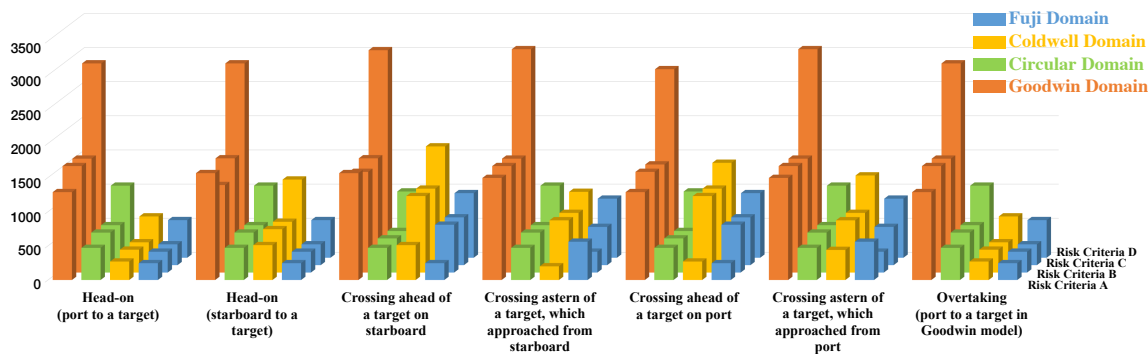
3) The risk criteria C (third line in fig.3.) is more reasonable for all the illustrated encounters. While, the risk criteria D (forth line in fig.3.) may lead to a too large spacing between ships, and a too large spacing usually means less economical and unnecessary.

4) The risk criteria A and B in fact analysis the same problem from different perspectives (from OS's perspective or TS's perspective). This asymmetrical domains in a same scenario may lead to a different assessment by OS and TS, which phenomena was first analysis by Jingsong, et al. [18] in a head-on encounter research. Even these different assessments always exist in practice, they will lead to a misunderstanding between involved ships and constitute a hazard.

By comparing all these four risk criteria, it might be concluded that the risk criteria C(Fig.2c) is more reasonable, economic, and fit well with the typical definitions of ship domain. While, it should be noted that all above conclusions are drawn under the very common situation. For some other special areas, such as harbor areas, busy inland river or narrow channels, there are some special changes in ship domain model and risk criteria to fit the situation[15, 16, 19].

**Table 1: Approximated minimum effective spacing for four domains  
(OS and TSs' length and beam are:  $L_1$ ,  $B_1$ ,  $L_2$ ,  $B_2$ , respectively. 1nm=1852m)**

Encounter/Safety condition	Domain by	Risk Criteria A	Risk Criteria B
Head-on (port to a target)	Circular	$3L_1-0.5B_1$	$3L_2-0.5B_2$
	Fuji	$1.6L_1-0.5B_1$	$1.6L_2-0.5B_2$
	Goodwin	$0.7nm-0.5B_1$	$0.85nm-0.5B_2$
	Coldwell	$1.75L_1-0.5B_1$	$1.75L_2-0.5B_2$
Head-on (starboard to a target)	Circular	$3L_1-0.5B_1$	$3L_2-0.5B_2$
	Fuji	$1.6L_1-0.5B_1$	$1.6L_2-0.5B_2$
	Goodwin	$0.85nm-0.5B_1$	$0.7nm-0.5B_2$
	Coldwell	$3.25L_1-0.5B_1$	$3.25L_2-0.5B_2$
Crossing ahead of a target on starboard	Circular	$3L_1-0.5B_1$	$3L_2-0.5L_2$
	Fuji	$1.6L_1-0.5B_1$	$4L_2-0.5L_2$
	Goodwin	$0.85nm-0.5B_1$	$0.85nm-0.5L_2$
	Coldwell	$3.25L_1-0.5B_1$	$6.1L_2-0.5L_2$
Crossing astern of a target, which approached from starboard	Circular	$3L_1-0.5B_1$	$3L_2-0.5B_2$
	Fuji	$4L_1-0.5L_1$	$1.6L_2-0.5B_2$
	Goodwin	$0.85nm-0.5L_1$	$0.85nm-0.5B_2$
	Coldwell	$1.75L_1-0.5L_1$	$3.9L_2-0.5B_2$
Crossing ahead of a target on port	Circular	$3L_1-0.5B_1$	$3L_2-0.5L_2$
	Fuji	$1.6L_1-0.5B_1$	$4L_2-0.5L_2$
	Goodwin	$0.7nm-0.5B_1$	$0.85nm-0.5L_2$
	Coldwell	$1.75L_1-0.5B_1$	$6.1L_2-0.5L_2$
Crossing astern of a target, which approached from port	Circular	$3L_1-0.5B_1$	$3L_2-0.5B_2$
	Fuji	$4L_1-0.5L_1$	$1.6L_2-0.5B_2$
	Goodwin	$0.85nm-0.5L_1$	$0.85nm-0.5B_2$
	Coldwell	$3.25L_1-0.5L_1$	$3.9L_2-0.5B_2$
Overtaking (port to a target in Goodwin model)	Circular	$3L_1-0.5B_1$	$3L_2-0.5B_2$
	Fuji	$1.6L_1-0.5B_1$	$1.6L_2-0.5B_2$
	Goodwin	$0.7nm-0.5B_1$	$0.85nm-0.5B_2$
	Coldwell	$1.75L_1-0.5B_1$	$1.75L_2-0.5B_2$



**Figure 3: Approximated minimum effective spacing (in meters) for four domains (OS and TSs' length and beam are: 160m, 20m, 200m, 30m, respectively. 1nm=1852m)**

## 2.2. Methods and Factors of Ship Domains

The definition and risk criteria determine the basic principles of ship domain, for instance the general shape, size and usage scenarios. Then, the method of ship domain presents the details of building a particular model, including specific shapes and boundaries, and values of parameters. In most cases, there are three main methods to determine a ship domain: experience-based, numerical and analysis-based. In practice, these three methods are not mutually exclusive, and more and more modern mathematical methods are approached to fix the complex scenarios[20, 21].

### 2.2.1 experience-based ship domain models

In experience-based methods, ship domain is regarded as a visualized representation of the experience of the crew, and the crew obey this domain during the navigation. The main work of experience-based method is finding these existing domains as well as the rules behind them, then developing the model for future use. AIS data is the most important resource for the experience-based method, it can be used to analysis crew's driving habits, summarize rules and validate the model.

Benefit from the well-functioning AIS data acquisition system all over the world, AIS data can be achieved and collection by researchers. Then, the parameters can be determined by analysing the navigation in the suitable water area. For instance, the length of the ship field in the crossing situation can be obtained by analysing the traffic flow in which ferries participate in the Fehmarn Belt strait[5], and similar with the width or angle of the domain. This kind of domain is often ellipse with an irregular boundary. And the ship does not locate in the centre of the ellipse, as the starboard side need more space for head-on and crossing encounters. By studying the AIS data, Wang and Hoong-Chor [16] draw a conclusion that there is a linear function between the size of the domain and the ship, and a quadratic function between the size of the domain and the ship's speed. Based on this assumption, the values of domain's length and speed can be determined precisely. In order to balance the impact on different domains, a smaller weight is assigned to the ships with larger minimal distances.

The experience-based ship domains are intuitive and data-based, they are powerful tools in modelling the collision risk situation and deepening the understanding of safety in navigation. However, the shape of this kind of ship domain is always irregular circle or ellipse with an unsmooth boundary, and the size is changing according to data. Obviously, it is difficult to use the model in a timely risk identification and decision-making in an intelligent system.

### 2.2.2 numerical ship domain models

For the deficiency of the experience-based method, people began to try to use the function fitting method to determine the ship domain. Benefit from the various modern regression functions, there are a plenty of choices to find the suitable method for the numerical ship domain model. The ship domain based on numerical method has a regular shape, which can reduce the computation and easy to use.

One of a typical numerical ship domain is presented by Zhu, et al. [22] based on a neural network. This research is constructed of the crew's experience data and a back propagation neural network (BPNN). The data is the assessments of different encounter scenarios collected from the crew, and the BPNN is developed to summarize the data and form the rules. The COLREGs, environment situations and OS and TS's ship size are also factors taken into account. Pietrzykowski presented a numerical fuzzy ship domain for open[23] and narrow[19] waters. These two papers are combinations and extensions of works by Jingsong, et al. [18] and Zhu, et al. [22] and the application of fuzzy neural networks and is the main innovation point. The different kinds of encounter situations and relative speed are added into the method, thus the ship domain turns into a changing model depending on the scenarios. The ship domains based on numerical method are usually polygons with a smooth boundary sometimes roughly resembling circles, the ship is in the aft section and port side of ship domain rather than the centre, as the star and fore side of ship need more space to move as a Give-way ship. The shape and size of the domain also change according to the encounter type: in case of head-on encounters the size of the domain is much smaller and narrower than crossing.

An advantage of the numerical method is that the whole area within the ship domain boundary can be function fitted. It was found that according to crew's experience the size of the ship domain is distributed exponentially with the change of the safety perceived by human[23]. By following this rule, the value of collision risk at any point within the ship domain can be calculated, and the area can be rebuilt with these risk value, which is often the first step of path planning.

### **2.2.3 analysis-based ship domain models**

The analysis-based ship domain model is established through theoretical analysis, and the historical data is used to modify the model. This type of model is theoretical, with a clear definition and a definite calculation method. The ship domain based on analysis has the simplest shape among the three methods, while the forms of the domain are also the most various.

N. Wang et al.[7, 8, 24] proposed a complete analytical domain model named Quaternion Ship Domain (QSD) and formed a dynamic analytical framework[8]. The QSD is consisted of four arc smooth connections (i.e. starboard, port, fore and aft), and all parameters are determined by ship's speed, length and manoeuvrability. By adding the factors such as environment, human and equipment, QSD has been further developed into a Dynamic QSD[8], in which the dynamic means the ship domain can be re-sizes and re-shapes according to conditions in a particular encounter situation.

The analysis-based method contributes a lot the developments of the concept of ship domain. The connotation of the concept is constantly being explored, and the scope of application is expanding. The domain has been divided into 'blocking area' and 'action area' to distinguish the different risk areas and to facilitate targeted decision making[3]. The 'blocking area' is the least effective space to keep own ship safe, and should not be violated by others. The 'action area' is a warning region for collision risk, all involved ships should take measures to reduce the risk. The 'action area' is more like a manoeuvre area which depends mostly on ship's manoeuvrability. This method of dividing the ship domain according to the function is also proposed in [25]. The scope of application of ship domain is mostly in ocean or coastal waters, Andrew, et al. [15] proposed a ship domain model for the River Thames to enable the analysis of the inland river traffic. All the size and shape of the domain has been changed to fit the particular water region and inland ship characteristics accordingly.

The analysis-based method gives researchers a lot of free space to construct ship domain models that best suit their needs. However, precisely because of this, these kinds of models are often lack of applicable and it is difficult to directly use it in other waters. In addition, the ship domain derived by this method also have to be tested by historical data (i.e. AIS data or crew's experience), otherwise it is not convincing.

## **3. PATH PLANING USING VVF METHOD**

### 3.1. the ship domain model designed for VFF

VVF method will be used in this part, in order to contribute the path planning in details. The purpose of building the target ships' potential field is to describe the impact of the target ships. Through the establishment of the target ship potential field, the own ship and the target ship can keep a safe distance by altering the course and the speed. For the target ship, the risk of the vicinity of the target ship in the vertical and horizontal distribution along the target ship is not uniform, taking the characteristics of the ship and the actual situation of navigation. For example, the main factor affecting the safety of a ship in the horizontal direction is the distance between the ships and the speed is also an important factor in addition to the distance. Therefore, the modeling of the target ship potential field will also be combined with the vertical and horizontal characteristics of different models of design. In this paper, the main innovation of the target ship potential field is the model used the ship's longitudinal influence as the skeleton, to extend the way to achieve the impact of the ship distribution of the description.

In this paper, an improved VFF model is proposed to model the risk field around a dynamic vessel. In contrast to traditional VFF, the repulsive force field in this paper has considered the effect of the relative velocity simultaneously. Such a feature increases the method's capability to deal with dynamic encounter collision situation, which permits the requirement of handling the status change of a sailing ship. To model a dynamic vessel, it is key point to form the field shape changing with ship's specific velocity. It is designed as follows: when the ship is travelling with low speed, a more circular field is constructed, otherwise a half-elliptical field at the fore side is used for a high speed ship. In addition, when own ship and target ship are both moving, the long axis changes along the relative velocity direction. The dimension of the new the repulsive force field is computed by calculating aft and fore sections respectively. By using the following two equations, it is continuous between the conterminous surfaces of the aft and fore sections.

The longitudinal potential of the target ship is calculated as follows. The local coordinate system is established based on the direction of the target ship body direction and the intersection of the ship's horizontal axis and the vertical axis. The entire longitudinal potential field distribution is a piecewise function. The calculation formula is shown as follows:

$$A_{ship} = \begin{cases} U_{ship} & p \in \beta \\ \frac{v_r}{K - D_s} & (p \in \alpha) \quad (v_r > 0) \\ 0 & (v_r \leq 0) \end{cases}$$

Where,  $U_{ship}$  is the ship potential constant, which represents the maximum value of the ship's potential field;

$v_r$  is the relative speed of the own ship and the target ship, when the speed of the two in the same direction, the speed of the own ship is greater than the target ship  $v_r > 0$ , on the contrary  $v_r < 0$ .

$K$  is the longitudinal distance between the own ship and the target vessel;

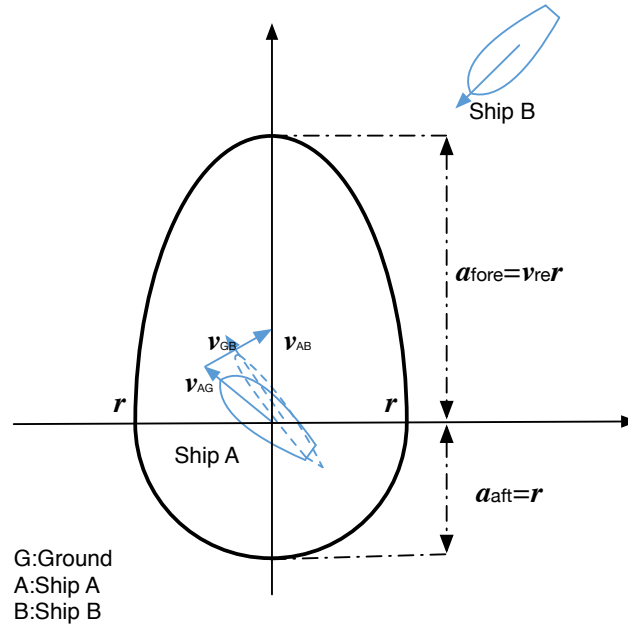
$D_s$  is a set safe distance, where  $D_s = v_r \cdot \Delta T + D_{smin}$ ,  $\Delta T$  is the system delay, which is related to sensor delay and calculation delay,  $D_{smin}$  is a set of extended safety distance.

When the point  $p$  is in the area  $\beta$ , The longitudinal potential field is distributed as a constant  $U_{ship}$ , Considering the relative velocity  $v_r$  which is the target ship and the ship, when  $v_r \leq 0$ , the relative distance between the target ship and the ship is gradually increased, and the longitudinal potential value is 0. When  $v_r > 0$ , it indicates that the distance between the ship and the target ship is gradually reduced. The more dangerous the potential value is, the more the relative velocity is, the more dangerous and the potential value is positively correlated with the relative velocity. The above-mentioned longitudinal potential distribution reflects the risk distribution of the target vessel in its longitudinal direction, the influence of the ship on its surrounding environment in its transverse direction, and its overall potential field is formed on the basis of longitudinal, which is shown in Fig.4.

The calculation of the target ship's potential is obtained by multiplying the longitudinal potential field  $A_{ship}$  by the transverse potential field. The horizontal calculation rule is a kind of Gaussian function. The whole calculation method is shown in the formula (6)

$$e_{mn} = A_{ship} \exp\left(-\frac{D^2}{2\sigma_v^2}\right)$$

Where,  $D$  is the potential distance of moving ship in different area which is shown in Fig.4.  $\sigma_v$  is the convergence coefficient of the ship's potential, and determines the horizontal influence range of the potential field.



**Figure 4: The Potential Distance of Moving Ship**

### 3.2. the VFF model for collision risk recognize

After building the new VFF model, the whole area can be digitally reconstructed by the target ships' risk field, as is depicted in Fig. 5. Every point in this area has a value of field strength, which is the sum of all the target ships' risk field. And the aim of own ship path planning is to find a trajectory which can pass the encounter situation safely and back to the original trajectory finally, as is depicted in Fig. 6. While, in order to find the way back to the original trajectory, it is required to set a destination and construct the attractive field about the destination.

The attractive field is used to attract own ship back to the original trajectory. As the attractive field is monotonically increasing with the distance, it is necessary to find a proper position point as a virtual destination. Firstly, there should be enough distance between the destination and own ship at start. Thus, arriving the destination means keeping away from the current collision risk scenario. Secondly, the destination cannot be too far away from own ship, in case the attractive field becomes too large at some place to analyses.



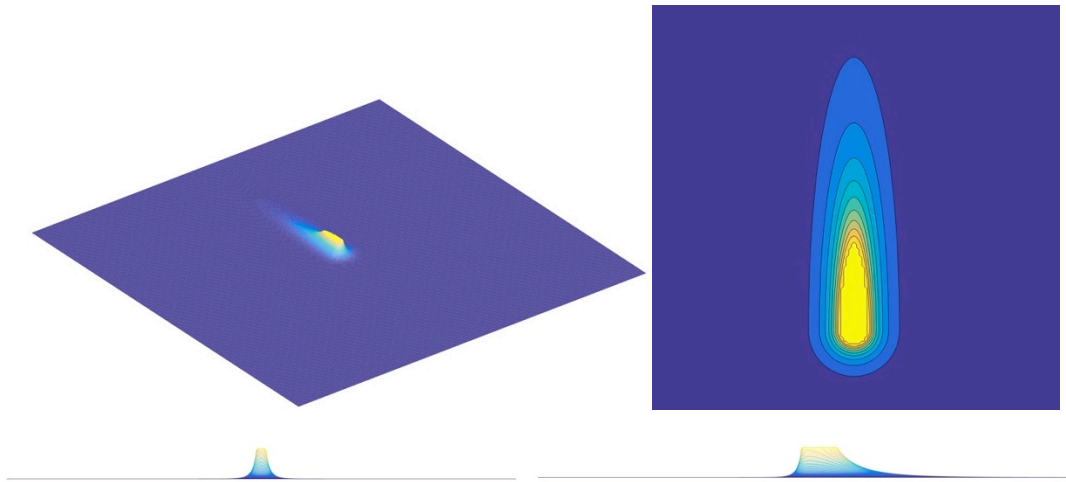


Figure 5: Four Visual Angles of the dynamic VFF of a moving ship

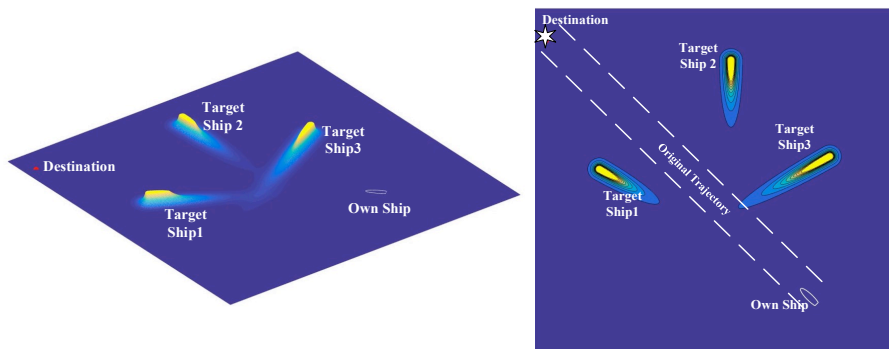


Figure 6: the repulsive fields of 3 target ships

For most cases, the  $U_{att}$  function has a quadratic form, which is monotonically increasing with the distance too fast. Moreover, the quadratic form brings great computation as the distance is much larger in ships' encounter situation than the common robot path planning scenario. Thus, this paper chooses another common  $U_{att}$  function which has the conical shape and linear form. The calculation formula is shown as follows:

$$U_{att} = \xi d$$

where  $d = d(q, q_{goal})$ , the current distance from destination;  $\xi$  is the attraction gain.

After constructing the attractive field, the whole VFF model of the multi-ship encounter scenario can be constructed with target ships' repulsive fields and virtual destination's attractive field. Fig. 6. can be improved into Fig. 7.

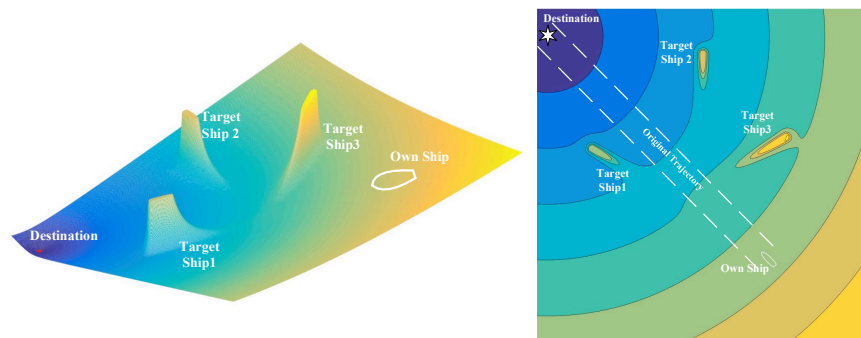


Figure 7: the VFF model of 3 target ships and own ship's virtual destination

#### 4. CONCLUSION

Ship domain is a concept which is widely used in research on collision avoidance and traffic engineering among others. The paper offers a systematic and critical review of the newer ship domain models and related research. It discusses multiple differences in approach to ship domain concept: from definitions and safety criteria, through research methodologies and factors taken into account, to sometimes mostly different results obtained by various authors. Then, an attempt of applying the risk based ship domain is made to build a new VFF in this paper. The aim of this approach is to establish a Virtual Field Force method(VFF) model with clear scientific meaning of risk and the ship domain model is used as a bridge during this process. The VFF method presented in this paper can be detailed and quantitative interpretation the collision risk of multi-ship encounter scenario. It can provide an essential basis for decision-making in multi-ship collision avoidance.

#### Acknowledgements

The first author has been supported by the China Scholarship Council (CSC) and the Fundamental Research Funds for the Central Universities (Grant No. WHUT2017-YB-035). The research was sponsored by grants from the Fundamental Research Funds for the Central Universities (WUT: 2018 III 061GX) and the Key Project in the National Science & Technology Pillar Program (Grant No.2015BAG20B05).

#### References

- [1] Y. Fujii, "Traffic Capacity," *Journal of Navigation*, vol. 24, pp. 543-552, 1971.
- [2] E. M. Goodwin, "A Statistical Study of Ship Domains," *Journal of Navigation*, vol. 28, pp. 328-344, 1975.
- [3] G. H. Dinh and N. K. Im, "The combination of analytical and statistical method to define polygonal ship domain and reflect human experiences in estimating dangerous area," *International Journal of e-Navigation and Maritime Economy*, vol. 4, pp. 97-108, 2016.
- [4] T. G. Coldwell, "Marine Traffic Behaviour in Restricted Waters," *Journal of Navigation*, vol. 36, pp. 430-444, 1983.
- [5] H. M. Gamborg, J. T. Koldborg, L. S. L. Tue, M. Kristina, R. F. Må, Lsted, and E. Finn, "Empirical Ship Domain based on AIS Data," *Journal of Navigation*, vol. 66, pp. 931-940, 2013.
- [6] E. V. Iperen, "Classifying Ship Encounters to Monitor Traffic Safety on the North Sea from AIS Data," *Transnav the International Journal on Marine Navigation & Safety of Sea Transportation*, vol. 9, pp. 51-58, 2015.
- [7] N. Wang, "An Intelligent Spatial Collision Risk Based on the Quaternion Ship Domain," *Journal of Ship Research*, vol. 56, pp. 170-182, 2012.
- [8] N. Wang, "A Novel Analytical Framework for Dynamic Quaternion Ship Domains," *Journal of Navigation*, vol. 66, pp. 265-281, 2013.
- [9] P. Zbigniew and U. Janusz, "The Ship Domain u2013 A Criterion of Navigational Safety Assessment in an Open Sea Area," *Journal of Navigation*, vol. 62, pp. 93-108, 2009.
- [10] Khatib, "Real-time obstacle avoidance for manipulators and mobile robots," *International Journal of Robotics Research*, vol. 5, pp. 500-505, 1986.
- [11] Y. Xue, D. Clelland, B. S. Lee, and D. Han, "Automatic simulation of ship navigation," *Ocean Engineering*, vol. 38, pp. 2290-2305, 2011.
- [12] C. K. Tam, R. Bucknall, and A. Greig, "Review of Collision Avoidance and Path Planning Methods for Ships in Close Range Encounters," *Journal of Navigation*, vol. 62, pp. 455-476, 2009.

- [13] S. Rui, Y. Liu, and R. Bucknall, "The angle guidance path planning algorithms for unmanned surface vehicle formations by using the fast marching method," *Applied Ocean Research*, vol. 59, pp. 327-344, 2016.
- [14] R. Song, Y. Liu, and R. Bucknall, "A multi-layered fast marching method for unmanned surface vehicle path planning in a time-variant maritime environment," *Ocean Engineering*, vol. 129, pp. 301-317, 2017.
- [15] R. Andrew, R. Ed, F. David, and P. David, "Practical Application of Domain Analysis: Port of London Case Study," *Journal of Navigation*, vol. 67, pp. 193-209, 2014.
- [16] Y. Wang and C. Hoong-Chor, "An Empirically-Calibrated Ship Domain as a Safety Criterion for Navigation in Confined Waters," *Journal of Navigation*, vol. 69, pp. 257-276, 2016.
- [17] R. Szlapczynski and J. Szlapczynska, "Review of ship safety domains: Models and applications," *Ocean Engineering*, vol. 145, pp. 277-289, 2017.
- [18] Z. Jingsong, W. Zhaolin, and W. Fengchen, "Comments on Ship Domains," *Journal of Navigation*, vol. 46, pp. 422-436, 1993.
- [19] Z. Pietrzykowski, "Ship's Fuzzy Domain – a Criterion for Navigational Safety in Narrow Fairways," *Journal of Navigation*, vol. 61, pp. 499-514, 2008.
- [20] R. Szlapczynski and J. Szlapczynska, "A method of determining and visualizing safe motion parameters of a ship navigating in restricted waters," *Ocean Engineering*, vol. 129, pp. 363-373, 2017.
- [21] J. Liu, Z. Feng, Z. Li, M. Wang, and L. R. Wen, "Dynamic Ship Domain Models for Capacity Analysis of Restricted Water Channels," *Journal of Navigation*, vol. 69, pp. 481-503, 2016.
- [22] X. Zhu, H. Xu, and J. Lin, "Domain and Its Model Based on Neural Networks," *Journal of Navigation*, vol. 54, pp. 97-103, 2001.
- [23] Z. Pietrzykowski and J. Uriasz, "The ship domain—a criterion of navigational safety assessment in an open sea area," *Journal of Navigation*, vol. 62, pp. 93-108, 2009.
- [24] N. Wang, "An Intelligent Spatial Collision Risk Based on the Quaternion Ship Domain," *Journal of Navigation*, vol. 63, pp. 733-749, 2010.
- [25] P. Krata and J. Montewka, "Assessment of a critical area for a give-way ship in a collision encounter," *Archives of Transport*, vol. 34, pp. 51-60, 2015.