Reliability Modeling of Phased Mission Multi-State Systems via a Scenario Inference Method

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Abstract: Due to the complexity of phased-mission multi-state systems (PM-MSSs), they enjoy barren research findings despite their universality in the real-world systems. To fill the research vacancy in this area, a novel hierarchical method, failure scenario tree (FST), was proposed in this paper based on failure mechanism dependence. Three kinds of composition logics were adopted according to the characteristics of different levels, including time order, fault order and event order. A case study was utilized to illustrate this method in details and the results showed that the evaluation of system reliability considering multi-state and multi-phase became closer to the engineering practice, which proved the method's effectiveness and availability.

Keywords: Reliability Modeling, PM-MSS, Failure Mechanism Dependence, Failure Scenario Tree.

1. INTRODUCTION

With the increase of system complexity, the coupling and propagation of failure mechanism had become more and more complicated, which increased the difficulty of system reliability modeling. Studying system reliability from this point of view has become a trend and several findings have been achieved in our previous studies [1, 2].

Multi-state system (MSS) is a kind of system in which both the system and its components may exhibit more than two states (or performance levels), ranging from perfect operation to complete failure [3]. Xing and Levitin [4] studied the reliability of MSSs that were subjected to propagated failure in terms of the global effect and the failure isolation effect. Wang et al. [5] studied systems with multiple dependent trigger components that were subjected to competing failure propagation and failure isolation events.

Phased-mission system (PMS) is a system in which multiple non-over-lapping phases or tasks of operations must be accomplished in sequence [6]. Zhang et al. [7] exploited the BDD-based model for reliability analysis of PMSs. Xing and Dugan [8] presented another modified model called TDD (Ternary Decision Diagram) to build models for generalized PMSs with two-level modular imperfect coverage.

Phased-mission multi-state systems (PM-MSSs) contain features of multi-state systems and phased-mission systems simultaneously. Few researches have been done in this area due to the complexity of the system. Li et al. [9] proposed a linear rule of damaging accumulation and a multi-level method based on BDD models for reliability modeling and analysis of PM-MSSs. However, BDD is a static method that hard to characterize the occurrence order of the mechanism and the failure order of the components in the system.

The process of reliability modeling is essentially the process of system failure scenarios inference. From this perspective, a novel hierarchical method, failure scenario tree (FST), is proposed in this paper based on failure mechanism dependence. FST could integrate the occurrence of internal system failures and external events into a tree logically and chronologically, and each sequence reaches a system terminal state with a certain probability. Thus, the system can be modeled from three dimensionalities: logic, time and probability.

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The remainder of this paper is organized as follows. Section 2 presents the representation and calculation of the failure mechanism correlations. Section 3 and 4 show the FST method for MSS and PMS respectively. Section 5 is a case study to illustrate the method in more details step by step. The conclusion of this paper is summarized in section 6.

2. FAILURE MECHANISM DEPENDENCE

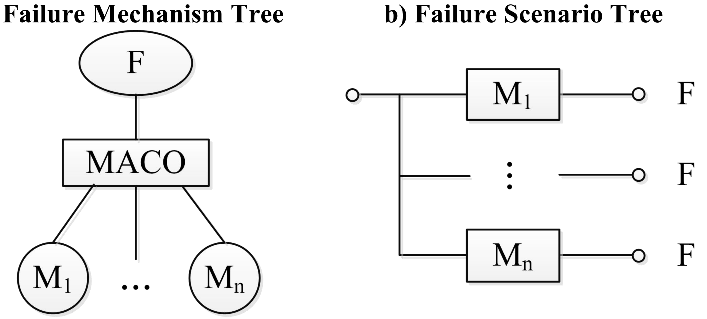
Failure dependence has been extensively treated in reliability modeling for complicated system in aerospace, aviation, naval and nuclear power plants system. For example, a two-component parallel system, when one of the components fails, the stress places on the surviving component will change. Dependent failure will increase joint-failure probabilities, and then reduce system reliability. Thus, for many complicated system, a modeling approach incorporating dependent failure resembles the true system reliability behavior in a more realistic manner. From engineering aspect, there are different types of failure mechanism correlations, including competition, trigger, acceleration, inhibition, and accumulation.

2.1. Competition

Some independent failure mechanisms have different development rates. System failure time will be determined by the failure time of the mechanism which develops to failure first. This process is competition, or these mechanisms have competition correlation. The failure mechanism tree and failure scenario tree of competition correlation are illustrated in Figure 1.

Figure 1: Competition Correlation

a) Failure Mechanism Tree **b) Failure Scenario Tree**



Failure probability of the system can be derived:

$$
\begin{aligned}
F(t) &= P(\zeta \le t) = 1 - P(\zeta \ge t) \\
&= 1 - P(\min\{\zeta_1, \zeta_2, ..., \zeta_n\} \ge t) \\
&= 1 - \prod_{i=1}^{n} P(\zeta_i \ge t) \\
&= 1 - \prod_{i=1}^{n} \left[1 - P(\zeta_i \le t)\right] \\
&= 1 - \prod_{i=1}^{n} \left[1 - \int_0^t f_i(t)dt\right]
\end{aligned} \tag{1}
$$

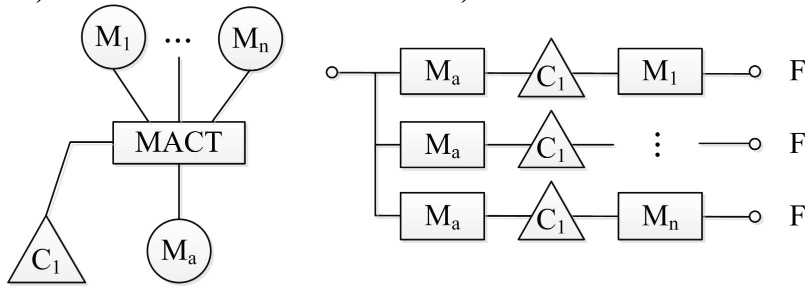
Where, ζ is system lifetime, ζ_i is the time of M_i from initiating to resulting in system failure, and $f_i(t)$ is failure distribution function of M_i.

2.2. Trigger

One failure developing to a certain degree will lead to another or many other failure mechanisms, this type of correlation is called trigger. For example, PCB deformation will trigger the crack of capacitor mounted on it. Trigger correlation can be expressed in Figure 2.

Figure 2: Trigger Correlation

a) Failure Mechanism Tree **b) Failure Scenario Tree**



Failure probability of this system is:

$$
\begin{aligned}
F(t) &= 1 - R(t) \\
&= 1 - P(\zeta > t) \\
&= 1 - P\left(\min\{t_a, T_C + t_1, T_C + t_2, \dots, T_C + t_n\}\right) \\
&= 1 - P\left(t_a > t, T_C + t_1 > t, T_C + t_2 > t, \dots, T_C + t_n > t\right) \\
&= 1 - \left[1 - P(t_a \le t)\right]\prod_{i=1}^{n}\left[1 - P(T_C + t_i \le t)\right] \\
&= 1 - \left[1 - F_a(t)\right]\prod_{i=1}^{n}\left[1 - F_i(t - T_C)\right] \\
&= 1 - \left[1 - \int_0^t f_a(t)dt\right]\prod_{i=1}^{n}\left[1 - \int_0^{t-T_C} f_i(t)dt\right]
\end{aligned}
\tag{2}
$$

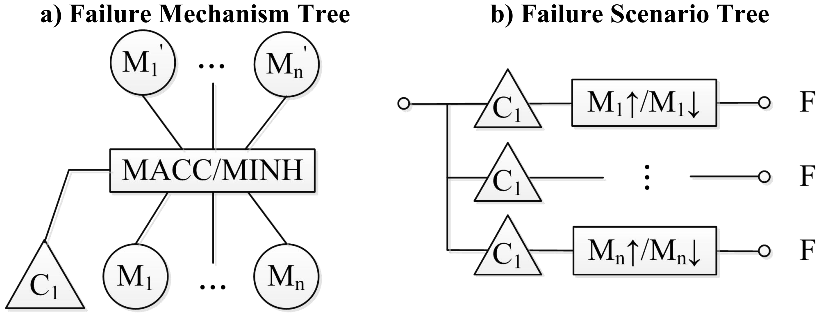
Where T_C is trigger time.

2.3. Acceleration and Inhibition

One failure mechanism developing to a certain degree will accelerate (or inhibit) the development speed of other failure mechanisms, this correlation is called acceleration (or inhibition). For example, heat dissipation of high power chips will accelerate the failure speed of adjacent components. In addition, rubber in high temperature is easy to become soft, which will inhibit embrittlement such as vitrification. Acceleration and inhibition are illustrated in Figure 3.

Figure 3: Acceleration or Inhibition Correlation

a) Failure Mechanism Tree **b) Failure Scenario Tree**



Failure probability of this system is:

$$
\begin{aligned}
F(t) &= P(\zeta \le t) = 1 - P(\zeta \ge t) \\
&= 1 - P(T_C + t_{r1} > t, T_C + t_{r2} > t, \dots, T_C + t_{rn} > t) \\
&= 1 - \prod_{i=1}^{n}\left[1 - F_{ri}(t - T_C)\right] \\
&= 1 - \prod_{i=1}^{n}\left[1 - \int_0^{t-T_C} f_{ri}(t)dt\right]
\end{aligned}
\tag{3}
$$

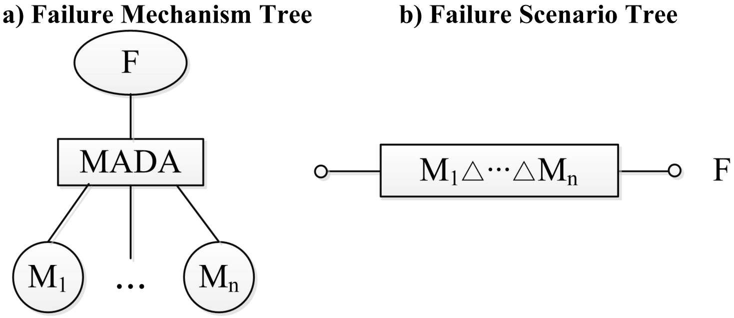
Where $f_{ri}(t)$ is failure distribution function of M_i after acceleration or inhibition.

2.4. Accumulation

Some kind of failure mechanisms may have the same effect on the failure site, component or system. The destructive effect will be accumulated and result in early failure. These mechanisms have accumulation correlation. For example, in electronic interconnection part, both thermal fatigue and vibration fatigue will result in crack of solder joint. Accumulation correlation is shown in Figure 4.

Figure 4: Accumulation Correlation

a) Failure Mechanism Tree **b) Failure Scenario Tree**



The lifetime of system is

$$\begin{aligned}\varsigma &= \frac{X_{th}}{\Delta X} = \frac{X_{th}}{\lambda_1 \Delta X_1 + \ldots + \lambda_i \Delta X_i} \\ &= \frac{X_{th}}{\lambda_1 \frac{X_{th}}{t_1} + \ldots + \lambda_i \frac{X_{th}}{t_i}} \\ &= \frac{1}{\frac{\lambda_1}{t_1} + \ldots + \frac{\lambda_n}{t_n}} = \frac{1}{\sum_{i=1}^{n} \frac{\lambda_i}{t_i}}\end{aligned} \tag{4}$$

Where X_{th} is the threshold of this system due to this kind of damage, ΔX is the accumulated damage in unit time, ΔX_i is the damage in unit time due to M_i, λ_i is a scaling factor of M_i, and t_i is the failure time due to M_i when it works alone.

Then failure probability of the system can be derived:

$$F(\mathrm{t}) = P(\varsigma \le \mathrm{t}) = P\left(\frac{1}{\sum_{i=1}^{n} \frac{\lambda_i}{t_i}} \le \mathrm{t}\right) \tag{5}$$

3. FST FOR MULTI-STATE SYSTEM

As mentioned before, FST could integrate the occurrence of internal system failures and external events into a tree logically and chronologically, and each sequence reaches a system terminal state with a certain probability. It can be seen that this method characterizes the system from three dimensionalities: logic, time and probability. There are two main aspects of the logic here. On the one hand, FST uses logical symbols to indicate fault mechanism dependencies, which had been discussed in the last section. On the other hand, FST needs to select corresponding composition logic according to the system type.

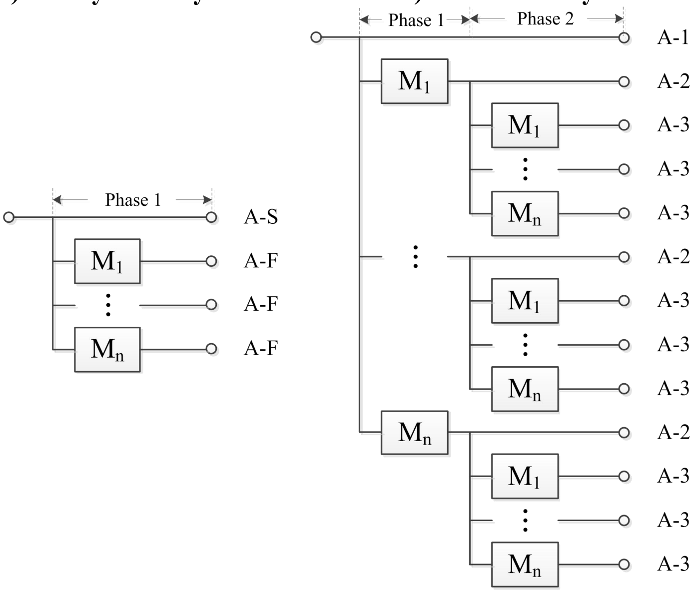
FST is a hierarchical method. The main concerns of multi-state system are: mechanism level, component level, subsystem level, and system terminal state. According to the characteristics of different levels, two kinds of composition logics are adopted, which are time order and fault order respectively.

3.1. Time Order FST

Time order FST, which could be abbreviated to time order scenario tree (TOST), is adopted for modeling at mechanism level. General models for binary-state system and MSS are provided in Figure 5.

Figure 5: TOST Models for Mechanism Level

a) Binary-State System **b) Multi-State System**



For a binary-state system, component A has only one phase from operation to fault. The competition of different mechanisms within the component leads to the emergence of multiple branches. As shown in Figure 5 b), component A has 3 states including operation (state 1), degradation (state 2), and fault (state 3). Each mechanism can lead to a change in the state of each phase. If there is a solid line between two branch points, this means that no mechanism occurs at this stage, or the mechanism does not cause the component to enter the next state. All sequences from the initial node to the final node are collectively referred to as the component's failure scenario.

3.2. Fault Order FST

Fault order FST, also called fault order scenario tree (FOST), is used at component and subsystem level. For some simple systems, subsystem level may be omitted. Here take the three-state series, parallel, and k/n systems in Figure 6 as an example. The corresponding FOST models are shown in Figure 7, where the notation X-*i* means the state *i* of X.

Figure 6: Simple Series, Parallel and, k/n Systems

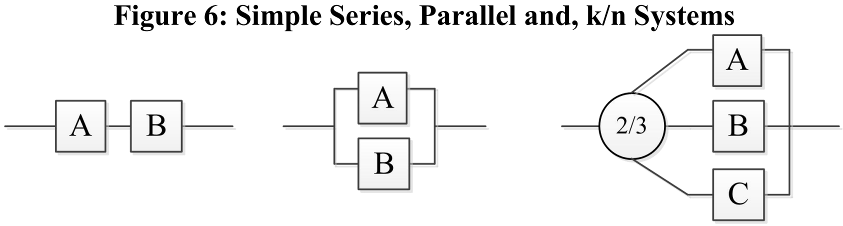
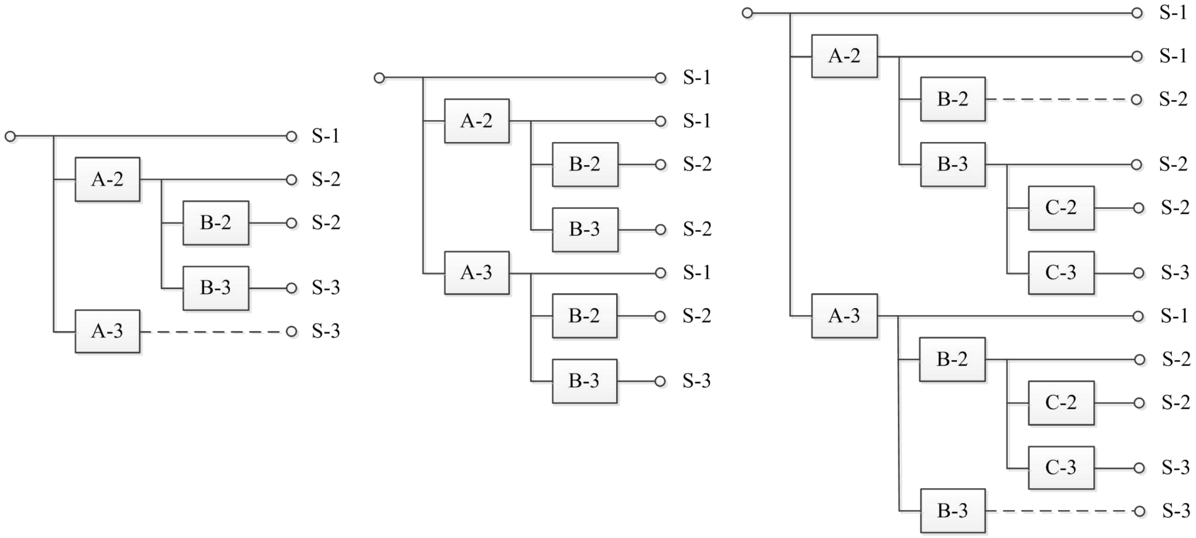


Figure 7: FOST Models for Three-State Series, Parallel, and k/n Systems



Fault order of the components needs to be determined before drawing FOST. Generally speaking, fault probability ranking can be used as the fault order to obtain the most likely failure scenario at certain time *t*. Since the operation state indicates that no fault occurred, it will not appear in the FOST.

At the first branch point, all possible failure states of the first component should be listed, which is also true at all layers after. If any component does not fail, it can be assumed that the subsequent components will not fail too, which is indicated by a solid line. Before completing the analysis of all the components, the state of the subsequent components does not need to be considered if there is a sequence that can directly determine the final state of the system, which is represented by dotted lines, such as sequence A-3 in the series system and sequence A-3, B-3 in the k/n system. The probability of each sequence is the product of the probabilities of all events in the sequence.

In general, both TOST and FOST use the breadth-first traversal search method, and reduce the number of sequences by applying failure mechanism dependences, determining fault orders in advance, and reaching the final states in advance. Then all possible valid and logistic scenarios can be obtained.

4. FST FOR PHASED-MISSION SYSTEM

4.1. Event Order FST

The main concerns of phased-mission system are: phase level, mission level, and total mission terminal state. Event order FST, also called event order scenario tree (EOST), is adopted for modeling at these levels. Figure 9 shows the EOST model of the mission in Figure 8, where the notation Ph*i*-X or MS-X means the system in phase *i* or the current mission is succeeded or failed.

Figure 8: Mission with Two Phases

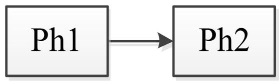
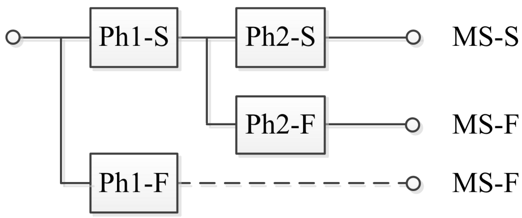


Figure 9: EOST Model for a Mission with Two Phases

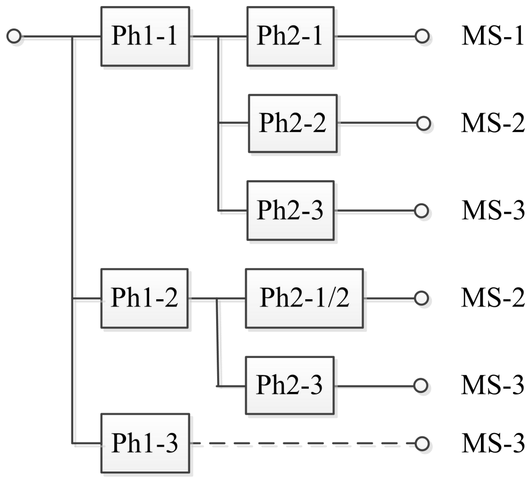


The main characteristic of the phase and mission levels is that the events order is determined. If the previous phase fails, the later phase will not be able to proceed. In EOST, the sequence of all phases succeed would be listed firstly. Then return to the previous branch point to analyze all of its possible branch information. The analogy is taken until the analysis of the first branch point is completed, thereby obtaining all possible failure scenarios. It can be seen that EOST uses the depth-first traversal search method.

4.2. Multi-State EOST

In general, PMS only considers two states situation. However, there will be multiple states at the phase and mission levels in PM-MSS. Then EOST should be upgraded to multi-state EOST (MS-EOST). Take the mission in Figure 8 as an example, and its MS-EOST model is shown in Figure 10, where the notation Ph*i*-*j* or MS-*j* means the system in phase *i* or the current mission is at state *j*.

Figure 10: MS-EOST Model for a Mission with Two Phases



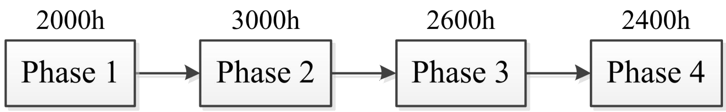
It should be noted that as long as Ph1 is at degradation state (Ph1-2), all non-faulted states of Ph2 (Ph2-1/Ph2-2) do not affect the final state of the mission and can be placed in the same sequence, thereby reducing the number of failure sequences.

5. CASE STUDY

5.1. Description

An electrical system, which is regarded as a PM-MSS, is required to perform a mission with four phases. Appellations of these four phases are simplified as numbers. The performing order and the duration of each phase are shown in Figure 11.

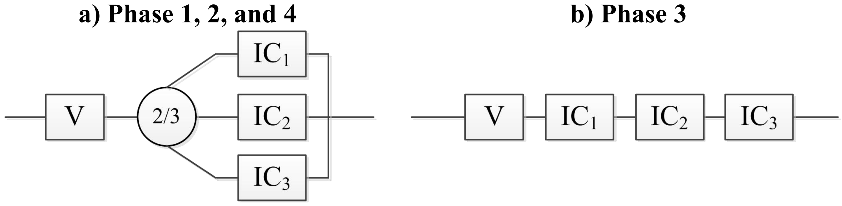
Figure 11: Mission with Four Phases



Different conditions of operating environment and load among different phases lead to the variety of reliability structures of the system, which are shown in Figure 12.

Figure 12: Reliability Structures of the System

a) Phase 1, 2, and 4 **b) Phase 3**



This system is composed of four components, one photocoupler V and three integrated circuits IC_1, IC_2, and IC_3. All components have three states including well-operating state (state 1), damaged state (state 2) and failed state (state 3), resulting in three states for the system (phase) and total mission. The definitions of the system states at each phase and the entire mission states are listed in Table 1.

Failure mechanisms and their correlations of the components are listed in Table 2, where VF is vibration fatigue, TF is thermal fatigue, TDDB is time-dependent dielectric breakdown, NBTI is negative bias temperature instability, and EM is electrical migration.

Table 1: State Definition

	Phase 1, 2, 4	**Phase 3**	**Mission**
State 1	V is in state 1 and at least two of three ICs are in state 1.	All components are in state 1.	All phases are in state 1.
State 2	V is in state 2 and no more than one IC is in state 3. Or V is in state 1 and at least one of the two best functioning ICs among three ICs is in state 2.	At least one of the components is in state 2 and none is in state 3.	At least one of the phases is in state 2 and none is in state 3.
State 3	V is in state 3 or at least two of three ICs are in state 3.	At least one of the components is in state 3.	At least one of the phases is in state 3.

Table 2: Failure Mechanism and Correlation

Component	**Mechanism**	**Correlation**	
V	Crack	Trigger by shock	Competition
	VF	/	
IC_1	TDDB	Accumulation	Competition
	NTBI		
	EM	/	
IC_2	Creep	Acceleration	
	EM		
IC_3	VF	Accumulation	Competition
	TF		
	EM	/	

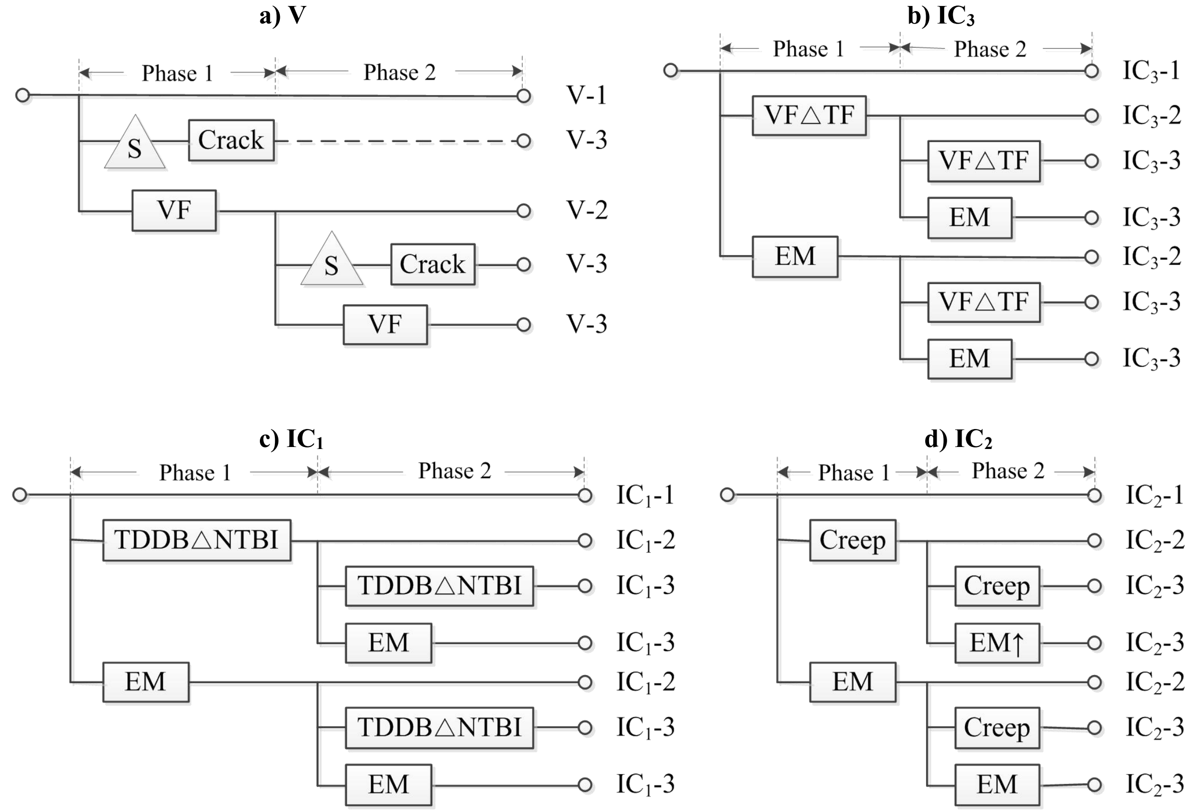
5.2. Modeling Generation

Referring to the proposed method described in the previous sections, the models of this case should be generated from three levels.

- Failure mechanism level

Failure mechanism level is the first level of modeling generation. As mentioned earlier, this level uses TOST method. The models of all components are shown in Figure 13.

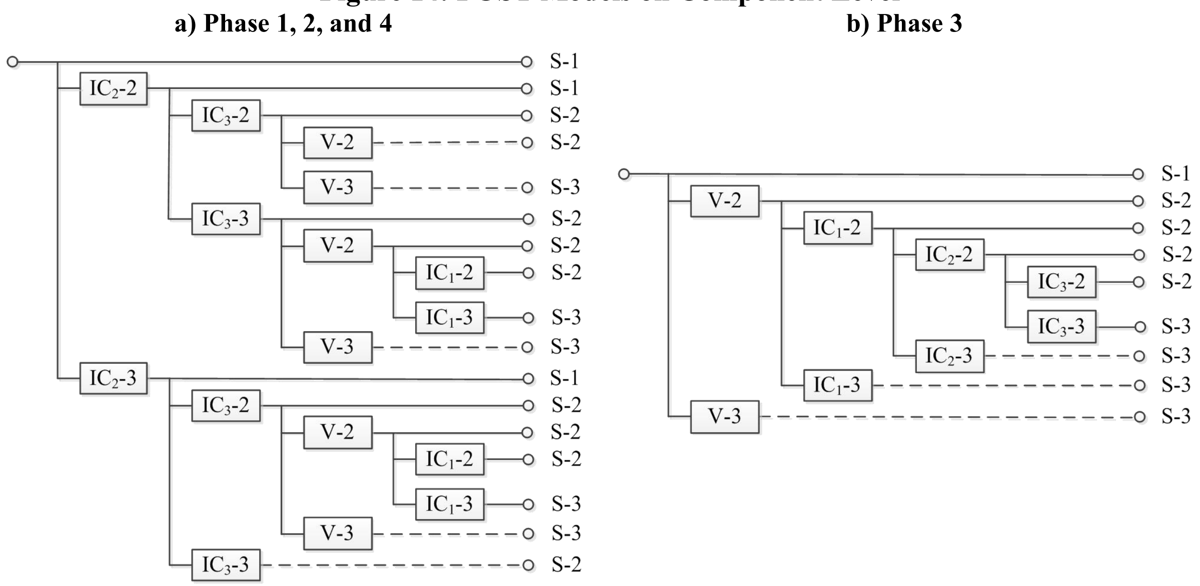
Figure 13: TOST Models on Mechanism Level



- Component level

In the four phases, there are two kinds of systems with different reliability structures that need to be modeled separately. FOST method is used on this level and fault order needs to be determined firstly. Assume that at a certain moment t_1 of phases 1, 2, or 4, the components' fault order based on failure rate (largest to least) is: IC_2, IC_3, V, and IC_1. And at a certain moment t_2 of phases 3, the components' fault order is: V, IC_1, IC_2, and IC_3. According to the state definitions in Table 1, the models are shown as Figure 14.

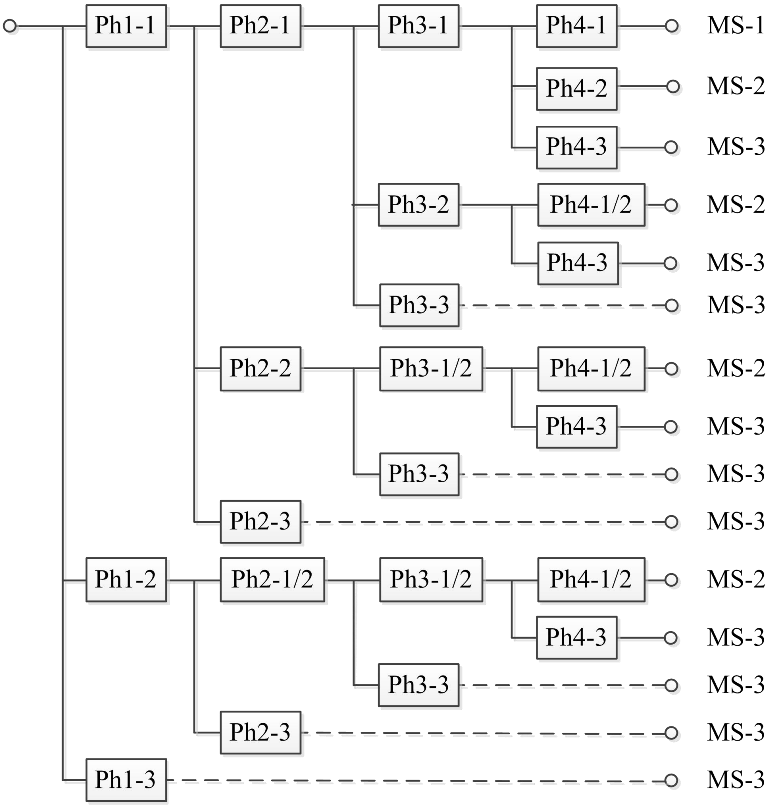
Figure 14: FOST Models on Component Level



- Phase level

MS-EOST method is used on this level for PM-MSS modeling. The reliability model of this level can be obtained as Figure 15.

Figure 15: MS-EOST Model on Phase Level



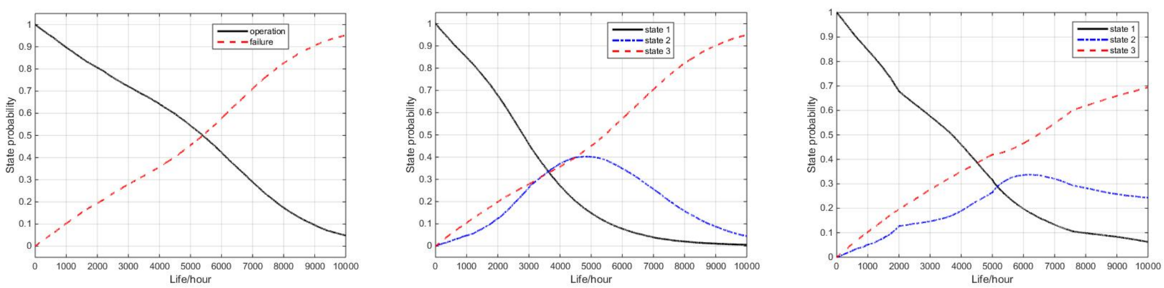
5.3. Simulation Result

Based on the models generated above, some expected simulation results can be obtained after computing level by level. To compare the differences between binary-state and multi-state as well as single-phase and multi-phase, three essential reliability curves are provided after finishing the modeling calculation in this paper.

Figure 16 a) shows the system reliability under binary-state and only performing the phase 1 during 0 to 10000, b) shows the three-state probability when the system only performs the phase 1, while c) is the ultimate results of the system state probability when the system suffers from a phased-mission requirement.

Figure 16: Reliability Curves of the Case System

a) Binary-State and Single-Phase b) Multi-State and Single-Phase c) Multi-State and Multi-Phase



From the comparison between these curves, some findings can be summarized as followings:

- The value of reliability of the binary-state condition is larger than that of the multi-state condition.
- The probability of state 2 is generally increased first and then decreased.
- The sum of all state probabilities at the same time is always equal to 1.
- The state probability curve of a multi-phase system is not as smooth as that under single-phase condition, and an inflection point often occurs when phase changed.
- The evaluation of system reliability and state probability considering multi-state and multi-phase becomes closer to the engineering practice.

6. CONCLUSION

In this paper, a novel hierarchical method, FST, was proposed to solve the problem of reliability modeling and analysis for PM-MSSs based on failure mechanism dependence. Three kinds of composition logics were adopted for different levels due to the complicated characteristic of PM-MSSs. TOST was mainly used on failure mechanism level, FOST was adopted on component and subsystem level, while EOST was applied on phase and mission level and was upgraded to MS-EOST when taking multi-state and multi-phase into account together.

In the last section of this paper, a specific PM-MSS was studied with the proposed method step by step. Three-level models were generated and reliability curves of the case system were obtained as well, which provided more details and proved the availability and effectiveness of this method.

Acknowledgements

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