A Bayesian Analysis of the Risk of Satellite Collisions and of Space Surveillance Improvements

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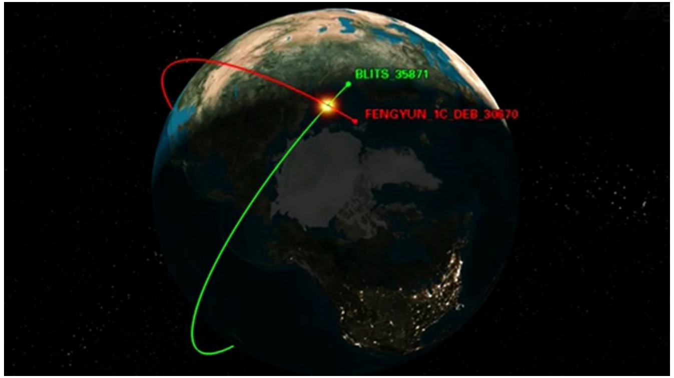
Bottom Line Up Front

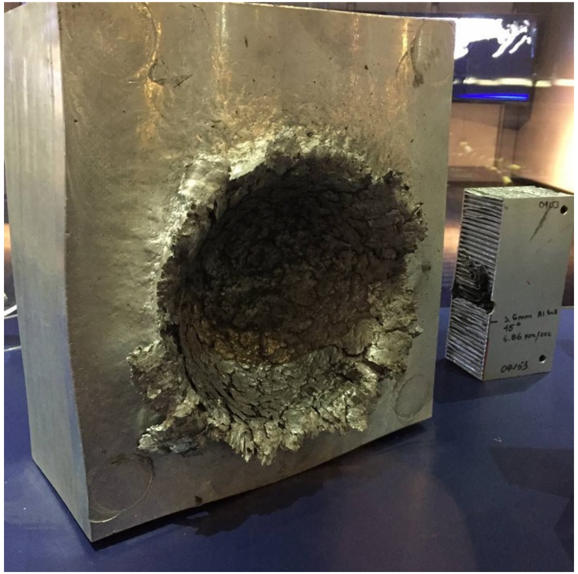
- Space is a heavily-utilized environment
- Satellites have a risk of unintended collisions (debris and meteorites)
- Space surveillance systems provide signals of potential collisions
- We use probabilistic risk analysis to quantify the risk reduction benefit of surveillance systems of various configurations
- Tradeoff: add larger but more expensive, or smaller but cheaper sensors



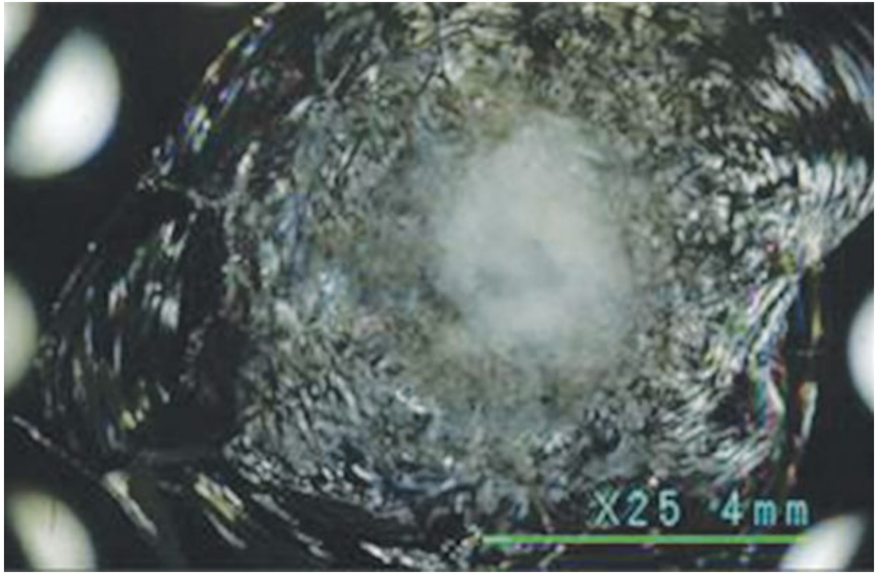
Source: Space.com

Unintentional Space Collisions and Space Surveillance

Space Collision Risks: The Impacts



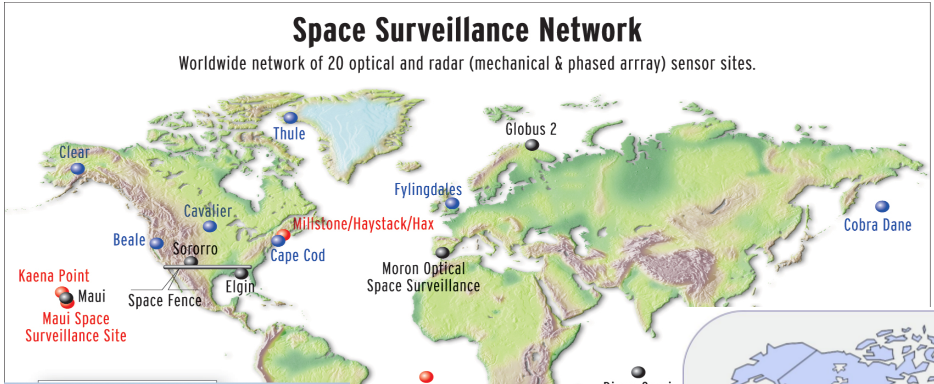
Impact crater from a 12mm plastic projectile, fired onto an aluminum plate at 15,000 mph



Example of window damage to the Space Shuttle Endeavour from debris particle impact

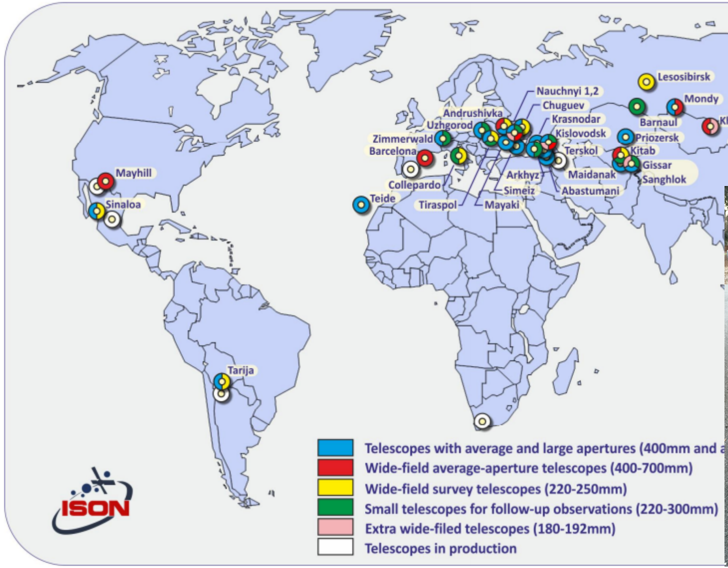
Source: NASA

Space Surveillance Networks



United States Space Surveillance Network (USSSN)
"Type A sensors"

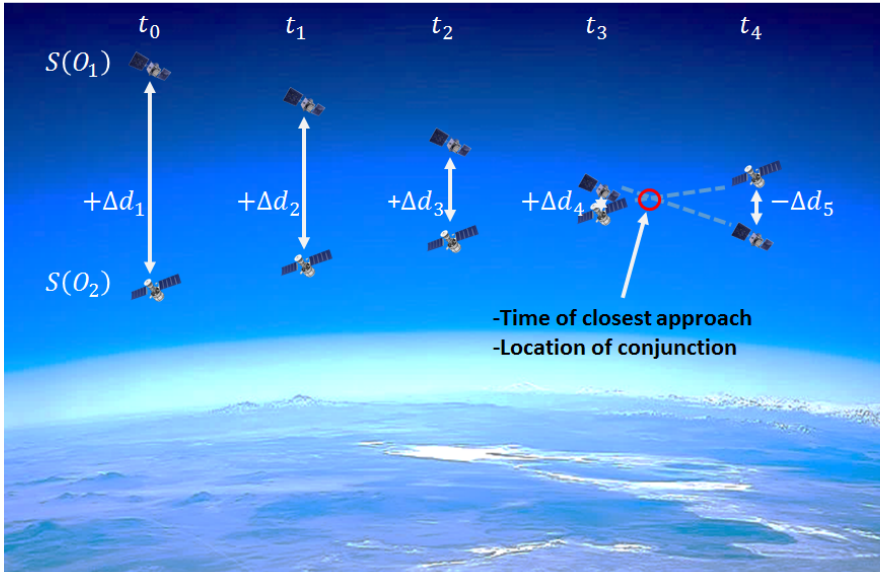
International Scientific Optical Network (ISON)
"Type B sensors"



Sources: SpaceNews, ISON

Conjunction (Collision) Assessment

- Conjunction assessment by interpolating between measured positions of orbiting objects
- Result: estimated location and time of the collision
  - Opportunity for a tactical decision to maneuver or not



Research Questions

- Question 1: What are quantitative measures of the value of information and, risk mitigation of data loss due to collisions, for space surveillance networks?
  - Incorporates two decisions, tactical and strategic, from different decision-makers with different risk attitudes
- Question 2: What is an optimal investment strategy for deploying new sensors, large or small, to minimize failure risk from satellite collision and thus data losses?
  - Fundamentally a strategic decision on how to manage space surveillance systems

Bayesian Model

Bayesian Model for Evaluating Monitoring Systems

- Probability of a true positive signal from A:

$$P("X_{ak}"|X_k) = 1 - P("\bar{X}_{ak}"|X_k) = 1 - P(fn_A)$$

- Probability of a true positive signal from B:

$$P("X_{bk}"|X_k) = 1 - P("\bar{X}_{bk}"|X_k) = 1 - P(fn_B)$$

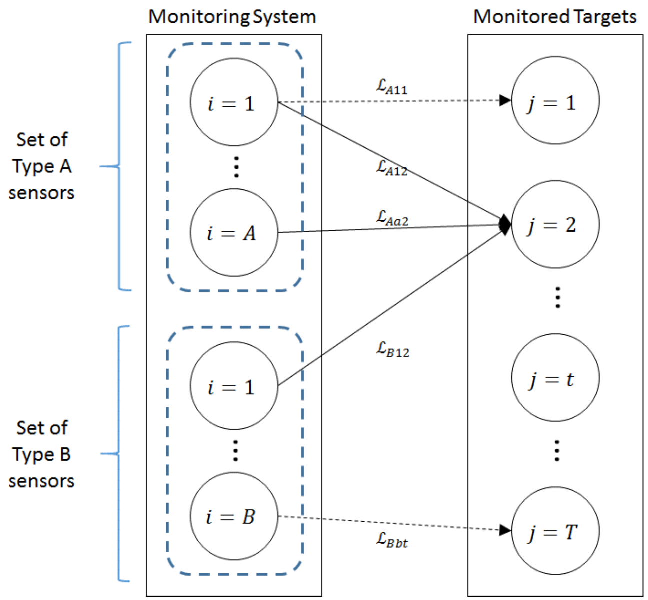
- Observational link function:

$$\mathcal{L}_{Aij} = \begin{cases} 1, & \text{if } \exists \text{ observation link between class } a \text{ and class } t \\ 0, & \text{if } \nexists \text{ observation link between class } a \text{ and class } t \end{cases}$$

$$\mathcal{L}_{Bij} = \begin{cases} 1, & \text{if } \exists \text{ observation link between class } b \text{ and class } t \\ 0, & \text{if } \nexists \text{ observation link between class } b \text{ and class } t \end{cases}$$

- Numbers of sensors monitoring targets C_{ik} from A:

$$N_{Ai} = \sum_{j=1}^{N_j} L_{Aij},\ N_{Bi} = \sum_{j=1}^{N_j} L_{Bij}$$



Posteriors for Failure

- General form of the posterior probability per time unit of an unintended collision is

$$P(collision|\{m\}) = \frac{P(collision) \times P(\{m\}|collision)}{P(\{m\})}$$

prior probability

likelihood function

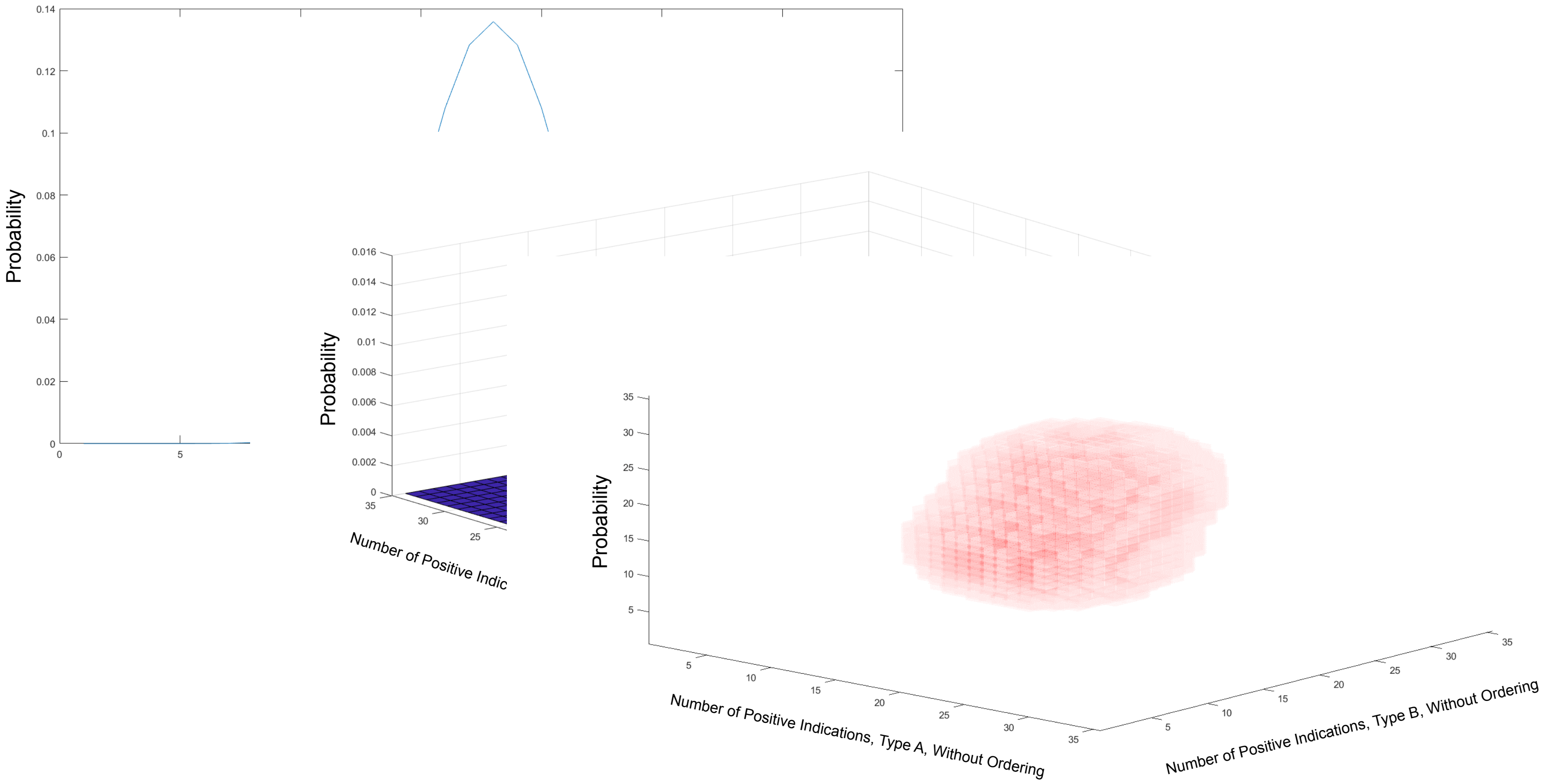
preposterior

for all sensors in A and B that observe satellite S_{ik}

- Posterior probability of losing a constellation and risk of data losses

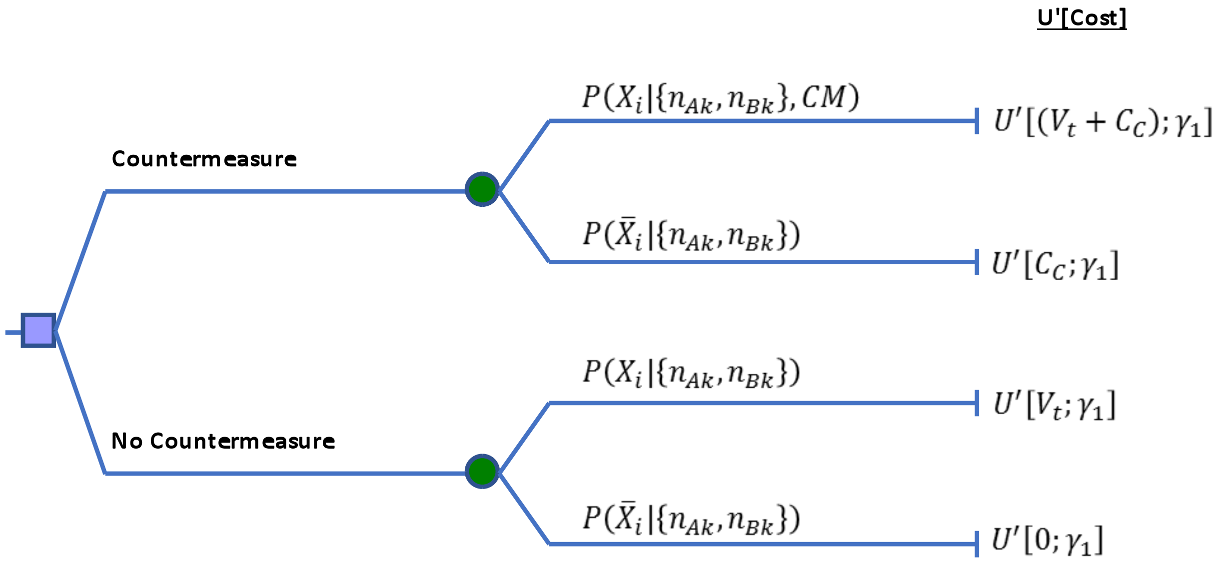
$$P(X_i|\{m\}) = \binom{N_i}{NF_i} \times P'(X_{ik})^{NF_i} \times \left(1 - P'(X_{ik})\right)^{N_i - NF_i}$$

Preposteriors for Many Sensor Types



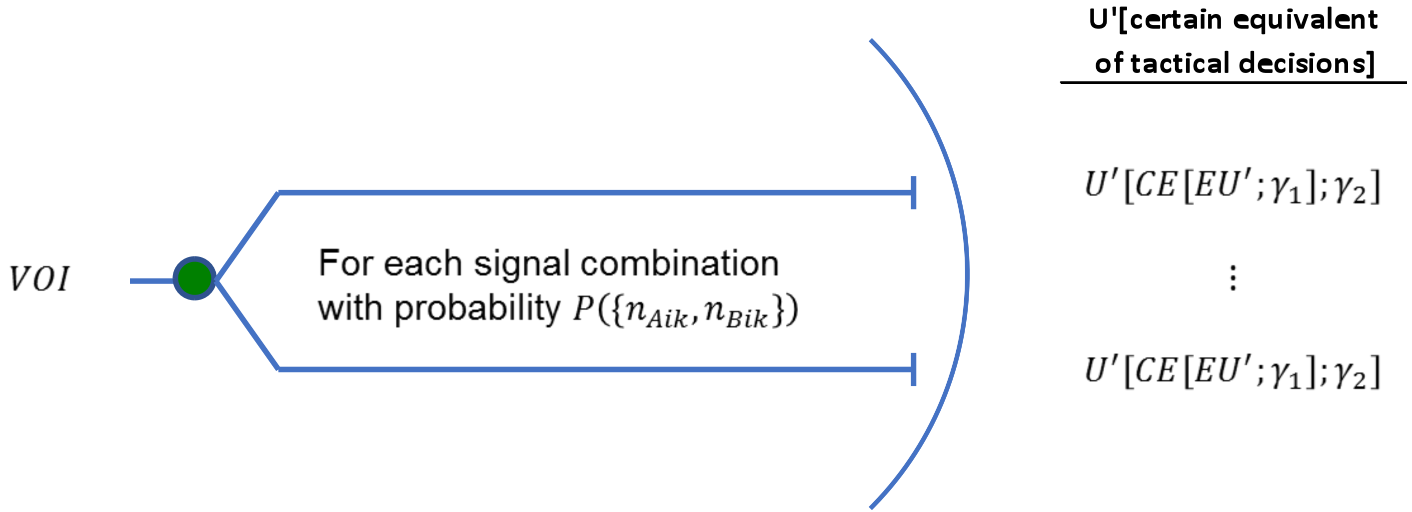
Value of Information for Satellite System

- Value of information is a reduction of the certain equivalent of the costs/ losses associated with a move countermeasure



Making Strategic Decisions

- Certain equivalent associated with the base monitoring system is



Alternatives: Addition of Large (Type A) or Small (Type B) Sensors

- Added benefit of modifying the baseline monitoring system with additional Type A or Type B sensors

$$Ben[A] = VOI_A - VOI$$

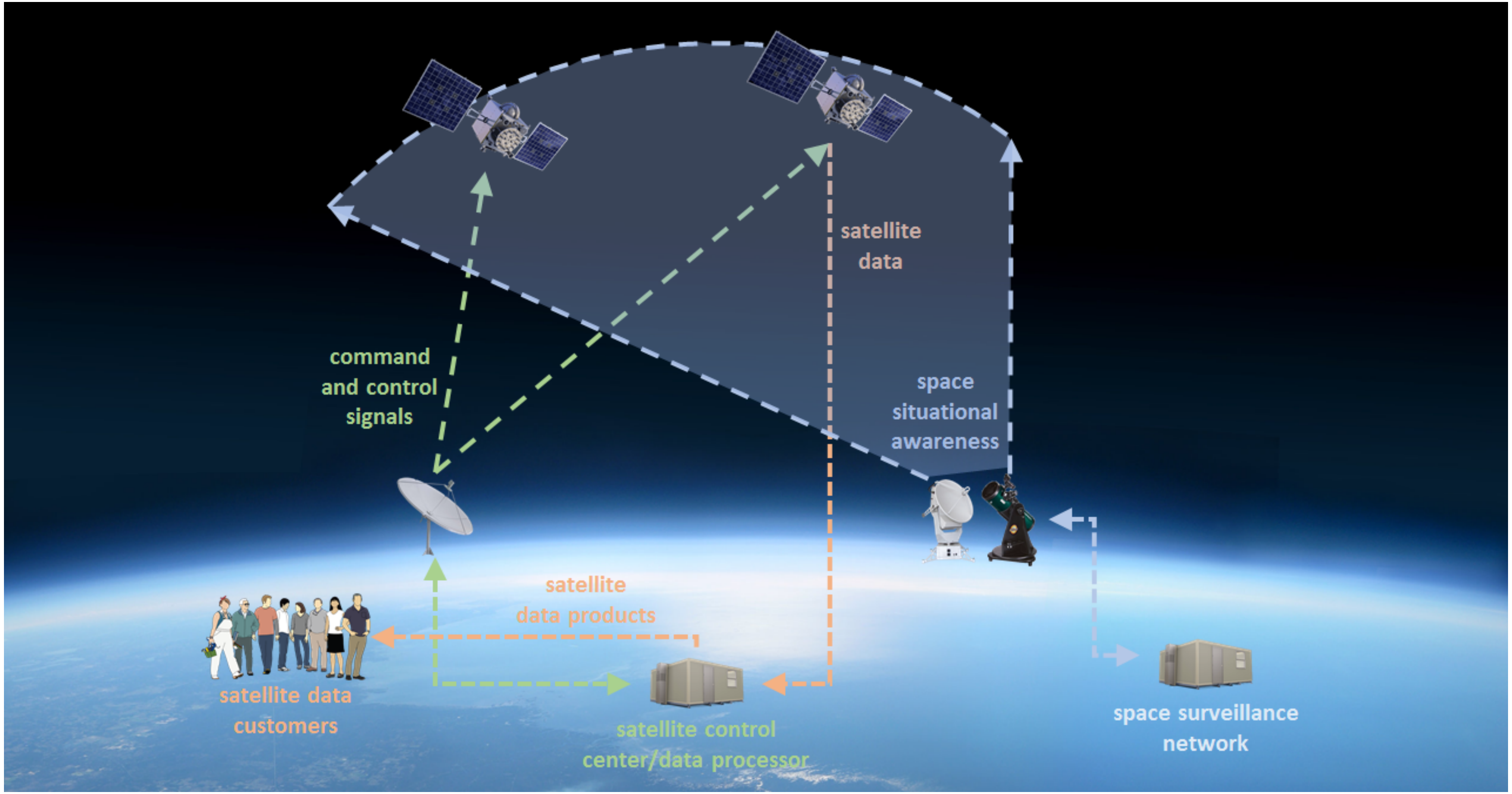
$$Ben[B] = VOI_B - VOI$$

- Strategic decision for optimal policy given by:

$$max(Ben[A], Ben[B])$$

Bayesian Model, Applied to Space Surveillance Systems

Model Applied to Space Surveillance Systems

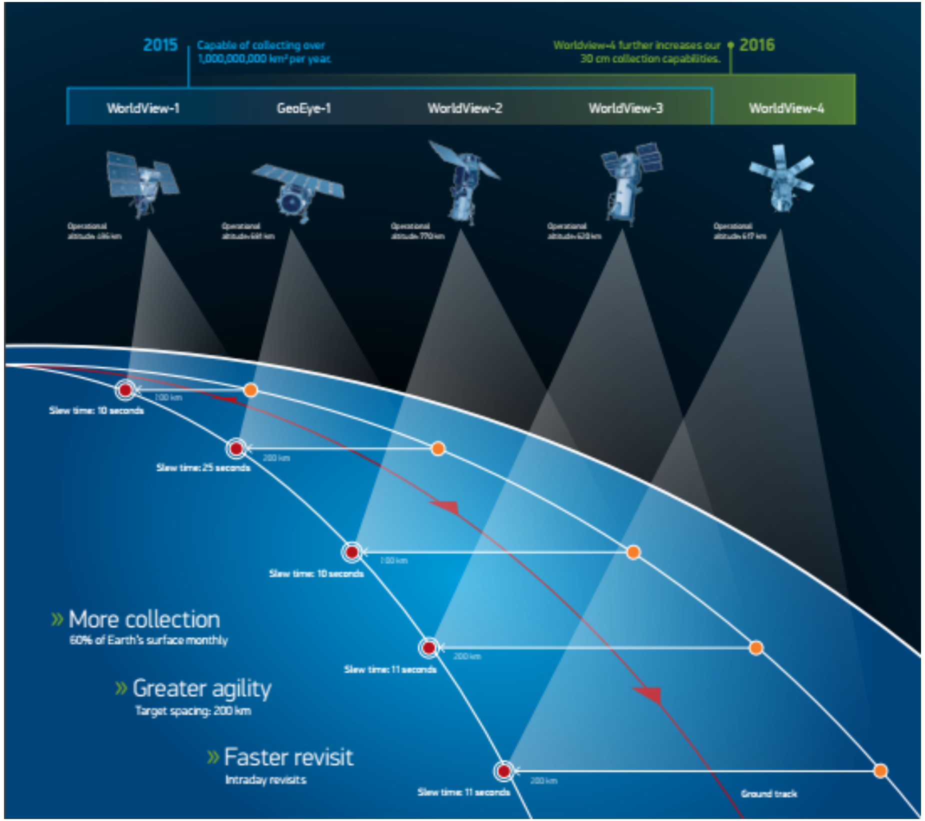


The DigitalGlobe System

Source: DigitalGlobe

Satellite Name	Mass (kg)	Perigee (km)	Ecc.	Inc. (deg)	Period (min)
GeoEye-1	1,955	671	1.06E-03	98.1	98.3
Worldview 1	4,500	491	2.19E-04	97.3	94.5
Worldview 2	2,800	765	1.40E-04	98.5	100.2
Worldview 3	2,800	612	1.43E-04	98.0	96.9
Worldview 4	2,485	617	2.15E-04	98.0	96.9

Summary of DigitalGlobe constellation



DigitalGlobe architectural overview

Inputs to Model, Space Surveillance of DigitalGlobe (Illustrative Numbers)

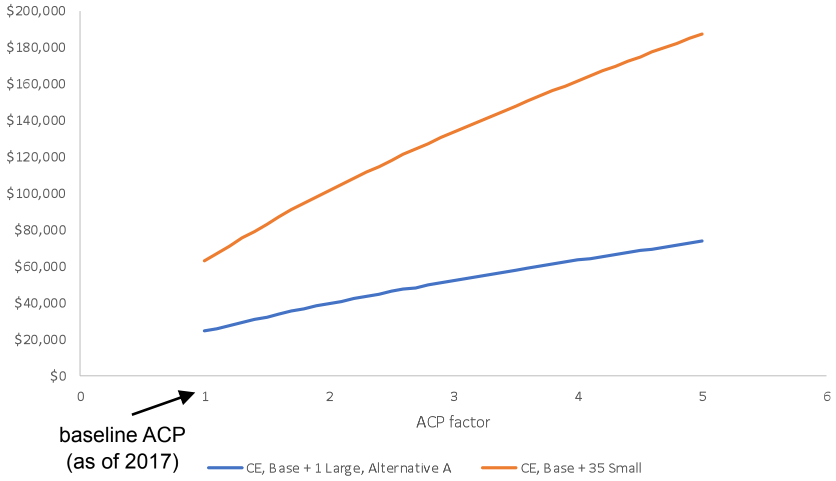
Sensor Class	True Pos	False Neg	False Pos	True Neg
Large SSN sensors observing LEO	0.8	0.2	0.2	0.8
Small SSN sensors observing LEO	0.6	0.4	0.4	0.6
Additional large sensors	0.8	0.2	0.2	0.8
Additional small sensors	0.6	0.4	0.4	0.6

Probabilities of collision detection, by sensor class

Illustrative Risk Analysis Results*, Monitoring of DigitalGlobe

Surveillance Architecture	Base certain equivalent, w/o monitoring	Posterior certain equivalent, with monitoring	Value
SSN + 1 Large Sensor	−$421,885	−$399,680	$22,204
SSN + 35 Small Sensors	−$421,885	−$355,271	$66,613

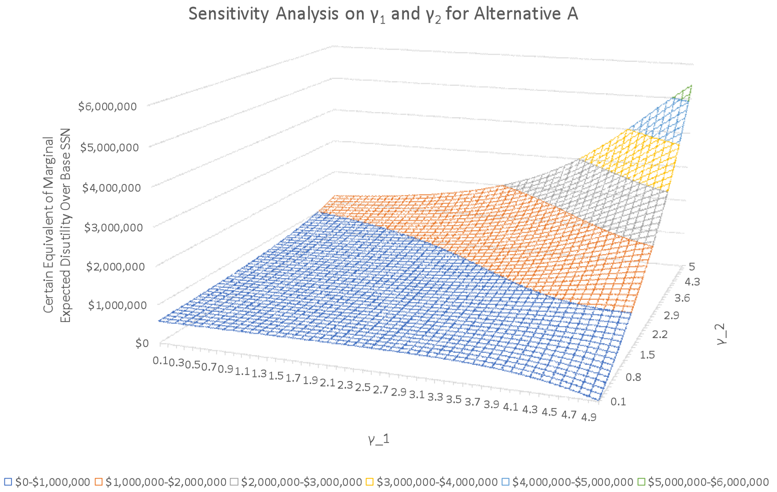
Summary of risk analysis, base SSN and hypothetical modifications, for DigitalGlobe satellite systems



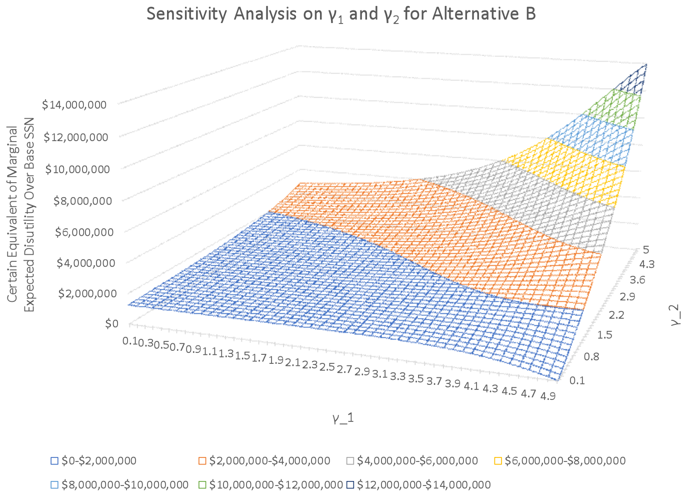
Risk analysis of base SSN and hypothetical modifications monitoring DigitalGlobe, repeated over a range of annual collision probabilities, from current levels (ACP factor = 1) to a five-fold increase over current levels

*assuming risk-neutral decision-makers

Sensitivity Analysis on Risk Attitude



Sensitivity analysis for Alternative A, varying γ_1 and γ_2 simultaneously over a range of 0 to 5



Sensitivity analysis for Alternative B, varying γ_1 and γ_2 simultaneously over a range of 0 to 5

Summary

Summary

- Development of a nodal framework for space monitoring systems
  - Rational decision-making for tactical and strategic decisions
  - Better characterization of value and risk of monitoring systems
  - Basis for "apples to apples" comparison of monitoring systems
- Proposal of a value framework that points to sources of value external to the existing monitoring system
  - Avoids self-pointing measures, which can be misleading
- Application of risk analysis model to space surveillance
  - Current analysis of space surveillance systems have almost no quantitative rigor

Thank You

Questions?