Risk Analysis and Decision Theory: An Extended Summary

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Abstract: We reconcile Kaplan and Garrick's seminal definition of risk with classical subjective expected utility, filling in the relevant gaps and providing a framework that is ready-to-use in applications. We show that Kaplan and Garrick's "frequency" format can be set in one-to-one correspondence with [26]'s utility theory. Kaplan and Garrick's "probability" format corresponds to the framework of [22] in which epistemic uncertainty is captured by a subjective probability over uncertain events. Finally, Kaplan and Garrick's "probability of frequency" format, the most general one, corresponds to the recently proposed framework of [13], which distinguishes aleatory and epistemic uncertainty in a Bayesian perspective. The classic Kaplan and Garrick's risk triplets are then cast in the powerful setting of axiomatic Decision Theory, with its solid behavioral foundations, allowing one to make explicit the often implicit decisions of a Risk Analysis.

Keywords: Decision Analysis, Risk Analysis, Decision Theory, Uncertainty Analysis.

1. INTRODUCTION

In the management of complex technological systems, the term risk analysis refers to the part of the policy-making process associated with the identification of scenarios and their likelihoods [14,20]. The location of a nuclear waste repository [17], the programming of a space mission [10,16,24], and the evaluation of design-changes in chemical and nuclear plants [11,12] are a few examples in which decision-making is informed by a risk analysis, in the so-called risk-informed decision-making [3]. This discipline has gained a significant amount of attention from both policymakers and the public over the past 30 years, as the interaction of technology and policy choices has become more predominant in the evaluation of trade-offs in a democratic society [4, p. 621].[1]

Over the years, Kaplan and Garrick's definition of risk [19] has become one of the pillars of risk analysis, guiding several key studies performed by national and international agencies and laboratories (for instance, Kaplan and Garrick's risk triplets are a structural part of NASA's recent risk management handbook, see, for example, [15]). From a theoretical viewpoint, the triplet structure introduced by Kaplan and Garrick remains in recent generalizations of the risk concept [1,6,7,8]. These works signal a common trait of risk analysis, that is, the consideration of risk as a self-standing concept, apart from an underlying decision-analytical background. This separation is considered attractive by some researchers (see the debates reported in [5]), insofar it permits the extension of Kaplan and Garrick's definition of risk to non-probabilistic approaches.[2] However, it has the drawback of vanishing the normative support that an underlying decision-analytical rationale brings to a risk analysis.

Indeed, in their seminal 1981's article Kaplan and Garrick maintain that risk must thus be considered always within a decision theory context [19, p. 25]. Several subsequent works discuss risk analysis from a decision-making viewpoint [2,4,18,28]. Both [20] and [14] underline that risk analysis and

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[1] For an early and critical review about risks and benefits of technological systems, we refer to [25]).
[2] The problem was already clear at the time of Kaplan and Garrick. As they state: *one often hears people say that we cannot use probability because we have insufficient data, in light of our current definitions, we see that this is a misunderstanding. When one has insufficient data, there is nothing else one can do but use probability* [19].

decision analysis are intertwined: a decision analysis can include a risk analysis component [20, p. 220]. Nonetheless, the decision-analytical background upholding Kaplan and Garrick's definition itself has not been investigated in depth to date.

This gap in risk analysis leads us to the decision side. Subjective expected utility originates in the seminal works of [26], [27], and [22], which have become the pillars of modern decision analysis [21,23]. This theory features a decision maker (DM) that evaluates acts whose consequences depend on states of the environment generated by mechanisms that are only partially known or understood. Each such mechanism corresponds to a probabilistic model that describes the frequency of the various states inherent to the phenomenon at hand. The information available to the DM allows her to posit a set of possible mechanisms, that is, of possible probabilistic models. In general, such set is not a singleton because information is not sufficiently accurate to pin down a single mechanism. In other words, the DM is uncertain about the true probability model. Thus, one considers two layers of uncertainty as follows [2]: the irreducible aleatory uncertainty (physical risk) about states, described by probabilistic models, and epistemic uncertainty about such models characterized by a prior probability over them. [13] show that, if the DM's preferences satisfy [22]'s axioms plus a consistency condition, then one obtains a subjective expected utility functional where the distinction of the two layers becomes meaningful.

The heart of the present work is the reconciliation of Kaplan and Garrick's definition of risk in its various aspects with the corresponding decision-theoretical rationales. We show that the notion of scenario in Kaplan and Garrick's risk triplets is in one-to-one correspondence with the decision-theoretical notion of event. We then show that Kaplan and Garrick's "frequency" format can be embedded in the [26] decision-theoretic framework and that Kaplan and Garrick's "probability" format is in correspondence with [22]'s expected utility framework, in which uncertainty is described by a subjective probability. Finally, we show that the "probability of frequency" format, where the aforementioned two layer distinction is applied, finds its natural axiomatic collocation within the recently proposed extension of subjective expected utility of [13]. As a side finding, [2]'s concept of unconditional model of the world finds its correspondence in the decision-theoretical notion of probabilistic reduction.

We illustrate the discussion through several examples. In particular, starting from seismic probabilistic risk assessment, we embed the three levels of Probabilistic Safety Assessment (PSA) in the decision-theoretical setup. Our numerical application concerns decision-making in space PSA. Since the late '90s, NASA uses probabilistic risk assessment in all of its programs and projects to support optimal management decision for the improvement of safety and program performance [24, p. 11]. In evaluating space missions, two consequences are typically considered, loss of crew and loss of mission. We address the decision-analytical rationale supporting the formulation of the problem in terms of an acceptability threshold on the probability of loss of crew (loss of mission), providing the overarching framework to the approach used in current practice.

For more details, we refer the interested reader to the working paper version of this paper [9], which will be soon available on the IGIER website.

2. CONCLUSION

Several applications foresee a risk analysis that supports an overall decision analysis problem. One of the cornerstones of risk analysis is represented by Kaplan and Garrick's risk triplets and their quantitative definition of risk. In this work, we have addressed what is the decision-theoretical rationale that supports the use of these risk triplets, in consideration of the well-known distinction between aleatory and epistemic uncertainty. We have seen that Kaplan and Garrick's framework can be set in one-to-one correspondence with suitable decision-theoretic rationales and their more general format is encompassed by the Classical Subjective Expected Utility framework. The work provides an approach ready to use in application, which is illustrated in [9] through the use of several case studies, among which the decision-process associated with the planning of a space mission.

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