A Jointly Optimization of Production, Delivery and Maintenance planning for multi-warehouse/muli-delivery problem

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Abstract: This paper develops à jointly optimization problem in order to establish an optimal production, delivery and maintenance strategy for a manufacturing system subjected to a random failure. The problem consists on several warehouses allow to satisfy random demands during a finite horizon, under service level. In order to assure an economical objective, we have determined the optimal production/maintenance plan and the economically delivery quantities plan considering the delivery time for each warehouse. The aim of the proposed approach is to show a jointly production/maintenance/delivery optimization, with a constrained stochastic production-delivery-maintenance planning problem under hypotheses of service level, delivery time for each warehouse and failure rate, which minimizes the total production, inventory, delivery and maintenance costs. A numerical example confirms the analytical results

Keywords: Delivery time, failure rate, random demand, service level, multi-warehouse.

1. INTRODUCTION

Throughout the last two decades, the determination of an optimal maintenance and production plan which minimizes the total cost including production, inventory and maintenance is one of the first actions of a hierarchical decision making process. This plan allows to finding the optimal production plan and maintenance strategy required by the company to manufacture their products which satisfy a random demand over future periods. [6] developed the linear decision rule and which has considered an important contribution for production planning decisions. This analytical rule determine the optimal solution for aggregate inventory, production and workforce levels by minimizing the quadratic cost functions subject to inventory and workforce balance equations. However, this work considers a production/inventory problem and not maintenance. [2] is among the first authors who studied the problem of integrated production and maintenance strategies. He studied the role of buffer stocks in increasing the system productivity. Also, [1,3,11] studied the strategy of a safety stock building to meet demands during periods of production interruption due to maintenance actions. A joint optimization of strategic stock and age type maintenance policy is proposed by [5]. In reality, the failure rate increases with time and the use of the equipment. Besides, in the literature, this phenomenon it’s rarely considered, except [7] who discussed the conditions of optimality of the hedging point policy for production systems in which the failure rate of machines depends on the production rate. Indeed, most of the researchers focus on a perfect manufacturing system and a perfect service level, and do not present the effect of the service level and the proportion of defective items on relevant performance measures and costs. Recently, [8, 9] studied a randomly failing manufacturing system which has to satisfy a random demand during a finite horizon given a required service level. More recently, [13] determined simultaneously the economical production planning, optimal maintenance strategy and the optimal delivery plan taken into account the delivery time, machine failures , random demand and withdrawal right to minimize the sum of inventory, maintenance, production and transportation costs. The authors studied the impact of delivery time and withdrawal right on optimal production/maintenance planning and transported quantity. Indeed, delivery time and transported quantity are important characteristics of manufacturing systems. Thus many manufacturers are working to reduce transportation delays such as the delivery time, which is the period of time that the part takes between a manufacturing store and a purchase warehouse (customer), and which usually has great impact on performance measures. studied the impact of delivery time on the optimal buffer level take into account to the machine failures and random demands, [4] presented a model for supply planning under lead time uncertainty and proposed a method to determine the optimal value of the planned lead time under lead time uncertainty. [10] developed a model for supporting investment strategies about inventory and preventive maintenance in an imperfect production system take into account the delivery time to the customer. In recent years, another important characteristic of transport is considered in the manufacturing system study such as transported quantity, which is the quantity of parts to be transported between the manufacturing store and the purchase warehouse [12]. However, this above works studied systems with single warehouse. In this paper we study a more complicate system consists on several warehouses allow to satisfy random demands. The originality of this work is that determined the optimal production/maintenance plan and the economically delivery quantities plan considering the delivery time for each warehouse. Indeed, in order to respect the service level, each warehouse should contain enough parts for satisfying customer demands. Thus, an optimal planning of the transported quantity between the manufacturing store and purchase warehouses should be determined based on the relationship with the production/maintenance planning and the service level.

The objective of this paper is to determine simultaneously the economical production planning, optimal maintenance strategy and the optimal delivery plan for different warehouses taken into account the delivery time, random demand to minimize the total cost of inventory, maintenance, production and transportation costs.

This remainder of this paper is organized as follows: Section 2 presents and formulates a general stochastic production, delivery and maintenance problem with the different production and maintenance policies. Section 3 develops the analytical expression of production and maintenance policies considering the influence of the delivery time on the production, delivery and maintenance plans. A simple numerical example is presented in section 4. Finally, the conclusion is given in Section 5.

2. PRODUCTION/MAINTENANCE/DELIVERY PROBLEM

2.1. Notation

The following parameters are used in the mathematical formulation of the model:

τ_i: delivery time for warehouse S_i
L: number of warehouse
Δt: length of a production period
H : number of production periods in the planning horizon
$H.\Delta t$: length of the finite planning horizon
$u(k)$: production rate of machine M during period k (k=0, 1,…, H-1)
$U=\{u(0), u(1), ..., u(H-1)\}$
$Q_i(k)$: delivery rate during period k (k=0, 1,…, H-1) for each warehouse
$Q_i=\{Q_i(0), Q_i(1), ..., Q_i(H-1)\}$
$\hat{d}_i(k)$: average demand during period k (k=0, 1,…, H) for each customer
$V_{di(k)}$: variance of demand during period k (k=0, 1,…, H) for each customer
$S(k)$: inventory level of S at the end of period k (k=0, 1,…, H)
$S_i(k)$: inventory level of S_i (i:0…L) at the end of period k (k=0, 1,…, H) for each warehouse
C_p: unit production cost of machine M
Cs: inventory holding cost of one product unit during one period at the first store S.
Cs_i: inventory holding cost of one product unit during one period at the warehouse S_i (i:0…L)
Cl: delivery cost
Q_v: delivery vehicle capacity
C_M: total maintenance cost
C_{pm}: preventive maintenance action cost

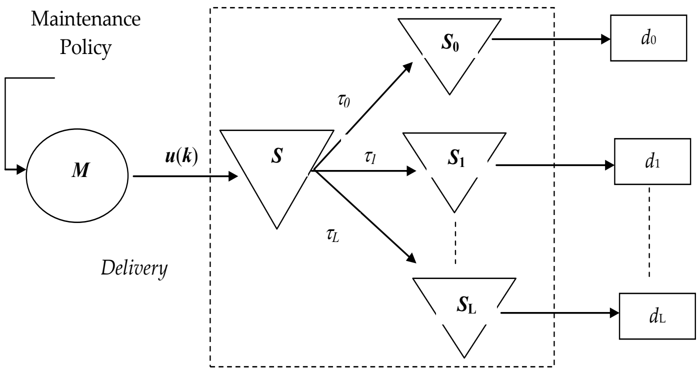
C_{cm}: corrective maintenance action cost
mu: monetary unit
U_{max}: maximal production rate of machine M
U_{min}: minimal production rate of machine M
θ_i: probability index related to each customer i satisfaction and expressing the service level.

2.2. Problem description

In this section, a joint optimization of production, delivery and maintenance planning problem is presented. In this proposed model, we assumed that the manufacturing system consists a single machine M which produces one type of product, a principal manufacturing store S (where the manufactured products are stored) and multi purchase warehouse (S_0, S_1,….S_L) (where the customer receives his demand (products)). Each warehouse aims to satisfy a several random demand under a given service level $\theta_{i,}$ over a finite horizon H. Hence the delivery time considered between the principal store S and each warehouse S_i (i=0,…,L), denoted by τ_i (i=0,…,L) (see figure1). In other words, if we suppose that the products leave S at period k, they will arrive at the period $k+\tau$ to the warehouse. Also, the products preparing for delivery is characterized by a cost called order preparation cost.

The machine M is subject to a random failure. The degradation degree of machine is influenced by the production rates, consequently, the failure rate $\lambda(t)$ increases with time and production rate $u(t)$.
Our objective lies in establishing the best production, delivery and maintenance strategy. To do this conjugated production, delivery and maintenance optimization, we minimize the sum of the inventory costs at the different stores, the manufacturing and delivery costs along with the costs associated with the maintenance strategy.

Figure 1: Problem description



2.3. Problem formulation

Aiming at organizing and optimizing the production system, our objective is to minimize the expected costs related to production, inventory, delivery and maintenance over the finite time horizon. It's assumed that the horizon is portioned equally into H periods with a length equal to Δt. Moreover, we assume that the fluctuation of the demands is a normal process with mean and variance given respectively by $\hat{d}_i$ and V_{d_i} and the demands are satisfied at the end of each period.
The stochastic problem as follows:

$$\underset{(U,Q,N)}{Min}\left(\left\{\sum_{k=0}^{H-1} f_k\left(S(k),S_0(k),....,S_L(k),u(k),Q_0(k),Q_1(k),.....,Q_L(k)\right)\right\}+\left\{\Gamma_M(U,N)\right\}\right) \quad (1)$$

Where

$f_k(.)$ denotes the expected production, inventory and delivery costs of each production period. $\Gamma_M(.)$ the total cost of preventive and corrective maintenance actions.

Subject to

The inventory balance equation level of the principal store S is given by the following equation:

$$S(k+1)=S(k)+u(k)-\sum_{i=1}^{L} Q_i(k) \quad (2)$$

(k=0, *1,…, H-1)*

The inventory level of each warehouse inventory S_i at the period $k+1$ is given by the following equation:

$$S_i(k+1)=S_i(k)+Q_i(k-\tau_i)-d_i(k) \quad (3)$$

(k=0, *1,…, H-1)* and *(i=1,…, L)*

The inventory level of each warehouse S_i at the period $k+1$ equals to the inventory level of S_i at period k plus the rate of products that arrives to S_i (i.e. $Q_i(k-\tau)$) minus the customer demand d_i at period k .

The service level requirement constraint for each warehouse at each period k is expressed by the following constraint.

$$\text{Prob}\left[S_i(k+1)\geq 0\right]\geq\theta_i$$

(k=0, *1,…, H-1)* and *(i=1,…, L)* (4)

The following constraint defines an upper and lower bounds on the production level during each period k.

$$U_{\min}\leq u(k)\leq U_{\max} \quad (5)$$

2.4. Production/Delivery/Maintenance policies

In this section, we represent a constrained production/delivery/Maintenance problem under service level, delivery time, and random demand using a HMMS model.

Inspired from the HMMS model, we had the idea to make emphasis on production rate, inventory levels and delivery rates in order to make the best and optimal production planning and maintenance strategy. Also, in our work, we make some changes on the model keeping its linear quadratic form. Furthermore, we take into account some constraints on the decision variables to make our approach more realistic and to ensure its applicability in real industrial cases.

2.2.1. Production/Delivery Policy:

The idea of the proposed model is the use of a quadratic cost function allows penalizing both excess and shortage in the inventory level.

The expected cost including production and holding costs for the period k is given by:

$$f_k(S(k),S_{i(i:0....L)}(k),u(k),Q(k)) = f_{u(k)}(u(k)) + f_{s(k)}(S(k),S_0(k),........,S_L(k)) + f_{Q(k)}(Q(k)) \quad (6)$$

Where the expected production cost for period k

$$f_{u(k)}(u(k)) = C_p \times E\left\{u(k)^2\right\} \quad (7)$$

The expected holding costs of period k

$$f_{s(k)}(S(k),S_0(k),........,S_L(k)) = C_s \times \left(E\left\{S(k)^2\right\}\right) + \sum_{i=1}^{L} C_{s_i} \times \left(E\left\{S_i(k)^2\right\}\right) \quad (8)$$

The expected transported cost for period k

$$f_{Q(k)}(Q_i(k)) = C_l \times \left(\sum_{i=0}^{i=L} E\left\{\left(\frac{Q_i(k)}{Q_v}\right)^2\right\}\right) \quad (9)$$

Note that *E{}* denotes the mathematical expectation operator.

The total expected cost of production, inventory and delivery over the finite horizon *H*.Δt can then be expressed as follows:

$$f(u) = C_s \times \left(E\left\{S(H)^2\right\}\right) + \sum_{i=0}^{L} C_{s_i} \times \left(E\left\{S_i(H)^2\right\}\right) + \sum_{k=1}^{H-1} \left[\begin{array}{l} C_p \times \left(E\left\{u(k)^2\right\}\right) + C_l \times \left(\sum_{i=0}^{i=L} E\left\{\left(\frac{Q_i(k)}{Q_v}\right)^2\right\}\right) \\ + C_s \times \left(E\left\{S(k)^2\right\}\right) + \sum_{i=0}^{L} C_{s_i} \times \left(E\left\{S_i(k)^2\right\}\right) \end{array}\right] \text{with } k \in \{0,1,...,H-1\} \quad (10)$$

2.2.2. Maintenance Policy

In this subsection, the resolution of maintenance planning problem consists in minimizing costs related to preventive and corrective maintenance. Other, to make correct and suitable decisions, it's important to determine the best scheduling of carrying out maintenance. The maintenance strategy considered in this work is a preventive maintenance with minimal repair. Preventive actions should be scheduled over the finite horizon H which divided equally into *N* parts of duration *T*. We suppose that performing a preventive action corresponds to times *k.T* (*k=1,2,....N*) consists in replacing some critical parts restoring the production unit to an as good as new condition. However, between successive preventive interventions, breakdowns may happen, minimal repair is performed. It is assumed that the repair and overhaul durations are negligible.
So, it is necessary to keep in mind and to consider that the status of machine depend on their production and transported plan variation. Otherwise, it is worth considering the influence of production on the degradation of machine and consequently in maintenance planning as it was studied in [8].

In this section, considering the joint optimization strategy, we aim at determining the optimal maintenance strategy which allows the firm minimizing its maintenance costs. Such a strategy is characterized by the optimal number *N** of preventive maintenance actions and the most adequate time between them noted *T**.

$$C_M\left(U\left(u(1),u(1),....u(H-1)\right),N\right) = C_{pm} \times (N-1) + C_{cm} \times \varphi_M\left(U,N\right) \quad (11)$$

Where $\varphi(U,N)$ is the average number of failures as a function of the production plan defined by the vector U and the number of preventive maintenance actions N.

3. ANALYTICAL STUDY

In this section, we would like to show an analytical approach that can be used to solve the above stochastic problem and that by transforming it to a deterministic equivalent problem. Some of these approaches are based on the certainty equivalent principle.

3.1. Deterministic Equivalent problem

An approach that transforms the stochastic problem into a deterministic equivalent is necessary. This deterministic problem maintains the main properties of the original problem.

Before proceeding, the following notation is introduced:

Mean variables:

$E\{S(k)\} = \hat{S}(k)$, $E\{S_i(k)\} = \hat{S}_i(k)$ $i:0....L$, $E\{u(k)\} = u(k)$, $E\{Q_i(k)\} = Q_i(k)$ $i:0....L$

Variance variables: $V_{u(k)} = V_{Q_i(k)} = 0; i:1....L$. (Note that this reflects the fact that the control variables $u(k)$ and $Q(k)$ are deterministic).

- The production, delivery and inventory costs simplified as:

Lemma1:

$$f\left(S(k),S_{i\{i:0...L\}}(k),u(k),Q_{i\{i:0...L\}}(k)\right)=\sum_{k=0}^{H}\left(C_s\cdot\hat{S}(k)^2+\sum_{i=0}^{L}C_{s_i}\cdot\hat{S}_i(k)^2\right)+\sum_{k=1}^{H-1}\left(C_p\times u(k)^2+C_l\times\sum_{i=0}^{L}\left(\frac{Q_i(k)}{Q_v}\right)^2\right)+\left[H+1\right]\cdot\frac{H}{2}\cdot\sum_{i=0}^{L}C_{s_i}\cdot\sigma_{d_i}^2 \tag{12}$$

- The inventory balance equation (2) can be reformulated as:

$$\hat{S}(k+1)=\hat{S}(k)+u(k)-\sum_{i=0}^{L}Q_i(k) \quad k=0,1,.....,H-1$$

Likewise, the inventory balance equation (3) can be reformulated as:

$$\hat{S}_i(k+1)=\hat{S}_i(k)+Q_i(k-\tau_i)-\hat{d}_i(k) \quad k=0,1,.....,H-1 \text{; and } (i=1,..., L)$$

- The service level constraint:

In this part, we introduce the service level constraint. To continue transforming the problem into a deterministic equivalent, the equation describing the service rate for the principal store and each warehouse can be transformed as follows [8]:

Lemma 2

For $k=0,1,..,H-1$ and for i=0,….,L we have:

$$\Pr ob\left(S_i(k+1)\geq 0\right)\geq\theta_i \;\Rightarrow\; \left(Q_i(k-\tau_i)\geq V_{d_i(k)}\times\varphi^{-1}(\theta_i)-S_i(k)+\hat{d}_i(k)\right) \quad k=0,1,....,H-1 \text{ and } i=0,1,....,L \tag{13}$$

φ : Cumulative Gaussian distribution function with mean $\hat{d}(k)$ and finite variance $V_{d(k)}$.

φ^{-1} : Inverse distribution function

3.2. Maintenance Cost

Motivated especially from the work [9], in our present work, we take into account the degradation of machine while forecasting the maintenance actions. Thus, the maintenance strategy depends strongly on production planning which is on accordance with the principle of joint production and maintenance planning.
We recall that the optimal maintenance strategy characterized by the optimal number N^* of preventive maintenance actions and the time between them T^*, as given by Eq. (14).

$$T^* = \frac{H}{N^*} \tag{14}$$

Recall that analytic expression of the total maintenance cost is as follows, with $N \in \{1, 2, 3.....\}$.

$$C_M(U,N) = (N-1) \cdot C_{pm} + C_{cm} \cdot \varphi_M(U,N) \tag{15}$$

Where $\varphi_M(U,N)$ corresponds to the expected number of failures that occur during the horizon *H,* considering the production rate in each production period Δt.

Furthermore, we assume that the equipment degradation is linear, we assume that $\lambda(t)$ represents the linear failure rate function at production period k is expressed as following :

$$\lambda_k(t) = \lambda_{k-1}(\Delta t) + \frac{u(k)}{U_{\max}} \cdot \lambda_n(t) \quad \forall t \in [0, \Delta t] \tag{16}$$

$\lambda_n(t)$: failure rate for nominal conditions which is equivalent to the failure rate with maximal production.

The average failure number over the horizon H .Δt is:

$$\varphi_M(U,T) = \sum_{j=0}^{N-1} \left[\sum_{i=In(j\times\frac{T}{\Delta t})+1}^{In\left((j+1)\times\frac{T}{\Delta t}\right)} \int_0^{\Delta t} \lambda_i(t) + \int_0^{(j+1)\times T - In\left((j+1)\times\frac{T}{\Delta t}\right)\times\Delta t} \lambda_{In\left((j+1)\times\frac{T}{\Delta t}\right)+1}(t)dt + \int_{(j+1)\times T}^{\left(In\left((j+1)\times\frac{T}{\Delta t}\right)+1\right)\times\Delta t} \frac{\left(\left(In\left((j+1)\times\frac{T}{\Delta t}\right)+1\right)\right)}{U_{\max}} \times \lambda_n(t)dt \right] \tag{17}$$

We now replace $T=H/N$:

$$\varphi_M\left(U,{}^{H}\!/_{N}\right)=\sum_{j=0}^{N-1}\left(\begin{array}{l}\left(In\left((j+1)\times\frac{H}{N\cdot\Delta t}\right)-In\left(j\times\frac{H}{N\cdot\Delta t}\right)\right)\times\Delta t\times\lambda_0(t_0)+\\ \frac{\lambda_0(\Delta t)\times\Delta t}{U_{\max}}\times\sum_{i=In\left(j\times\frac{T}{\Delta t}\right)+1}^{In\left((j+1)\times\frac{H}{N\cdot\Delta t}\right)}\sum_{l=1}^{i-1}u(l)\,dt\;+\;\frac{1}{U_{\max}}\cdot\sum_{i=In\left(j\times\frac{H}{N\cdot\Delta t}\right)+1}^{In\left((j+1)\times\frac{H}{N\cdot\Delta t}\right)}\int_0^{\Delta t}u(i)\cdot\lambda_0(t)\,dt+\\ \sum_{l=1}^{In\left(\frac{(j+1)\times H}{N\cdot\Delta t}\right)}\frac{u(l)}{U_{\max}}\cdot\lambda_0(\Delta t)\cdot\left((j+1)\times\frac{H}{N}-In\left(\frac{(j+1)\times H}{N\cdot\Delta t}\right)\times\Delta t\right)\\ +\int_0^{(j+1)\times\frac{H}{N}-In\left(\frac{(j+1)\times H}{N\cdot\Delta t}\right)\times\Delta t}\frac{u\left(In\left(\frac{(j+1)\times H}{N\cdot\Delta t}\right)+1\right)}{U_{\max}}\cdot\lambda_0(t)\,dt\\ +\frac{u\left(In\left(\frac{(j+1)\cdot H}{N\cdot\Delta t}\right)+1\right)}{U_{\max}}\times\int_{In\left((j+1)\cdot\frac{H}{N\cdot\Delta t}\right)\times\Delta t}^{(j+1)\frac{H}{N}}\lambda_0(t)\,dt\end{array}\right)\qquad(18)$$

4. NUMERICAL EXAMPLE

Let us consider a system that produces one type of products to meet the delivery for L=2 warehouses that will satisfy random demands below. Basing in the models described in previous sections, we will establish the optimal production plan and the optimal maintenance strategy minimizing the total cost over a finite planning horizon: H=18 periods each of period length Δt=1. We supposed that the standard deviation of each demand of product is the same for all periods and for each demand $\sigma_{di(\{i:1,2\})}=1.2$ and the initial inventory level we assume that $S(0)$=0.

- Lower and upper boundaries of production capacities: Umin=0 and Umax=44.
  C_p *=2 mu*, *Cs =0.2 mu/k*, Cs_i*=0.2 mu/k* {i:1,….L=2}, *Cl=12,* Q_v=6, $S_i(0)=40$ *with* $\{i:1,2\}$
- We assume that customer satisfaction degree, associated with each warehouse stock constraint, is equal to 90% (θ_i=0.9 (i=1, 2)).

Concerning the system reliability, we suppose that the failure time of machine M has a degradation law characterized by a Weibull distribution. To calculate the failure rate and average number of failures functions given by equations 16 and 17, we assume that the nominal degradation follows a Weibull distribution $W(\alpha,\beta)$ with scale and shape parameters are respectively $\beta=100$ and $\alpha=2$ given by :

$$\lambda(t)=\frac{\alpha}{\beta}\cdot\left(\frac{t}{\beta}\right)^{\alpha-1}$$

The cost associated with a corrective and preventive maintenance action are respectively $C_{cm}=3000$ mu $C_{pm}=500$ mu (monetary unit).

The average demand is presented in table 1 below:

Table 1: Average demands for customer of warehouse 1

$d_1(0)$	$d_1(1)$	$d_1(2)$	$d_1(3)$	$d_1(4)$	$d_1(5)$
15	17	15	15	15	14
$d_1(6)$	$d_1(7)$	$d_1(8)$	$d_1(9)$	$d_1(10)$	$d_1(11)$
16	14	16	15	15	15
$d_1(12)$	$d_1(13)$	$d_1(14)$	$d_1(15)$	$d_1(16)$	$d_1(17)$
15	15	15	13	15	15

Table 2: Average demands for customer of warehouse 2

$d_2(0)$	$d_2(1)$	$d_2(2)$	$d_2(3)$	$d_2(4)$	$d_2(5)$
16	13	15	15	14	16
$d_2(6)$	$d_2(7)$	$d_2(8)$	$d_2(9)$	$d_2(10)$	$d_2(11)$
16	16	14	15	15	14
$d_2(12)$	$d_2(13)$	$d_2(14)$	$d_2(15)$	$d_2(16)$	$d_2(17)$
15	16	14	16	14	14

The optimal delivery time for each warehouse is presented in figure 2. Other hand, The economically production and delivery plans for each warehouse 1 and 2 are presented respectively in table 3, table 4 and table 5 and the optimal maintenance scheduling and figure 3.

Table 3: Optimal Production Plan

$u^*(1)$	$u^*(2)$	$u^*(3)$	$u^*(4)$	$u^*(5)$	$u^*(6)$
44	29	25	32	32	42
$u^*(7)$	$u^*(8)$	$u^*(9)$	$u^*(10)$	$u^*(11)$	$u^*(12)$
41	42	36	23	14	44
$u^*(13)$	$u^*(14)$	$u^*(15)$	$u^*(16)$	$u^*(17)$	$u^*(18)$
30	18	19	36	27	25

Table 4: Optimal delivery plan for warehouse 1 for $\tau^*_1=3$

$Q_1^*(1)$	$Q_1^*(2)$	$Q_1^*(3)$	$Q_1^*(4)$	$Q_1^*(5)$	$Q_1^*(6)$
25	25	25	25	22	9
$Q_1^*(7)$	$Q_1^*(8)$	$Q_1^*(9)$	$Q_1^*(10)$	$Q_1^*(11)$	$Q_1^*(12)$
16	21	7	20	23	22
$Q_1^*(13)$	$Q_1^*(14)$	$Q_1^*(15)$			
11	16	12			

Table 5: Optimal delivery plan for warehouse 2 for $\tau^*_2=3$

$Q_2^*(1)$	$Q_2^*(2)$	$Q_2^*(3)$	$Q_2^*(4)$	$Q_2^*(5)$	$Q_2^*(6)$
16	12	15	16	25	17
$Q_2^*(7)$	$Q_2^*(8)$	$Q_2^*(9)$	$Q_2^*(10)$	$Q_2^*(11)$	$Q_2^*(12)$
10	25	7	8	20	13
$Q_2^*(13)$	$Q_2^*(14)$	$Q_2^*(15)$			
23	12	25			

Figure 2: Total cost in function of the delivery time τ_1 and τ_2

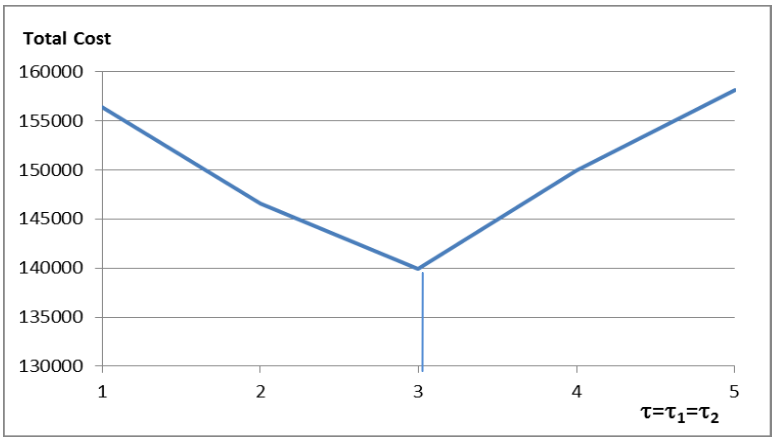
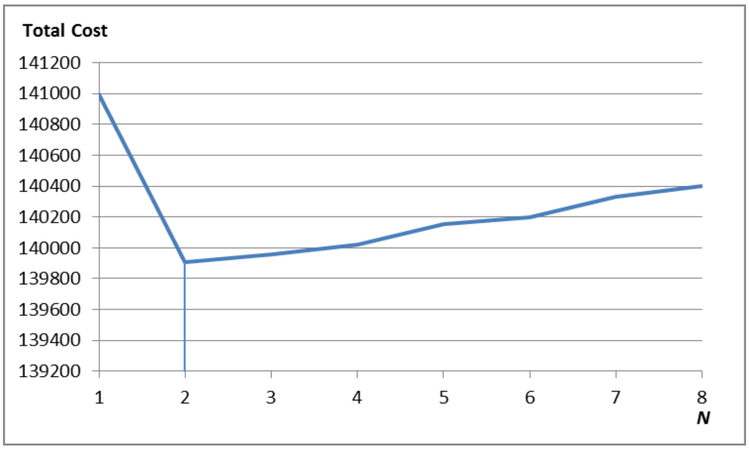


Figure 3: Total cost in function of preventive maintenance actions number *N*



The above tables 3-5 illustrate the optimal production and delivery plans of the minimum total cost for different values of the delivery times τ_1 (for warehouse 1), τ_2 (for warehouse 2) and the number N of PM actions to be performed.

In what follows, we interest to find the values of τ_1, τ_2 and N which corresponds to the lowest total cost value. Figure 2, Figure 3 show the total cost (production, inventory, delivery and maintenance) in function of the delivery times τ_1 and τ_2 and number of preventive maintenance actions N. we can see that the lowest total cost value corresponds to $\tau^*=3$ for the first and second warehouse and $N^*=2$. Thus, the optimal delivery time denoted by $\tau_1{}^*=3$ for the first warehouse and $\tau_2{}^*=3$ for the second warehouse and the optimal number of preventive maintenance actions denoted by $N^*=2$. Therefore, over the finite horizon H of 18 months, Two preventive maintenance actions should be done, i.e. for every period equals to $T^*=H/N^*= 9$ *tu* a preventive maintenance action should be done.

5. CONCLUSION

This paper dealt an approach involving in optimizing a constrained stochastic production, delivery and maintenance planning problem for several warehouse considering a several delivery time, a several random demand, a several service level and a randomly failing production system. The contribution of this study is that it formulates and solves the related stochastic production/delivery/maintenance problem under a service level. An optimization has been performed obtaining an optimal production, delivery plans as well as the corresponding preventive maintenance intervals taking into account the influence of production rates on the machine degradation and the delivery time on the problem optimization.

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