Estimating Farmer's risk aversion

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Abstract: In the early days of safety, together with his famous diagram, Farmer introduced a form of risk aversion. The first objective of the paper is to propose a general formulation of risk aversion along Farmer's thinking. This theoretical framework is particularly well suited when accident severity cannot be mitigated and prevention efforts are aimed at reducing probabilities, as is the case with nuclear safety. This is shown to go beyond the expected utility theory. The second part of the paper reports on an attempt at estimating Farmer's risk aversion as perceived by a panel of nuclear safety professionals. This tends to confirm Farmer's views.

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1. THE VALUE OF EXPECTED VALUE

1.1. Early criticism of expected value

In the early days of probabilities, in the 17-18th century, wealthy people had plenty of time to play card games or engage into lotteries. The moral problem was: how much is it fair to ask for a lottery ticket? The general thinking of scientists of that time was to evaluate lotteries by their expected value. For instance, if a true coin was tossed and the winner would pocket 100 upon heads coming up, the fair value of this lottery was: ½ x 100 = 50. It was honest to ask a potential player to pay 50 for such a lottery ticket both players then having the same probability of winning or losing; the lottery was then changed from a potential swindle to a "pure" game of chance.

Even nowadays, many feel that, faced with a probabilistic outcome, the expected value is the natural and the best quantification with *one* figure. But this was criticized at as early as 1738! Consider the following game:

> Toss a true coin until heads come up; if this happens on the first throw, you win 2 and the game stops; if this happens the second time the coin is tossed, you win 4 and the game stops; in general, if this happens at the n^{th} stage, you win 2^n.
>
> The expected value of this lottery is: $2 \times \frac{1}{2} + 4 \times \frac{1}{4} + \cdots + 2^n \times \left(\frac{1}{2}\right)^n + \cdots$ which is worth $1 + 1 + \ldots + 1 + \ldots = \infty$!

This paradoxical game found a solution when Daniel Bernoulli remarked that:

> *The determination of the value of an item must not be based on the price, but rather on the utility it yields…. There is no doubt that a gain of one thousand ducats is more significant to the pauper than to a rich man though both gain the same amount.*

Bernoulli proposed the logarithm of a sum of money as the utility of the said monies which implies that an "additional ducat" is worth less and less as gains increase. The expected value of the utility of the above lottery is no longer infinite. This solution was published in 1738 in the Commentaries of the Imperial Academy of Science of Saint Petersburg; hence its name as the Saint Petersburg Paradox.

1.2. Expected utility and risk aversion

This approach was later generalized by Non Neumann and Morgenstern as the Expected Utility theory (EU). When utility U(x) of a sum x boils down to x, EU simply judges according to expected values. EU is thus (much) more general.

When judging probabilistic outcomes on the basis of their expected value, if amounts increase by, say, 10%, the value also raises by 10%. (In sophisticated terms, the elasticity of risk with respect to losses is 1). This is no longer the case with EU. It could be lower or larger than 10%. With a decreasing utility of gains, the utility function is concave, such as the logarithm proposed by Bernoulli and the increase in the value of the lottery is lower than 10%. (The elasticity of the value of the lottery with respect to potential gains is lower than 1).

The concavity of the utility function models an important behavior: risk aversion. This gives rise to risk premiums. On financial markets, if you have a wife, two kids and a few loans, you probably wish to limit the risks you bear when investing your savings. The returns you can ask for are then lower than what you might earn if you were prepared to take larger risks; the difference is called the risk premium. A risk premium is earned by those who accept to take larger risks and is paid by those who wish to limit their risk.

The larger your risk aversion, the larger the risk premium you are ready to pay in order to run lower risks. When you have zero risk aversion, your utility function has zero concavity, it is a straight line, the value of risk is the expected value. You are said to be risk neutral, a limit case of risk aversion.

This model is applicable to prevention. With risk neutrality, you only prevent up to the expected value of losses; in contrast, a risk averse decision process results in (much) more prevention; one could use the term prevention premium. Here is a trivial example:

> With risk neutrality, it is equivalent to prevent a grain of sand weighing 1 g falling onto your head with probability 10% and preventing a 1 kg stone falling on your head with probability 10^{-4}. Indeed the expected value is 0.1 g in both cases. The result is minimal prevention.

Or, to paraphrase Bernoulli:

> *The determination of the value of a risk must not be based on the expected losses, but rather on the disutility it yields…. There is no doubt that a loss of one thousand ducats with probability 10^{-3} is more significant to prevention than a certain loss of one ducat.*

More generally, using averages is a profoundly misleading way of approaching risks. It denies any specific character to the disastrous effect of a sudden lump sum loss by simply asserting that it is equivalent to fraction losses incurred more frequently. A fundamental feature of disasters is thus swept under the rug.

1.3. Beyond expected utility

Such risk aversion only relates to losses; in EU theory risk remains linear with respect to probabilities; elasticity of risk with respect to probabilities is identical to 1.

> In formal terms, let w be wealth in the absence of risk and U(w) its utility without risk. Let x be a potential loss incurred with probability p. The existence of such a risk leads the situation to be valued as the expected utility:
>
> $$(1\text{-}p)\ U(w) + p\ (U(w\text{-}x) = U(w) – p\ [U(w) – u(w\text{-}x)]$$
>
> The value of risk is thus p [U(w) – u(w-x)], the expected disutility of the risk materializing. This can be written px µ(x) where µ is a multiplier of expected values which quantifies risk aversion. Conversely, if the value of risk can be expressed as px µ(x), it can be shown that the decision process is EU. Therefore EU is entirely characterized by: "the elasticity of risk with respect to probabilities is identical to 1".

Therefore, despite arguments existing in favor of EU, there is a need to investigate risk aversion with respect to probabilities.

2. FARMER'S RISK AVERSION

As early as 1967, Farmer [1] analyzed risks in a quantitative fashion in order to determine suitable locations for NPPs. At the time, his work was construed as a "probability approach", in contrast with general siting criteria such as "more than 50 km away from any town". His major contribution was seen as a quantification of health risks; however, much progress has been made in risk quantification since this historical paper and what is mainly retained now is the "Farmer diagram".

The idea was first put forward as the left diagram in Figure 1 where consequences appear as Curies released on the x-axis and probabilities appear as return years on the y-axis. The line CD pictures risks (x,p) which are deemed equivalent. In Farmer's words:

> It is seen on Fig. 2 that the area A is one of low risk and area B one of high risk and all parallel lines of equal slope -1 join points of equal risk in terms of curies per year. One such line might be used as a safety criterion by defining an upper boundary of permissible probability for all fault consequences.

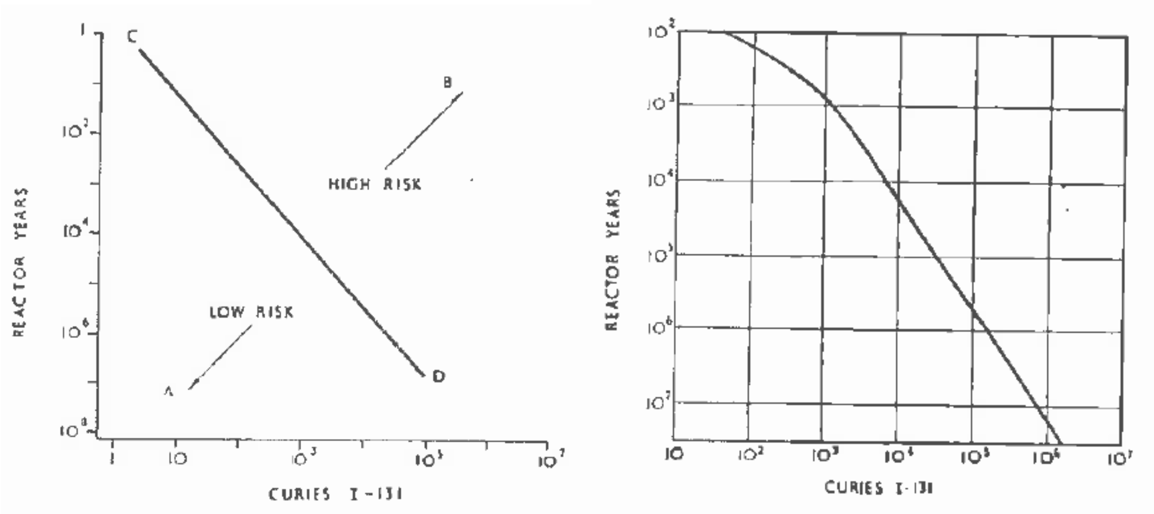

FIG. 2. Probability-consequence diagram

FIG. 12. Proposed release criterion

Figure 1 : Farmer's original diagrams

A few pages later, Farmer refines this general idea as follows:

> 7. A BOUNDARY LINE AS A CRITERION
>
> Ih the introductory section of this paper it was suggested that a useful standard for a reactor might be provided by a boundary line indicated as CD on Fig. 2. Although this line joins points of equal risk in curies per year, it may not represent an equal risk of casualties†. Furthermore, it is likely that most people would apply a relatively heavier penalty against the possibility of a large release than a small release. This would lead to a boundary line of greater negative slope. The one now chosen has a slope of -1.5 so as to reduce by three orders of magnitude the frequency of an event whose severity increases by two orders of magnitude.

The diagram is log-log; therefore the first "line of equal risk" implies px = constant since it has a slope of -1; the expected value of risk is constant along the line. The simplest corresponding value of risk is[1]:

$$V(x,p) = px$$

The lines of equal risk then form a network of parallel lines of slope -1 in log-log coordinates, as shown on the above left diagram. This depicts risk neutrality.

In the second diagram to the right, which represents the actual quantification suggested by Farmer, the equal risk lines can similarly be represented by:

$$V(x,p) = p^{0.66}x$$

Let's generalize by:

$$V(x,p) = p^{1-a}x$$

For a = 0, this formulation boils down to risk neutrality. As a increases, the reduction in probability Δp required to offset an increase in severity Δx increases [$(1-a)\, dp/p + dx/x = 0$]. For a = 1, the value of risk is independent of its probability of occurrence which is certainly excessive for most prevention specialists. Thus "a" can be conceived as a risk aversion with $0 \leq a < 1$.

This formulation generalizes the well-known Farmer risk diagram in two ways.

1. It is no longer the separation of the risk plan into two zones – the acceptable and unacceptable risk zones[2] – which is a rather gross decision aid in the view of economists. It now gives rise to a risk value formula, a choice function or a (dis)utility function which allows much finer distinctions.
2. The formula for the risk value involves parameter "a" which I propose to call the Farmer Risk Aversion (FRA).

Let me address a paradox. The above value of risk, $V = px.p^{-a}$, is the expected value of losses multiplied by p^{-a}. For standard probabilities of disaster such as 10^{-n}, the multiplier is 10^{na} and may rise to impressively high values; for instance, if a = 0.33 as suggested by Farmer, and $p = 10^{-6}$, the value of risk is 100 times the expected value of losses. For $p = 10^{-9}$, this multiplier is 1000 while it is only 10 for $p = 10^{-3}$. The lower the probability the higher the multiplier. In other words, as safety measures reduce risk (for disasters with fixed damages $x = x_0$), the decision process allocates proportionally more and more funds for safety which may appear odd.

However, if one wanted to have multipliers decreasing with decreasing values of probabilities, the FRA would have to be negative. The value of risk would then be lower — possibly much lower — than the expected value of risk which would represent a risk loving prevention behavior…

These surprising features now being acknowledged, notice that resulting prevention behaviors remain logical. When losses x are not modified:

$$\frac{dV}{V} = (1-a)\frac{dp}{p} \quad (1)$$

[1] In general, $V(x,p) = f(px)$. Imposing $V(x,1) = x$, which is fairly natural, implies $f(u) = V(u,1) \equiv u$

[2] More sophisticated versions include an « amber zone » between the green and red zones.

Since a < 1 the value of risk always varies in the same direction as p; a reduction in probability always results in a reduction in risk.

If a is fixed, the percentage reduction of risk is a fixed proportion of the percentage reduction in probability which is not unsatisfactory. And finally, although p^{-a} can become large, reductions in risk are by no means negligible.

> Numerical example: x = 100b, p = 10^{-6}, expected value of risk is 100k/year. With a = 0.33, the risk value is 100 times larger at 10m/year. Dividing probabilities of occurrence by 10 reduces p to 10^{-7}; the value of risk is then reduced by a factor of 4.67 to reach 10k.214 = 2.1m/year.

There is no particular reason why the FRA should be constant over the entire range of risks. In general, it could vary according to x and p. We then have the following variation of risk according to variations in the level of p:

$$\frac{dV}{V} = \underbrace{\frac{d(px)}{px}}_{\text{Risk neutral Variation}} + \underbrace{a(p,x)\frac{dp}{p}}_{\text{Effect of Risk}} \qquad (2)$$

In general, one can always set V(x,p) = px μ(x,p) — which is simply a definition of the multiplier μ. One then immediately derives:

$$\frac{\mathrm{dV}}{\mathrm{V}} = \frac{\mathrm{d(px)}}{\mathrm{px}} + a_x(\mathrm{x,p})\frac{\mathrm{dx}}{\mathrm{x}} - a_p(\mathrm{x,p})\frac{\mathrm{dp}}{\mathrm{p}} \qquad (3)$$

With $$a_x(x,p) = \frac{x\mu'_x}{\mu} \quad a_p(x,p) = \frac{p\mu'_p}{\mu}$$

When losses x cannot be mitigated, (3) boils down to (2). When both a_x and a_p are nil, we obtain expected value; when $a_p = 0$ and $a_x \neq 0$ we have EU. The general case features both a_x and a_p non-nil.

Therefore, testing for $a_p = 0$ is a way to test the expected value theory (in addition, testing $a_x = 0$ would test zero risk aversion, i.e. risk neutrality).

3. ELICITATION OF THE FRA

I hope to have shown the above framework to have theoretical value. In practical applications, it would be particularly useful to address questions raised by the prevention of disasters where potential losses x are high and occurrence probabilities p are low. For ordinary risks with “limited” losses and “non- negligible” probabilities, values of a_x and especially a_p could be low or even negligible.

It does, however, remain theoretical as long as values of the FRA cannot be justified. Hence an attempt to find out whether my colleagues at IRSN did exhibit FRAs different from 0 and whether these values might be influenced by the severity of the accident.

Procedures designed to quantify behaviors and preferences have been developed in environmental economics. It has been found that asking direct questions such as: “How much would you be ready to pay for, or think the nation should invest in, the reduction of such risk?" do not provide satisfactory answers. More indirect questions are needed.

3.1. The questionnaire and its validation

Equation (1) directly suggests questions such as: "In order to divide risk by a factor k, by how much would it be necessary to divide its probability of occurrence?" Notice that risk is not defined in such a question and thus refers to "a feeling" of professionals. The answer of respondents should not result from a computation or formula but correspond to (deep) values and (authentic) choice behaviors.

The following questionnaire was proposed:

In order to divide risk by	2	probabilities should be divided by	?
In order to divide risk by	3	probabilities should be divided by	?
In order to divide risk by	5	probabilities should be divided by	?
In order to divide risk by	10	probabilities should be divided by	?

These questions were asked for the severe accident used by IRSN in its estimates of nuclear accident costs in France [2] and then for the major accident referred to in these studies.

In order to test reactions to such an unusual query, the questionnaire was first proposed to 4 colleagues well acquainted with IRSN accident cost estimates. This was performed during face to face interviews which lasted up to half an hour. Interviewees were allowed to ask any question and offer any comments they wished. Results were encouraging. For the severe accident, elicited FRAs ranged from 0.1 to 0.7 with an average of 0.35. For the major accident (with radioactive releases comparable to those of the Fukushima disaster), they ranged from 0.3 to 0.7 with an average of 0.41. FRAs were significantly higher for the major accident than for the severe accident, except for one person.

Although interviewees thought the questionnaire was difficult to answer, the procedure encountered no major difficulty and largely comforted Farmer's intuition.

3.2. The procedure and its difficulties

The questionnaire was then proposed to two groups of professionals who attended a 45 mn presentation of IRSN accident cost estimates. They were not aware of the "survey" and simply attended to get acquainted with accident costs. They were all simultaneously confronted with a printed questionnaire, between the presentation itself and the ensuing Q&A session. After a short general briefing, the questionnaires were handed out and respondents had 5-10 mn to write down their answers.

Results differed from the preparatory face to face interviews. Respondents felt it was extremely difficult to answer; a majority of persons felt they were passing some sort of test, as in school, and were supposed to find "the right answer" although it had been clearly stated that this was not the case. I call the corresponding answers SF — for "deriving from a scholarly formula". The most frequent such routine is to answer with the very same factor as in the question, in which case a = 0, i.e. risk neutrality. Another is to square the factor resulting in a = 0.5; another is to multiply the factor by 10; and so forth.

Overall, it is difficult to judge whether risk neutral answers really mean a fully-conscious risk neutrality or simply result from a difficulty with the notions involved and the search for a simple formula to answer. This could be examined directly with the persons involved ex-post (if they did fill up their name on the questionnaire).

The global feedback — a great difficulty to answer the questionnaire —, suggests that such elicitations are better not conducted through such paper questionnaires but should involve face to face interviews. Several French scholars declare this is a general phenomenon when complex risk questions are involved [3]. For example, they conducted studies of risk perception around a French NPP and say global paper questionnaires give very poor answers. In their experience, it is necessary to acculturate people to the said risk through one-hour interviews. They report that perceptions change in the course of the interview. And also change with time, which was observed by conducting interviews with the same people once a year over several years.

Proceeding through interviews is clearly much more costly in terms of time and resources. The need to ensure comparable standards and interviewing procedures may also make the results easier to criticize. In any case, the results will tend to be protocol dependent…

3.3. Results

In total, 37 questionnaires were received. Details are given in the appendix.

Full risk neutrality ranges from 0% to 30% depending on whether answers which duplicate factors included in the question are interpreted as a routine without deep significance or as fully-conscious expressions of risk neutrality. This means that risk aversion is dominant among IRSN professionals. Risk neutrality appears significantly more plausible for controlled radioactive releases than for the massive releases implied by a major accident: +20% above the previous 30% resulting in 50% risk neutral answers for controlled releases, in the hypothesis most favorable to risk neutrality.

Another general conclusion is that risk aversions elicited in this exploratory attempt are quite high. When leaving aside answers pointing to risk neutrality, they tend to be above the Farmer figure of 0.33, even for the severe accident (between 0.4 and 0.45 in average). There are no answers with positive but low FRAs such as, for instance, a = 0.05; thus, there is a gap between risk neutrality and risk aversion; this reinforces the interpretation that risk neutral answers do not fully correspond to risk neutrality *stricto sensu*.

And a third conclusion is that risk aversion does increase with the severity of the accident as would be expected. It is globally confirmed with FRAs above 0.6 for the major accident.

4. CONCLUSION

This paper suggests that risk aversion with respect to probabilities is a subject worth further investigation. A fairly general formulation is suggested which could perhaps help put the problem on the agenda. The opposing view would argue that EU is well grounded in a solid axiomatic, namely that of Savage [4]. This has been the consensus for many years because the axioms of Savage appear “natural” when formulated. However, many experiments have shown that they do not correctly describe actual behaviors with respect to risk. The intuition put forth by Farmer, not an economist but an engineer, goes back to 1967 when challenging EU was basically unheard of (see however Allais [5]).

The heuristic experiment conducted among IRSN professionals seems to support Framer's intuition and thus constitutes a further argument for considering risk aversion with respect to probabilities.

References

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Appendix

In a first session 19 answers were collected among which 7 were SF with a = 0. Some of these clearly stated that: "Risk is probability". Their answer was thus the same for the two different accidents of largely different severity and simply consisted in copying the factors in the question.

Another 3 respondents copied the factor in the same way for the severe accident (a = 0), but multiplied these factors by 10 for the major accident thus coming up with a ≈ 0.6. There is an obvious suspicion that such answers result from a "formula" rather than a "pure feeling" but this should ideally be tested directly with the respondents.

4 respondents offered identical answers for the severe and the major accident. Their FRAs were thus not influenced by severity of consequences. However, they were fairly high at 0.6, 0.55, 0.4 and 0.2.

The other 5 answers resulted in average FRAs of 0.45 and 0.6 for the two accidents.

In a second session, 18 questionnaires were filled up with 4 risk neutral answers. 5 respondents were neutral for the severe accident (a = 0), but used another routine for the major accident, mainly the "squares" routine; their FRAs were quite high for the major accident, ranging from 0.5 to 0.7. Two questionnaires exhibited equal non-zero FRAs for the two accidents, with very high aversions of 0.75 and 0.85.

The other 7 respondents showed an average FRA of 0.4 for the severe accident and 0.75 for the major accident.