

Seismic Quantification Enhancements for getting CDF/LERF Distribution from the Point Estimates Results

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Abstract: Technical requirements of the standard ASME/ANS RA Sa-2009 for capability category 2 imply appropriate consideration of uncertainty and combination of random failures with seismic failures. The paper presents how to develop the plant state mean fragility from the point estimate results that includes random failures. The plant state *CDF/LERF* components corresponding to each acceleration range are divided by the corresponding hazard frequency resulting discrete points of the mean plant state fragility. Furthermore using relationships presented in Ref. [2] β_U and β_R can be recovered and full plant state fragility parameters are obtained. Finally *CDF/LERF* distribution is developed.

Keywords: Seismic PSA, Random Failures, Fragility.

1. INTRODUCTION

High quality PSA that can be used as basis for various risk informed applications should comply with the technical requirements of the standard [1]. Seismic PSA should consider combination of random failures with seismic failures and also should properly consider variability associated to seismic hazard and seismic fragility functions. To address these high level requirements *CDF/LERF* distributions should be calculated at sequence level separate for random failures and seismic failures and after that combined to obtain in sequence *CDF/LERF* distribution and finally union of all significant sequences *CDF/LERF* define the plant state distribution of *CDF/LERF*. Special quantification tools are needed to perform such seismic quantification analysis.

Importance and sensitivity analyses are conducted to identify significant contributors and accident sequences. For practicality these analyses are performed for the point estimate level (mean *CDF/LERF*) and normal PRA software can be used (e.g. CAFTA, RISKSPECTRUM, etc.). The paper presents how plant state mean fragility can be obtained from the point estimates results. Also it presents an iterative process for approximating plant state mean fragility with a lognormal fragility that convolved with the mean hazard curve produce same mean *CDF* as the point estimate. The iterative process display the level of approximations introduce by this method and its impact to the final results. The equivalent lognormal plant state fragility allows recovering β_U and β_R by analytical solutions developed by the author [2]. Furthermore the full variability of the plant state fragility is obtained allowing to propagate both hazard and fragility variability in seismic *CDF* distribution.

2. EXTRACTION OF THE END STATE MEAN FRAGILITY

It is a common practice in S-PSA to split the hazard range of interest in several acceleration bins. Hazard frequency is calculated for each bin and point estimate results *CDF/LERFs* are obtained for each acceleration bin in a similar manner as for internal events PSA. Final mean *CDF/LERF* is obtained by simply summation of results obtained for each acceleration bin. In this analysis consideration of random failures is straightforward and does not require special quantification tools. This step of analysis is carried out to support importance, sensitivity and ranking analyses aimed to identify the main contributors and significant sequences. Observing S-PSA point estimated results the author identified a way of extracting points belonging to the mean plant state fragility (as many as number of acceleration bins are used in the analysis). These points belonging to the mean plant state

fragility are obtained by dividing *CDF/LERF* for each acceleration bin to the corresponding hazard frequency.

Furthermore the author developed analytical relation to develop lognormal fragility equations crossing/bounding plant state fragility points. If only seismic failures are considered it is possible to find a single mean lognormal fragility crossing the points obtained from the plant state mean *CDF/LERFs* (as described above). If random failures are considered the plant state fragility is not lognormal distributed anymore but can be closely bounded by lognormal fragilities and an equivalent lognormal plant state fragility can be obtained. The equivalent lognormal plant state fragility has the property to produce the same mean *CDF/LERF* (by convolution with the mean seismic hazard) as compared to direct point estimates results. The approximation can be graphically displayed how close bounded the mean plant state fragility (close range of β_C and A_m values).

There are many advantages of getting the equivalent lognormal plant state fragility function:

- a. First allows extraction of the plant *HCLPF* in various cases:
 - 1) considering seismic failures, random failures and human errors
 - 2) considering only seismic failures (same as in SMA) and compare both margin estimates
- b. Second allows recovery of the full plant state fragility function by recovering β_R and β_U from the mean plant state fragility. If the mean plant fragility is lognormal than we have analytical solution to extract β_R and β_U and is possible to expand to the full fragility variability [2] and after that to convolve with the hazard curves for getting *CDF/LERF* distribution.

All these are showed in the following sections using an illustrative example.

3. ILLUSTRATIVE EXAMPLE

3.1. Description of the Illustrative Example

The basis for the illustrative example is presented in Tables A1 to A3 of the Annex A. Table A1 describes significant sequences corresponding to all acceleration bins (resulted after importance, sensitivity and ranking analysis are performed). Some of the sequences contain only seismic failures since other sequences include random failures and/or human errors associated to recovery actions. Table A2 presents fragility parameters for seismic basic events. The conditional probability of failure for each basic event corresponding to each seismic bin (S1 to S6) is calculated based on fragility parameters presented in Table A2. Table A3 presents random failures and human errors that appears in the sequences presented in Table A1. Finally Table A1 presents partial seismic *CDF* values corresponding to each acceleration bin and the sum of partial *CDF* values build the total seismic *CDF*. Table A1 allows calculation of contribution of the random failures as well as the contribution of the human errors to the final results. For this illustrative example we get:

<i>CDF</i> -without RF	1.09E-5	
<i>CDF</i> -without HERR	7.00E-05	
<i>CDF</i> including RF and HERR	1.68E-05	<i>CDF</i> without HERR and RF = 6.5E-5
HCLPF with RF and HERR	0.33g	HCLPF without HERR and RF = 0.27g

Can be observed that *CDF* without RF is 1.55 less than the *CDF* when RF is considered. Also *CDF* without HERR (no recovery actions are credited) is 4.16 higher than *CDF* when HERR is considered. A qualitative observation shows that in case of SMA ignoring both random failures and human errors associated to recovery actions lead to a conservative estimate of seismic margin. Also it will be un-conservative to credit operator recovery actions and not considering combination of seismic failures with random failures.

3.2. Recover Plant State Lognormal Equivalent Fragility from the Point Estimate Results

The process for recovering the plant state equivalent lognormal fragility is described using the illustrative example presented in Annex A. Table 1 summarize results of the illustrative example containing:

- definition of each acceleration bin (S1 to S6),
- acceleration value within each bin for which fragility point associated to that bin is defined,
- hazard frequency for each acceleration bin,
- partial seismic *CDF* for each acceleration bin and total seismic *CDF*,
- plant state fragility points obtained by dividing partial seismic *CDFs* by the hazard frequency,
- relative *CDF* contribution of each acceleration bin.

It should be noted that partial *CDF* values and plant state fragility points include the effect of seismic failures, random failures and human errors.

Table 1: Calculation of Plant State Fragility Points based on Partial *CDFs*

Bin	Frag acc. Point	Acceleration Bin		H(a1)	H(a2)	Hazard. Freq.	Partial <i>CDF</i>	Plant State Frag.	Contrib.
		a1	a2						%
S1	1.5E-01	0.1	0.2	5.53E-01	5.83E-03	5.47E-01	1.4E-07	2.6E-07	0.85
S2	2.2E-01	0.2	0.3	5.83E-03	4.07E-04	5.43E-03	1.0E-06	1.9E-04	6.13
S3	3.6E-01	0.3	0.4	4.07E-04	6.15E-05	3.45E-04	7.9E-06	2.3E-02	46.94
S4	4.7E-01	0.4	0.6	6.15E-05	4.29E-06	5.72E-05	4.8E-06	8.4E-02	28.64
S5	7.5E-01	0.6	0.8	4.29E-06	6.49E-07	3.64E-06	1.9E-06	5.2E-01	11.16
S6	1.0E+00	0.8	1.2	6.49E-07	4.53E-08	6.04E-07	1.1E-06	1.0E+00	6.27
Total Seismic <i>CDF</i>							1.68E-5		

The hazards curves and relative contribution of each acceleration bin to the total *CDF* are presented in Figure 1. Figure 2 presents the plant state fragility (before and after fine adjustment of the fragility acceleration points within each bin). The adjustment of fragility acceleration point is done in order to reduce deviation from lognormal plant state fragility introduced by the random failures including the conditions that the mean *CDF* is conserved. The following equation is used to obtain lognormal fragility parameters crossing the fragility acceleration points corresponding to each acceleration bin:

$$A_{m-i} = \frac{a}{e^{(\Phi^{-1}(F(a))\beta_c)}} \quad (1)$$

where: *a* – corresponds to fragility acceleration point for each bin *S-i* and *F(a)* is the plant state fragility for *S-i* (see Table 1). Using equation (1), β_c is iterated until A_{m-Si} values (corresponding to acceleration bins with important contribution to *CDF*) get close enough to the plant state fragility points shown in Table 1.

The iterative process is illustrated in Table 2 and Figure 3. Solution converges to lognormal fragility parameters that closely bound the plant state fragility points. The equivalent plant state lognormal fragility parameters are those that gives the same point estimated seismic *CDF*. In other words convolution between the mean hazard curve with the equivalent plant state lognormal fragility produce same seismic *CDF* as the one obtained by point estimate shown in Table 1 or Table A1 for Annex A.

Table 2 Numerical results of iterations for getting Plant State Lognormal Mean Fragility

A_m	<i>CDF</i> Contrib. %	<i>Bc</i> Iterations 1 to 4			
		0.25	0.40	0.33	0.341
A_{m-S1}	0.85	5.26E-01	8.07E-01	7.86E-01	8.31E-01
A_{m-S2}	6.13	5.35E-01	6.78E-01	7.11E-01	7.40E-01
A_{m-S3}	46.94	5.93E-01	6.34E-01	6.96E-01	7.12E-01
A_{m-S4}	28.64	6.63E-01	6.25E-01	7.41E-01	7.52E-01
A_{m-S5}	11.16	7.43E-01	5.65E-01	7.41E-01	7.41E-01
A_{m-S6}	6.27	6.63E-01	5.62E-01	5.81E-01	5.70E-01

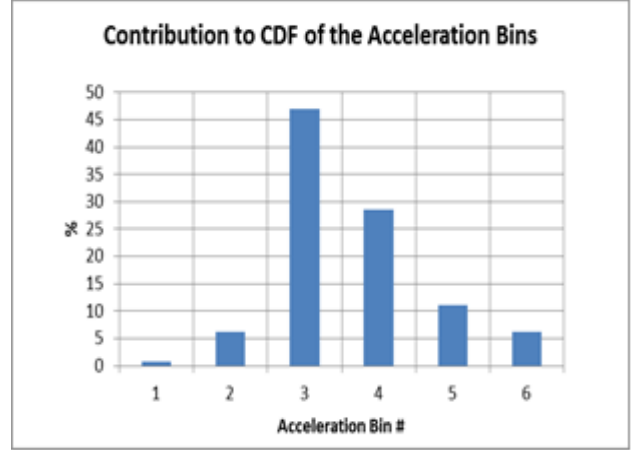
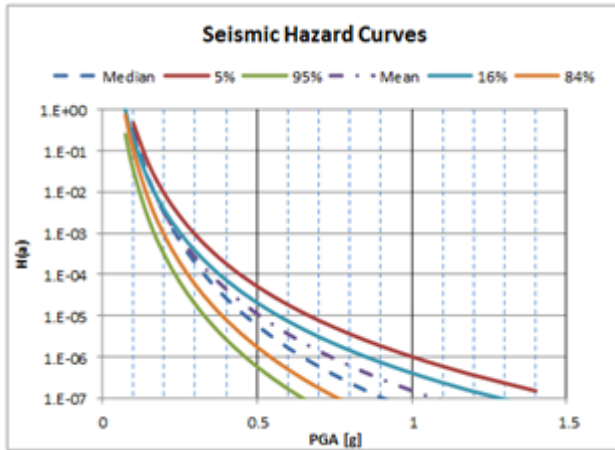


Figure 1 Seismic hazard curves and relative contribution of acc. bins to total seismic CDF

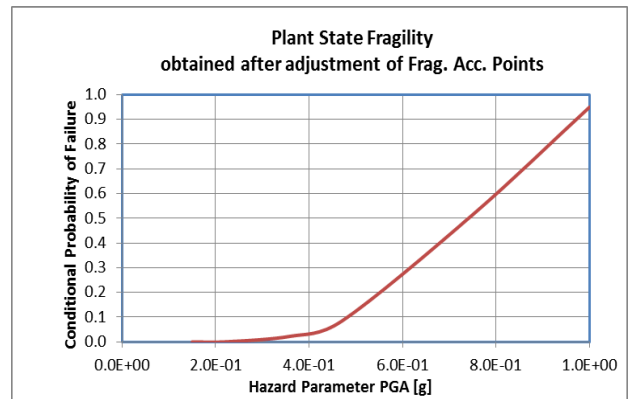
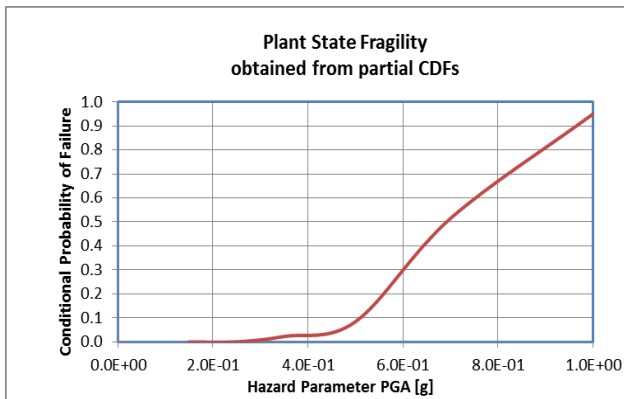


Figure 2 Plant state fragility obtained from partial CDFs divided by hazard frequency for each acc. bin.

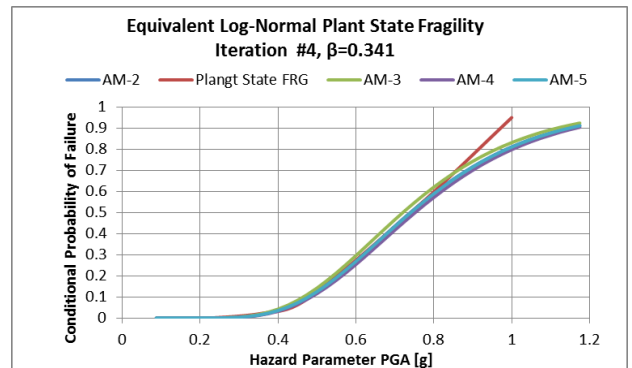
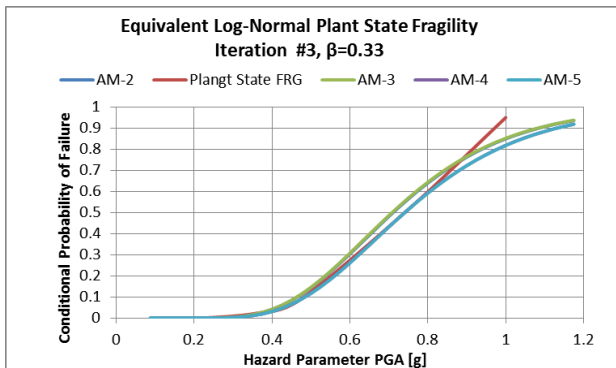
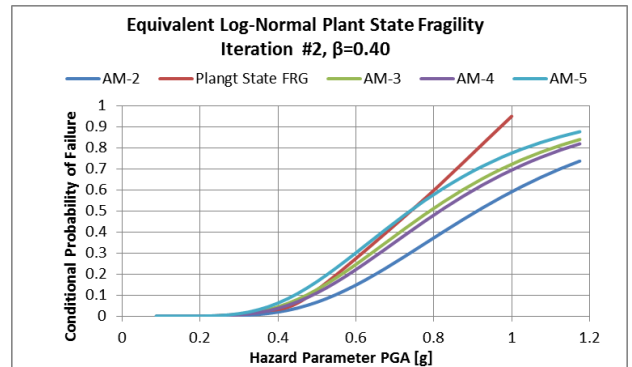
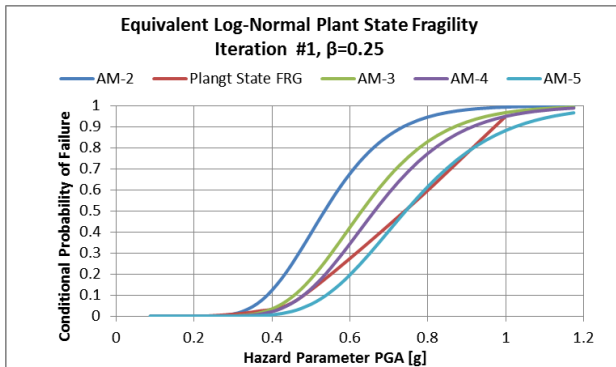


Figure 3 Graphic Results of Iterations for getting equivalent lognormal plant state fragility

The median capacity A_m corresponding to the equivalent lognormal plant state fragility is calculated as weighted sum of A_{m-si} for each acceleration bin corresponding to the last iteration. The relative acceleration bin contribution to CDF is used as weighting factors. In this example, using the proposed iterative process the following plant state lognormal fragility parameters have been obtained:

- $A_m = 0.72g$ and $HCLPF=0.325g$, $\beta_C=0.341$ (including the effect of random failures).
- $CDF = 1.68E-5$ (by convoluting the derivative of the mean hazard and mean fragility) is the same as the one shown in Table 1 obtained by point estimate considering and random failures and human errors.

This shows that the equivalent lognormal mean plant state fragility obtained in this way is conserving the point estimate CDF value. Also Table 2 and Figure 3 shows numerically and graphically the approximation range (for A_m and β_C) introduced by this procedure.

3.3. Recover β_U and β_R from the Equivalent Plant State Mean Fragility

As presented in [2] analytical relationships have been develop to obtain β_R and β_U based on the mean fragility parameters (A_m , $HCLPF$ and β_C):

$$\beta_U = \frac{B + \sqrt{B^2 - 4AC}}{2A} \quad \beta_R = \frac{B - \sqrt{B^2 - 4AC}}{2A} \quad (2)$$

$$A = 2 \quad B = 2 \frac{\ln\left(\frac{HCLPF}{A_m}\right)}{\phi^{-1}(0.05)} \quad C = \left(\frac{\ln\left(\frac{HCLPF}{A_m}\right)}{\phi^{-1}(0.05)}\right)^2 - \beta_C^2 \quad (3)$$

The equivalent lognormal plant state mean fragility parameters obtained in Sub-Section 3.2 are:

$$A_m = 0.72 \quad \beta_C = 0.341 \quad HCLPF = 0.325$$

Using equations above plant state fragility parameters and equations (2) and (3) we get:

$$A = 2 \quad B = 0.963 \quad C = 0.1152$$

$$\beta_U = 0.258 \quad \beta_R = 0.223$$

Finally the equivalent lognormal plant state fragility function (considering all variability) is defined by the following parameters:

$$A_m = 0.72; \beta_U = 0.258; \beta_R = 0.223$$

The above described process lead to the equivalent lognormal plant state fragility function consistent to point estimate results that includes contribution of random failures and human errors. Plant state fragility function can be further used to calculate seismic CDF distribution.

3.4. Seismic CDF distribution

Figure 4 illustrates the required input needed for calculation of the seismic CDF distribution. Equation (4) can be used to develop a number of plant state fragility corresponding to different confidence levels " Q " (typical 25 to 50 fragility curves) – also associated probability distribution parameters are needed. Same number of seismic hard curves should be available from PSHA study or should be developed based on available PSHA results.

$$F(a) = \phi \left(\frac{\ln\left(\frac{a}{A_m}\right) + \beta_U \phi^{-1}(Q)}{\beta_R} \right) \quad (4)$$

Each fragility curve is convolved with each hazard curve resulting the *CDF* value and corresponding distributions parameters. This pair of values defines one *CDF* distribution point. For all combinations of fragility and hazard curves the seismic *CDF* distribution is obtained as shown in Figure 4 and 5.

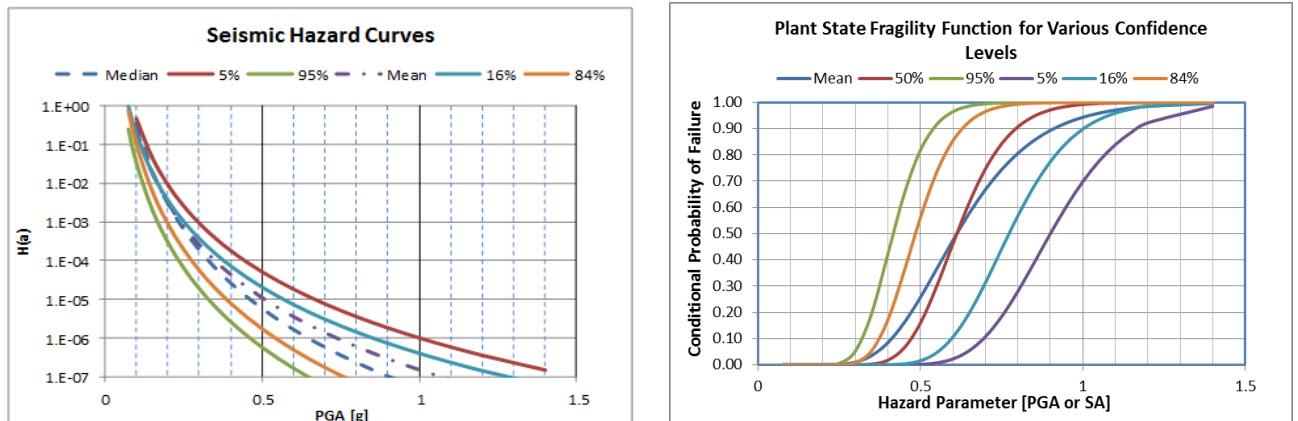


Figure 4 Input for Seismic Risk Quantification

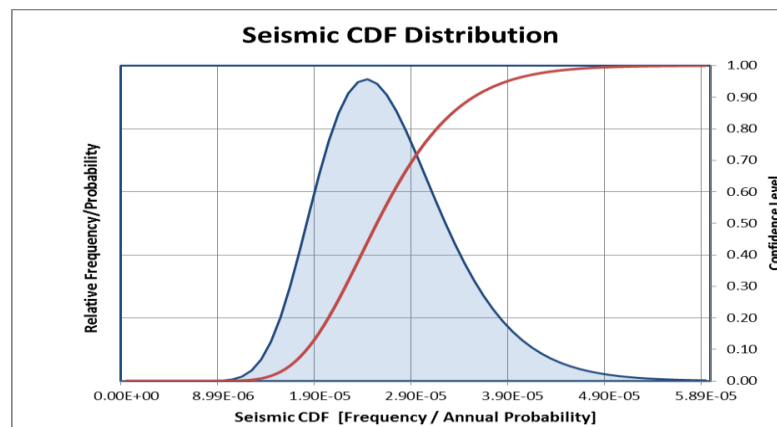


Figure 5 Seismic *CDF* Probability Distribution

4. CONCLUSIONS

The plant state point estimate of seismic *CDF/LERF* is obtained for each acceleration bin and summation of partial *CDFs/LERFs* give the total seismic *CDF/LERF* corresponding to the acceleration range of interest. Plant state fragility points are simply obtained by dividing partial *CDFs/LERFs* with the hazard frequency corresponding to each acceleration bin. Due to the random failures these points does not belong to a lognormal mean fragility.

Equivalent lognormal mean plant fragility can be defined for each acceleration bin using equation (1) and iterating β_C until these fragilities closely bound all plant state fragility points (that have significant contribution to *CDF/LERF*). The equivalent plant state lognormal mean fragility parameters are obtained as a weighted sum of median capacity values A_m for each acceleration bins and β_C values form the last iteration. Second condition used in the iteration process is that the equivalent lognormal end state mean fragility produce same mean *CDF* as compared with the one obtained initially for the point estimate analyses results presented in Annex A and Table 1.

The approximation introduce in this process can be displayed numerically (variation of A_m capacity for different acceleration bins) and graphically (bounding of plant state fragility points). The equivalent plant state lognormal fragility obtained at the end of this process has the property to conserve the mean seismic *CDF* and closely cross the plant state fragility points (corresponding to significant acceleration bins) that include the effect of random failures.

There are many advantages developing equivalent lognormal plant state mean fragility. One of advantage is that the lognormal fragility can be expanded to the full fragility function – recovering β_R and β_U using analytical relationships developed by the author [2] and finally the seismic CDF distribution can be obtained. Also seismic margin estimates can be easily developed including contribution of the random failures and human errors. The author believes that point estimates with these enhancements converge to results obtained by the accurate quantification process described at the beginning of Section 1.

5. REFERENCES

- [1] External Hazard PRA Standard ASME/ANS RA-Sa-2009.
- [2] O. Coman, “Some useful enhancements for Seismic PRA” ANS PSA 2013 International Topical Meeting on Probabilistic Safety Assessment and Analysis, Columbia, SC, Sept. 2013.
- [3] IAEA/ISSC EBP-WA2 – Draft Technical Report on “Seismic Probabilistic Safety Assessment Implementation Guidelines” (Draft 3 January 2014).

Annex A

ILLUSTRATIVE EXAMPLE

Table A1: Significant sequences/cutsets including random failures, human errors and partial CDFs for each acceleration Bin.

Seq. Freq.	Acc. Bin	H-Freq	S-IE	S-F1	SF-2	RF	HERR	DESC	Partial CDF
3.2E-09	S1	5.5E-01	1.0E-02	8.4E-04	3.4E-02		2.0E-02	A1*B1*HER1	CDF-S1 1.4E-07
1.4E-07	S1	5.5E-01	1.0E-02	8.4E-04			3.0E-02	A1*B1*RF2	
4.6E-07	S2	5.4E-03	5.4E-02	4.6E-02	3.4E-02			A1*B1*E1	CDF-S2 1.0E-06
1.5E-09	S2	5.4E-03	2.4E-02	2.6E-04			4.5E-02	T1*C1*HER2	
2.6E-07	S2	5.4E-03	2.4E-02	1.0E-02	1.9E-01			T1*A1*C1	
3.0E-07	S2	5.4E-03	5.4E-02	3.4E-02			3.0E-02	A1*E1*HER3	
7.0E-09	S2	5.4E-03	2.4E-02	1.9E-01	3.6E-03	7.5E-02		T1*C1*E3*RF3	
7.1E-07	S3	3.5E-04	1.6E-01	1.8E-01		7.3E-02		E1*A1*RF5	CDF-S3 7.9E-06
2.2E-06	S3	3.5E-04	1.6E-01	5.4E-01			7.5E-02	E1*C1*HER5	
2.2E-06	S3	3.5E-04	1.9E-01	2.4E-01	1.4E-01			A1*B1*E1	
2.8E-06	S3	3.5E-04	2.0E-01	5.4E-01		7.5E-02		T1*C1*RF3	
1.7E-10	S3	3.5E-04	2.0E-01	1.6E-03	1.6E-03			T1*E2*C2	
9.4E-09	S3	3.5E-04	1.9E-01	1.6E-03	8.7E-02			A1*E2*B2	
1.7E-09	S3	3.5E-04	5.7E-03	1.6E-03	5.4E-01			M1*E2*C1	
1.7E-10	S3	3.5E-04	5.7E-03	1.6E-03			5.5E-02	M1*E2*HER4	
1.7E-06	S4	5.7E-05	3.4E-01	6.2E-01	1.4E-01			A1*B1*E1	CDF-S4 4.8E-06
1.1E-06	S4	5.7E-05	4.3E-01	6.2E-01		7.5E-02		T1*C1*RF3	
9.6E-09	S4	5.7E-05	4.3E-01	2.0E-02	2.0E-02			T1*E2*C2	
1.4E-07	S4	5.7E-05	3.4E-01	2.0E-02	3.7E-01			A1*E2*B2	
3.9E-10	S4	5.7E-05	2.3E-02	2.0E-02		1.5E-02		M1*E2*RF4	
5.2E-10	S4	5.7E-05	2.3E-02	2.0E-02	2.0E-02			M1*E2*C2	
1.3E-09	S4	5.7E-05	2.3E-02	2.0E-02		5.0E-02		M1*E2*RF7	
1.1E-06	S4	5.7E-05	4.3E-01	6.2E-01		7.5E-02		T1*C1*RF3	
6.1E-08	S4	5.7E-05	2.3E-02	6.2E-01			7.5E-02	M1*C1*HER5	
6.5E-07	S4	5.7E-05	3.4E-01	6.2E-01			5.5E-02	E1*C1*HER4	
1.4E-06	S5	3.6E-06	6.4E-01	9.0E-01	6.8E-01			A1*B1*E1	CDF-S5 1.9E-06
2.1E-07	S5	3.6E-06	8.4E-01	9.0E-01		7.5E-02		T1*C1*RF3	

Seq. Freq.	Acc. Bin	H-Freq	S-IE	S-F1	SF-2	RF	HERR	DESC	Partial CDF
3.8E-08	S5	3.6E-06	8.4E-01	1.1E-01	1.1E-01			T1*E2*C2	
1.9E-07	S5	3.6E-06	6.4E-01	1.1E-01	7.3E-01			A1*E2*B2	
1.8E-08	S5	3.6E-06	5.0E-02	1.1E-01	9.0E-01			M1*E2*C1	
4.3E-07	S6	6.0E-07	8.0E-01	9.9E-01	9.0E-01			A1*B1*E1	CDF-S6 1.1E-06
3.1E-07	S6	6.0E-07	9.6E-01	9.9E-01	5.4E-01			T1*C1*E3	
8.0E-08	S6	6.0E-07	9.6E-01	3.7E-01	3.7E-01			T1*E2*C2	
1.7E-07	S6	6.0E-07	8.0E-01	3.7E-01	9.5E-01			A1*E2*B2	
7.0E-08	S6	6.0E-07	3.1E-01	3.7E-01	9.9E-01			M1*E2*C1	
Total CDF									1.7E-05

Notes:

H-Freq = Seismic event frequency for acceleration bin #1, ...6

S-IE = conditional probability of seismic initiating event

SF-1, SF-2 = seismic failures conditional probabilities

RF = Non-seismic random failure probability

HERR = Human error associated to operator's recovery actions

Table A2: Seismic Fragility Functions

<i>BE</i>	<i>A_m</i>	<i>HCLPF</i>	<i>β_U</i>	<i>β_R</i>	<i>β_C</i>
A1	0.30	0.15	0.23	0.20	0.3
A2	0.54	0.25	0.25	0.22	0.33
A3	0.99	0.35	0.34	0.29	0.45
B1	0.45	0.20	0.26	0.23	0.35
B2	0.56	0.25	0.26	0.23	0.35
T1	0.38	0.15	0.30	0.26	0.40
M1	1.28	0.40	0.38	0.33	0.50
M2	2.01	0.50	0.45	0.39	0.60
M3	1.61	0.45	0.42	0.36	0.55
E1	0.57	0.20	0.34	0.29	0.45
E2	1.14	0.45	0.30	0.26	0.40
E3	0.96	0.30	0.38	0.33	0.50
C1	0.34	0.15	0.26	0.23	0.35
C2	1.14	0.45	0.30	0.26	0.40
C3	1.59	0.50	0.38	0.33	0.50

Table A3: Random Failures

Non Seismic Random Failures	
RF1	2.50E-03
RF2	2.00E-02
RF3	7.50E-02
RF4	1.50E-02
RF5	7.30E-02
RF6	2.00E-02
RF7	5.00E-02
RF8	3.00E-03

Human Errors	
HER1	2.50E-02
HER2	2.50E-02
HER3	3.00E-02
HER4	5.50E-02
HER5	7.50E-02