A Computer Program for Evaluating the Alpha Factor Model Parameters Using the Bayesian Operation

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Abstract: The assessment of common cause failure (CCF) is necessary for reducing the uncertainty during the process of probabilistic safety assessment. A basic unavailability assessment method is an approach for the quantitative analysis of CCF modeling using Bayesian probability, in which the estimation of parameters is more accurate by combining the failure information from system, component and cause level. This study describes the CCF evaluation program which has been developed for assessing the α-factor common cause failure parameters. Examples are presented to demonstrate the calculation process and necessary databases are presented. As a result, the posterior distributions for α-factors model parameter are obtained using the conjugate family distributions as well as general distributions for conducting a numerical estimation.

Due to the fact that CCF is one of the significant factors to affect both core damage frequency and large early release frequency, the appropriate evaluation for the relevant parameters is essential, though there are rare the CCF data. In the previous study, the Multiple Greek Letter model (MGL) had been used for modeling the common cause failures in the OPR 1000 reactors. In the future modeling for the reactors, the α-factors approach might be employed for simulating the common cause failures as well as it will be quantified using the computer program developed by the C# language. The main operation to quantify the α-factors parameters is Bayesian which combines the prior distribution and the likelihood function to produce the posterior distribution. It is expected that this program might contribute to enhancing the quality of probabilistic safety assessment and to reducing common cause failure uncertainty.

Keywords: common cause failure, Multiple Greek Letter model, α-factors model, Bayesian, parameter uncertainty, data analysis

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1. INTRODUCTION

Since the quantitative analysis about the Common Cause Failure (CCF) was first attempted in WASH-1004 [1], the various quantitative CCF models have been suggested. These models for the CCF analysis were synthetically analyzed in NUREG/CR-4780 [2-3] and have been mostly used in the probability safety assessment (PSA). The Common Cause Failure events occur when one or more components fail simultaneously or around same time due to a common cause. Due to the fact that CCF is one of the significant factors to affect Core Damage Frequency and Large Early Release Frequency, systematic and accurate evaluation of the CCF parameter is essential. However, it is practically impossible to perform CCF analysis accurately with the lack of CCF data just for one Nuclear Power Plant because CCF is rare event. To address this problem, NUREG/CR-5485 [4] reported the various CCF parameter estimation methods based on the Bayesian method, which can reduce the CCF parameter uncertainty using CCF data not only of the reference Nuclear Power Plants but also of other NPPs.

Up to now, PSA in Korea has performed the CCF analysis using beta factor model or Multiple Greek Letter (MGL) model and mainly used the CCF parameter data provided by USNRC. The CCF parameter data provided by USNRC, however, is currently the CCF parameter data of Alpha-Factor Model (AFM). Thus, the CCF parameters used for the CCF analysis in Korea have used MGL parameters converted form AFM parameters to MGL parameter. In this case, the PSA researchers have not performed the Bayesian updating for reducing uncertainty of the MGL parameters due to the fact that conversions of these CCF parameters cannot know the MGL parameters distribution. For this reason, therefore the MGL parameter distribution has been used popularly to perform PSA for Korean plants.

In this study, therefore, the Bayesian-based CCF parameters distribution estimation program using α-factors parameter data has been developed for evaluating the CCF parameters and applied for the system of the reference plant to obtain the uncertainty data of the parameters in MGL model. The calculation algorithm of this program using the Bayesian method has been developed and programmed by the C# language in this study.

2. METHODOLOGY

2.1. CCF parametric models

The Bayesian-based CCF parameters estimation program developed in this study consists of various CCF models. Multiple Greek Letter (MGL) model and Alpha-Factor Model (AFM) as shown in table 1 are important in this study. The estimation approaches of each parameter of the CCF model are different. However, these CCF models have same value when parameters of the MGL model and AFM model can be converted to parameter of Basic Parameter Model (BPM) following the conversion formulas described in table 2. Depending on the reason of these, the developed program in this study uses the relation of MGL and AFM.

Table 1. Parameters of the CCF model

CCF models	**CCF parameter**	
	Non-Staggered testing (NS)	Staggered testing (S)
MGL	$\rho_k^{NS} = \sum_{i=k}^{m} in_i \Big/ \sum_{i=k-1}^{m} in_i$	$\rho_k^{S} = \sum_{i=k}^{m} n_i \Big/ \sum_{i=k-1}^{m} n_i$
	$2 \le k \le m$ $(\rho_1 = 1, \rho_2 = \beta, \rho_3 = \gamma, \rho_4 = \delta, \ldots, \rho_{m+1} = 0)$	
AFM	$\alpha_k = n_k \Big/ \sum_{k=1}^{m} n_k$	
	$1 \le k \le m$	

Table 2. CCF conversion parameters to Basic Parameter Model

CCF models	**CCF parameter**	
	Non-Staggered testing (NS)	Staggered testing (S)
MGL	$Q_k^{NS} = \frac{\left(\prod_{i=1}^{k} \rho_i^{NS}\right) \cdot \left(1 - \rho_{k+1}^{NS}\right) \cdot Q_t^{NS}}{{}_{m-1}C_{k-1}}$ $= \frac{Q_t^{NS}}{{}_{m-1}C_{k-1}} \cdot \frac{kn_k}{\sum_{k=1}^{m} kn_k}$	$Q_k^{S} = \frac{\left(\prod_{i=1}^{k} \rho_i^{S}\right) \cdot \left(1 - \rho_{k+1}^{S}\right) \cdot Q_t^{S}}{{}_{m-1}C_{k-1}}$ $= \frac{Q_t^{S}}{{}_{m-1}C_{k-1}} \cdot \frac{n_k}{\sum_{k=1}^{m} n_k}$
	$1 \leq k \leq m$ $(\rho_1 = 1, \rho_2 = \beta, \rho_3 = \gamma, \rho_4 = \delta, \ldots, \rho_{m+1} = 0)$	
AFM	$Q_k^{NS} = \frac{k}{{}_{m-1}C_{k-1}} \cdot \frac{\alpha_k}{\alpha_t} \cdot Q_t^{NS}$ $= \frac{Q_t^{NS}}{{}_{m-1}C_{k-1}} \cdot \frac{kn_k}{\sum_{k=1}^{m} kn_k}$	$Q_k^{S} = \frac{k}{{}_{m-1}C_{k-1}} \cdot \alpha_k \cdot Q_t^{S}$ $= \frac{Q_t^{S}}{{}_{m-1}C_{k-1}} \cdot \frac{n_k}{\sum_{k=1}^{m} n_k}$
	$1 \leq k \leq m$	

2.2. CCF Multiplier

The conversion formulas in Table 2, however, cannot be practically used due to fact that the total failure probability of each component (Q_t) is unavailable generally. Due to the fact that the CCF events rarely occur, the values of Q_t and Q_1 are almost the same as shown in Eq. (1).

$$Q_t \approx Q_1 \qquad (1)$$

Therefore, for calculation of the failure probability involving k specific components (Q_k), the CCF multiplier (M_k) is first calculated. Q_k is converted to the approximated equations to be practically used as shown in Eq. (2). In these equations, M_k and Q_1 are CCF multiplier and single failure probability, respectively. The program developed in this study was designed to easily calculate CCF multipliers for both MGL and AFM models as shown in Table 3.

$$Q_k \approx M_k \cdot Q_1 \qquad (2)$$

Table 3. CCF Multipliers Formula

CCF models	CCF parameter	
	Non-Staggered testing (NS)	Staggered testing (S)
M G L	$M_k^{NS} = \frac{\left(\prod_{i=1}^{k} \rho_i^{NS}\right) \cdot \left(1 - \rho_{k+1}^{NS}\right)}{{}_{m-1}C_{k-1}}$ $= \frac{1}{{}_{m-1}C_{k-1}} \cdot \frac{kn_k}{\sum_{k=1}^{m} kn_k}$	$M_k^{S} = \frac{\left(\prod_{i=1}^{k} \rho_i^{S}\right) \cdot \left(1 - \rho_{k+1}^{S}\right)}{{}_{m-1}C_{k-1}}$ $= \frac{1}{{}_{m-1}C_{k-1}} \cdot \frac{n_k}{\sum_{k=1}^{m} n_k}$
	$1 \leq k \leq m$ $(\rho_1 = 1, \rho_2 = \beta, \rho_3 = \gamma, \rho_4 = \delta, \ldots, \rho_{m+1} = 0)$	
A F M	$M_k^{NS} = \frac{k}{{}_{m-1}C_{k-1}} \cdot \frac{\alpha_k}{\alpha_t}$ $= \frac{1}{{}_{m-1}C_{k-1}} \cdot \frac{kn_k}{\sum_{k=1}^{m} kn_k}$	$M_k^{S} = \frac{k}{{}_{m-1}C_{k-1}} \cdot \alpha_k$ $= \frac{1}{{}_{m-1}C_{k-1}} \cdot \frac{n_k}{\sum_{k=1}^{m} n_k}$
	$1 \leq k \leq m$	

2.3. Bayes' Theory

Up to now there has been no detail guidance for estimating the CCF parameters using the Bayesian method in an uncertainty analysis. In this study the Bayesian method has been introduced and applied for reducing the uncertainty of the common cause failures. The basic equation of the Bayesian theory [5-6] is described as follows.

$$\pi(\theta / E) = \frac{L(E/\theta)\pi_0(\theta)}{\int L(E/\theta)\pi_0(\theta)} \quad (3)$$

Where, $\pi(\theta / E)$ is posterior distribution of θ given evidence E, $\pi_0(\theta)$ is distribution of θ prior to the evidence, and $L(E / \theta)$ is likelihood function or the probability of the evidence E for the given θ.

Table 4. Conjugate family distributions of Bayes' operation

Likelihood	Conjugate prior	Posterior
Binomial Distribution	Beta distribution	Beta distribution
Multi-nominal distribution	Dirichlet distribution	Dirichlet distribution

The likelihood function for common cause failure parameters depends on the common cause failure models and has the corresponding conjugate distribution as shown in Table 4. The posterior distribution is proportional to the product of prior distribution and the likelihood function as shown in Eq. 4. By applying the normalization factors into denominator, the Bayesian data analysis for the common cause failure parameters can be performed to result in determining the posterior distribution as shown in Eq. (2).

$$\pi(\theta / E) \propto L(E/\theta) \cdot \pi_0(\theta) \quad (4)$$

Bayesian analysis can be used as an efficient data analysis method among moments method, the curve fitting method, maximum likelihood method and Bayes' method.

2.4. Parameter distributions

The C# program named COCAP (Common Cause Failure Parameter Estimation for PSA), has been developed and applied to obtain the common cause failure parameters distribution. The parameters distribution of the common cause failure models can be obtained using the Beta conjugate family as sown in Table 4. The Bayesian approach results are shown in Table 5. It involves the systematic approach about how to combine the prior distribution and the likelihood function to produce the posterior distribution. Thus, the developed program, COCAP, can not only estimate the CCF parameter distribution but also perform the Bayesian updating of the CCF parameters of the important systems in the reference nuclear power plants.

Table 5. Distribution of the CCF parameter

CCF models	CCF parameter distribution	
M G L	Beta distribution (a, b)	
	Non-Staggered (NS) testing	Staggered (S) Testing
	$a = a_0 + \sum_{i=k}^{m} in_i$ $b = b_0 + \sum_{i=k-1}^{m} in_i - \sum_{i=k}^{m} in_i$	$a = a_0 + \sum_{i=k}^{m} n_i$ $b = b_0 + \sum_{i=k-1}^{m} n_i - \sum_{i=k}^{m} n_i$
	$2 \le k \le m$	
A F M	Beta distribution (a, b)	
	$a = a_0 + n_k$ $b = b_0 + \left(\sum_{k=1}^{m} n_k \right)$	
	$1 \le k \le m$	

3. RESULTS

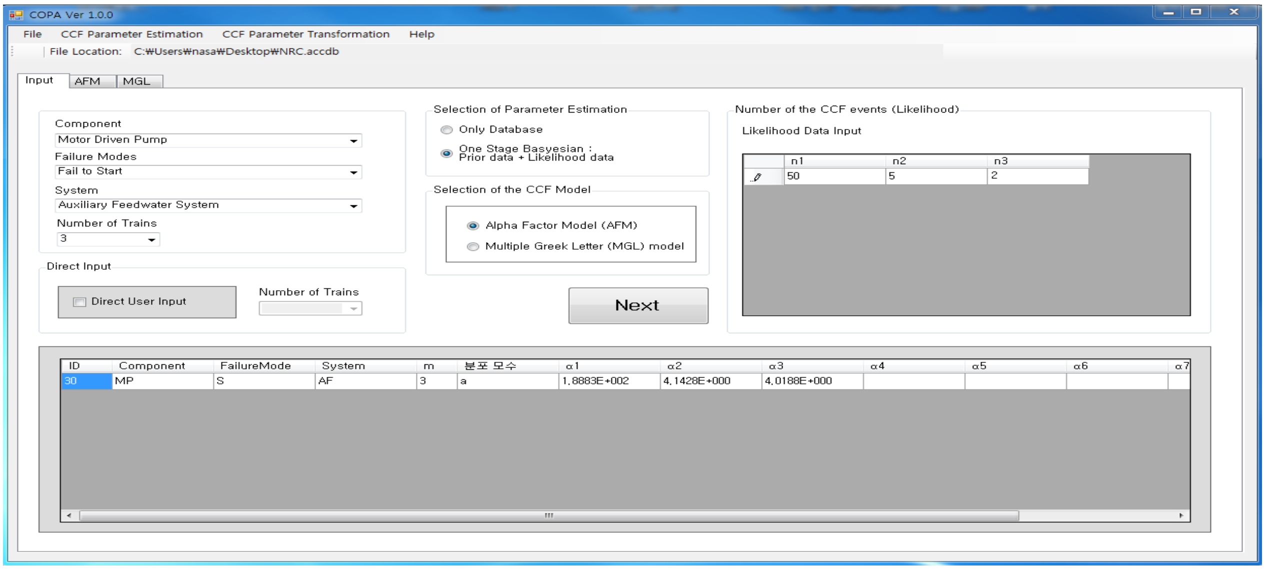
3.1. A Computer Program, COCAP

To evaluate the parameter distributions and the common cause failure uncertainty, a computer program named COCAP (Common Cause Failure Parameter Estimation for PSA), has been developed using the C# language. Figure 1 shows the main screen of the program developed in this study. This program might contribute to evaluating easily the common cause failure analysis related to the Multiple Greek Letter model and Alpha-Factor Model. The developed program involves the Common Cause Component Group (CCCG = m) which may be applied from a value of 2 to 8. The output data of the user-selected CCF parameters offered in this program provides prior, likelihood and posterior distribution for the parameters of the Multiple Greek Letter model and Alpha-Factor Model as shown in Figure 2. Each distribution provides the confidence interval such as 5 %, 50 %, and 95 %, mean values for the Beta distribution (a, b).

The COCAP computer program consists of four functions which are Input module, Conversion module, Run module and Help module as shown in table 6. The Input module can deal with USNRC Alpha-Factor Model CCF parameters [7] in which users can select the appropriate CCF parameters considering the relevant system, failure mode, type of component, and system size. The Conversion function module can convert from AFM parameters to MGL parameters easily. The common cause failure multiplier function supplies the function to calculate the multiplying values for the CCF parameters selected by the analyzing user. The Bayesian analysis function module supplies the function to perform the Bayesian-updating for the CCF parameter when the user allocates the actual number of the CCF events in likelihood menu of the computer program.

Table 6 : Main functions used in the computer program, COCAP

Main function	Description
Input data	Connect database
	Selection of NRC CCF data (2007)
Conversion	AFM to MGL conversion (CCF parameter)
	MGL to AFM conversion (CCF parameter)
Bayesian analysis	Bayesian analysis of AFM/MGL parameter distribution
	AFM/MGL multiplier calculation



File Location: C:₩Users₩nasa₩Desktop₩NRC.accdb

Component: Motor Driven Pump

Failure Modes: Fail to Start

System: Auxiliary Feedwater System

Number of Trains: 3

Direct Input

☐ Direct User Input

Number of Trains:

Selection of Parameter Estimation

○ Only Database

● One Stage Basyesian : Prior data + Likelihood data

Selection of the CCF Model

● Alpha Factor Model (AFM)

○ Multiple Greek Letter (MGL) model

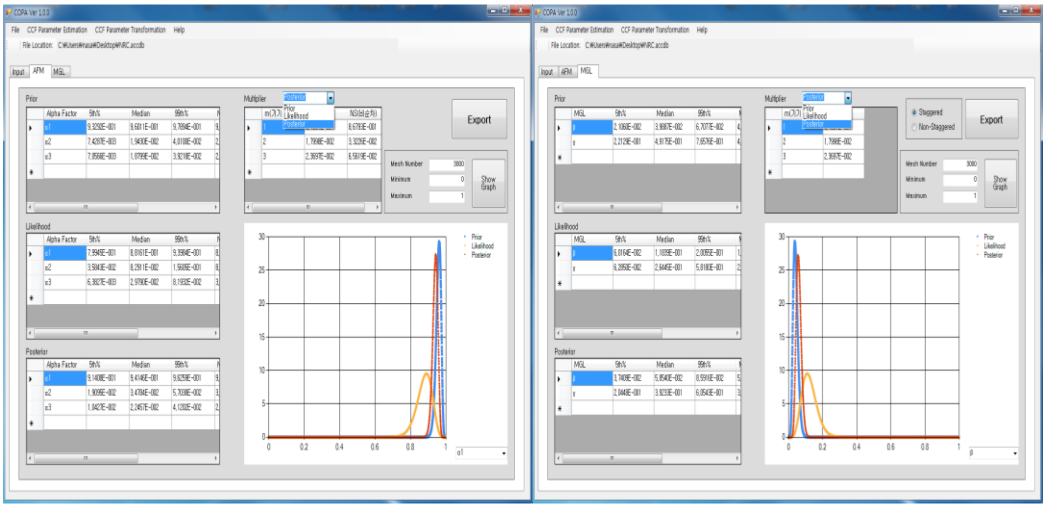
Number of the CCF events (Likelihood)

Likelihood Data Input

n1	n2	n3
50	5	2

ID	Component	FailureMode	System	m	분포 모수	α1	α2	α3	α4	α5	α6	α7
30	MP	S	AF	3	a	1,8883E+002	4,1428E+000	4,0188E+000				

Figure 1. Main screen of the program



(a) AFM parameter output frame **(b) MGL parameter output frame**

Figure 2. Parameter distribution Output obtained from the program

3.2 Execution of the computer program

The common cause failure parameters have been evaluated using the program, COCAP developed in this study under the particular assumed conditions. For demonstrating the program execution process, the prior data of the auxiliary feedwater system, motor driven pump, and “fail to start” for CCCG=3 are utilized. The USNRC CCF parameter estimations used in this study are shown in Table 7 while the likelihood data are shown in the Table 8. To execute the developed program, the parameters data of the CCF models have been calculated individually using the MGL model and AFM model and they are compared each other. Table 9 shows the Bayesian updating results showing that the calculated CCF multiplier values are exactly identical regardless of the model types depending on the conditions. It is shown that the program, COCAP might evaluate the parameter distributions of the MGL model and AFM model properly without any problems.

Table 7. Prior data

CCCG	Fail mode	α-factors		Alpha 1	Alpha 2	Alpha 3
m=3	Fail to Start	Parameter of beta distribution	a	1.89E+02	4.14E+00	4.02E+00
			b	8.16E+00	1.93E+02	1.93E+02
		Percentiles	5%	9.33E-01	7.43E-03	7.06E-03
			50%	9.60E-01	1.94E-02	1.88E-02
			95%	9.79E-01	4.01E-02	3.92E-02
		Mean		9.59E-01	2.10E-02	2.04E-02

Table 8. Likelihood data

CCCG	Fail mode	α-factors		Alpha 1	Alpha 2	Alpha 3
m=3	Fail to Start	Parameter of beta distribution	a	5.00E+01	5.00E+00	2.00E+00
			b	7.00E+00	5.20E+00	5.50E+01
		Percentiles	5%	7.99.E-01	3.58.E-02	6.38.E-03
			50%	8.82.E-01	8.29.E-02	2.98.E-02
			95%	9.40.E-01	1.56.E-01	8.19.E-02
		Mean		8.77.E-01	8.77.E-02	3.51.E-02

Table 9. Posterior data and CCF multiplier

CCF models		CCCG	Fail mode	CCF parameters		Alpha 1	Alpha 2	Alpha 3
AFM		m=3	Fail to Start	Parameter of beta distribution	a	2.39.E+02	9.14.E+00	6.02.E+00
					b	1.52.E+01	2.45.E+02	2.48.E+02
				Percentiles	5%	9.14.E-01	1.91.E-02	1.04.E-02
					50%	9.41.E-01	3.48.E-02	2.25.E-02
					95%	9.63.E-01	5.70.E-02	4.12.E-02
				Mean		9.40.E-01	3.60.E-02	2.37.E-02
				CCF multiplier	**NS**	**8.86E-01**	**3.32E-02**	**6.56E-02**
					S	**9.40E-01**	**1.80E-02**	**2.37E-02**
CCF models		**CCCG**	**Fail mode**	**CCF parameters**		**β**	**γ**	
MGL	Non-Staggered	m=3	Fail to Start	Parameter of beta distribution	a	1.00.E-01	3.62.E-01	
					b	1.31.E-01	4.97.E-01	
				Percentiles	5%	1.67.E-01	6.32.E-01	
					50%	1.32.E-01	4.97.E-01	
					95%	3.63.E+01	1.81.E+01	
				Mean		2.39.E+02	1.83.E+01	
				CCF multiplier	**NS**	**8.86E-01**	**3.32E-02**	**6.56E-02**
	Staggered	m=3	Fail to Start	Parameter of beta distribution	a	3.74.E-02	2.04.E-01	
					b	5.85.E-02	3.92.E-01	
				Percentiles	5%	8.59.E-02	6.05.E-01	
					50%	5.97.E-02	3.97.E-01	
					95%	1.52.E+01	6.02.E+00	
				Mean		2.39.E+02	9.14.E+00	
				CCF multiplier	**S**	**9.40E-01**	**1.80E-02**	**2.37E-02**

4. CONCLUSION

The common cause failures in nuclear power plants are one of the significant factors to affect both the values of Core Damage Frequency and Large Early Release Frequency. Several investigators have made many efforts to attempt to reduce the common cause failure uncertainty. In this study a methodology using the Bayesian operation has been developed and applied for reducing the uncertainty of the common cause failures. The Bayesian-based CCF parameters estimation program called COCAP has also been developed for evaluating the common cause failures parameters. It is shown that the COCAP program can obtain the parameter distributions for various common cause failure models for both the non-staggered and the staggered tests. It is also shown that this program might contribute to assessing the safety measures such as core damage frequency and large early release frequency. It is expected that the developed program might contribute to supplying the efficient risk assessment procedures by evaluating the uncertainty of the relevant common cause failures parameters of the analyzing systems.

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