Stress-Dependent Weibull Shape Parameter Based on Field Data

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Abstract: The Weibull shape parameter is often assumed to be constant, with no dependency on stress. However, some cases exist, in which it is a function of stress. If the stress-dependency is not considered, vague assumptions of the Weibull shape parameter may lead to inaccurate results, e. g. for reliability prediction or demonstration testing purposes. Drawbacks in choosing an adequate parameter are e.g. extensive testing at a specific stress level, or insufficiently established mathematical descriptions.
This paper presents an approach which allows a stress-dependent derivation of the Weibull shape parameter based on field data. In order to do so, simulations of the customer behavior and additional information from the customers themselves are used. Linking the occurred failure with the corresponding stress-level is thus possible.

Keywords: Non-constant Weibull Shape Parameter, Stress-dependency, Field Data, Customer Behavior, Automotive Engineering.

1. INTRODUCTION

The Weibull distribution is commonly used in mechanical engineering for the characterization of the failure behavior of specific components and systems. The failure behavior of certain parts in combination with certain failure modes can be described by using a shape parameter. A specific value is often assumed to be constant, e.g. in the case of roller bearings. However, for other components such as gears and shafts, the shape parameter is non-constant, as it also depends considerably on the load [1]; in this case, the values are within a specific range, e.g. known from literature. Yet the dependency often cannot be described by a function. Hence, regarding a reliability prediction for future products under consideration of these dependencies, certain challenges need to be faced:
Firstly, to describe the dependencies by a statistically representative function, extensive testing would be required for each failure mode of a given component, which leads to considerable costs. The correlation between the component's behavior under testing conditions and field use has to be known. This is to ensure that the shape parameter is the same under both conditions. Secondly, gaining relevant customer data for automotive applications in the private sector is generally difficult, as strict legal constraints apply. In case of a failure, conclusions on the actual load history, i.e. the stress endured, can hardly be drawn systematically. Therefore, the failure of interest is allocated to the set of failures under field conditions, resulting in a single shape parameter for failures actually stemming from different load scenarios.

The goal is to find a method which allows for describing the shape parameter more realistically based on the load history actually experienced in the field and declared by the user. The field data on hand could then be used more efficiently and effectively; expenditures for testing could be minimized even further. The stress-dependent determination of the shape parameter will lead to a more realistic reliability prediction for the given failure mode to be fed back as input for future applications or developments. It can also be used for reliability demonstration testing. Its benefits are clarified by a brief example regarding the success run (see [2]): Assuming a reliability of required lifetime $R = 0.9$, a confidence level $P_A = 0.9$, a lifetime ratio $L_V = 0.6$, an acceleration factor $r = 3$ and a "constant" shape parameter $b = 1.5$, results in a required sample size $n = 12$. If the shape parameter can be described more precisely concerning lower stress in the field ($b_{field} = 1.3$) and higher stress in test ($b_{test} = 1.7$) the sample size will be reduced to $n = 8$.

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In order to make field data usable for efficient analysis, case-dependent characteristics need to be gathered, e.g. driving style and driver type [3]. In case of an occurred failure these data are gathered by collecting information directly from the customer. By means of a sum of square error approach this information is matched with statistical data. In order to classify the user data, simulations for different load scenarios are implemented. The simulated load time functions are used to calculate damage values by means of the damage accumulation hypothesis [4]. Based on these results, the required rules for correctly combining the customer data are derived. The lifetime characterizing unit (e.g. miles, hours or load cycles) is then linked with the stress-intensity. The data used are multiple censored data. A Weibull analysis of the stress-specific data using Sudden-Death approaches [1] leads to stress-specific shape parameters derived from field data.

The Section below discusses briefly the shape parameter and the assumption of its stress-dependency. The third Section describes the customer´s role and the way how inference from the customer´s behavior about the applied stress can be drawn. Section 4 introduces the entire approach step-by-step whereas Section 5 and 6 provide a short example and conclusion.

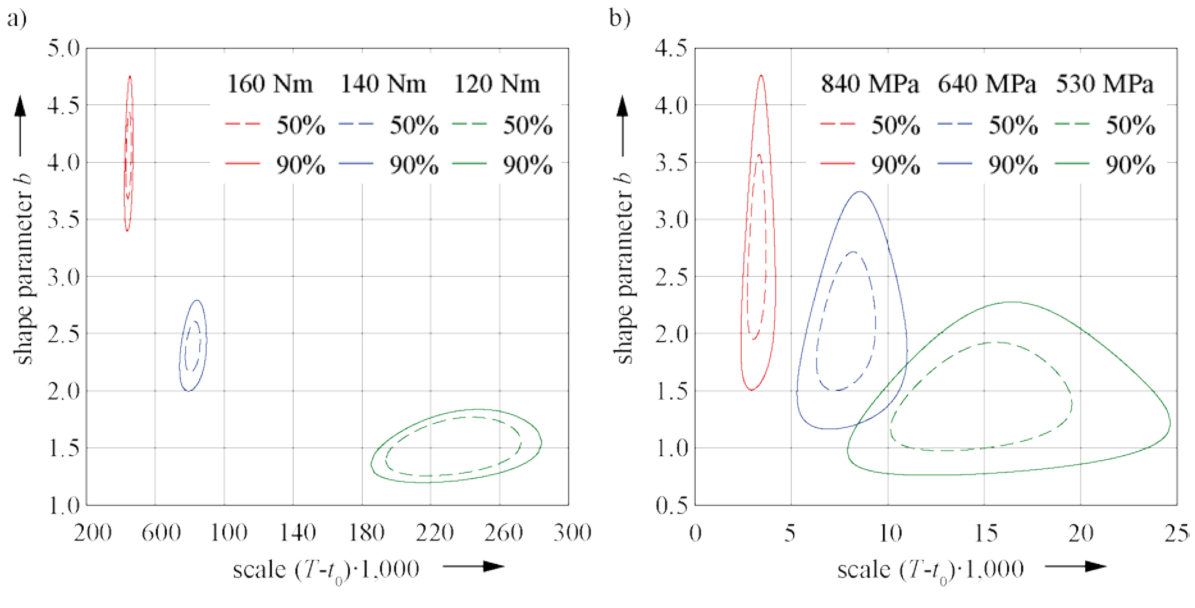
2. STRESS-DEPENDENT SHAPE PARAMETER

The Weibull distribution is determined by its parameters: the shape parameter b, the characteristic life (or scale parameter) T and – in case of a three parameter Weibull – the failure free time t_0. All these parameters are dependent upon geometry, material, machining and stress [1]. In this paper, the focus is on the stress-dependency of the shape parameter.

Experiments with some components such as gears and shafts have shown that the shape parameter depends significantly on the load [1]. In many cases, a higher stress yields a higher shape parameter. However, a steep Weibull does not immediately go along with high stress. A large shape parameter corresponds with a small variation in the times to failure. This fact is for instance used to control the quality of turbine blades made from metal as purer metal results in steeper shape parameter than dirtier metals [5]. The stress-dependency is shown by several conducted experiments:

Maenning [6] demonstrated a stress-dependency for shafts made from C35. The observed failure mode is crack due to fatigue. He proved his findings by experiments on 19 stress levels from 295 up to 385 MPa with at least 20 specimens in each level. A higher stress results in a larger shape parameter.

Figure 1: Stress-Dependent Weibull Shape Parameter as well as the Characteristic Life Including their Confidence Intervals; a) Brodbeck and b) Groß



The same dependency is determined for gears with failure mode crack. Groß [7] tested three samples with 12 specimens each at 530, 640 and 840 MPa. He used gears made from 42CrMo4V. Brodbeck [8] considered carburized steel 16MnCr5 gears and carried out extensive tests at 120, 140 and 160 Nm with each 100 specimens. Results from Groß and Brodbeck are summarized in Figure 1. Here, a three parameter Weibull analysis is applied.

Beier [9] revealed a similar positive correlation for gear pairs made from plastic. In this case the observed failure modes were crack due to fatigue and additional wear. He investigated at least 8 specimens in different variants. The occurred mechanisms are discussed separately.

Nelson [10] states that experience confirms the dependency between the shape parameter and stress levels in terms of metal fatigue and even roller bearings. Bergling on the other hand [11] assumes a constant shape parameter for roller bearings. Additionally, for some electrical insulation Nelson suggests a negative correlation, i.e. a higher shape parameter at lower stress.

3. APPLIED STRESS BY THE CUSTOMER

Field data is generally one of the most important resources that can be used by any reliability program. No data will better demonstrate the true reliability of a product, nor identify the failure modes that exist in the field. Thus, a more comprehensive reliability database can be created by incorporating field usage data. Particularly, information about customer behavior helps to differentiate the individually occurring stress that leads to failure. For instance, Lucas et al. [12] introduced a complementary FRACAS approach, which focuses on differentiated usage conditions linked to failed and non-failed units, in terms of the oil and gas industry.

Normally, in automotive engineering, important data on operating and environmental conditions are missed to draw interference about the applied stress leading to the specific failure. On the other hand, in case of leasing models, e.g. car-sharing or car rental, gathering stress-related data should be possible. Leopold [13] analyzed several sources of reliability data during operating time. A common drawback is the missing link between the applied stress and an occurred failure, especially for non-commercial passenger cars. For this reason, the influences on stress from customer behavior are discussed first, followed by a proposed adequate procedure which is implemented in the entire approach in Section 4 to challenge this fact.

3.1. Influences on Stress from Customer Behavior

In general, the stress on a certain part depends on the operating and environmental conditions of the product. Both are immediately affected by customer behavior. In the context of automotive engineering, particularly powertrain components, the resulting stress is a combination of the driver, the road and the vehicle. Depending on the actual product design, the externally applied load on the product is broken down into single parts as stress. Thus, the applied stress, as a function of the individual load, ultimately leads to wear and fatigue failures at different times [3].

The driver's - respectively the customer's - influence includes the driving style and the loading. In a wider sense the road type can be inferred as well, as the driver certainly chooses the road. The driver decides where the product is used and the ways in which it is used. These external criteria are listed in Table 1. These are in line with the 3F method introduced by Kücükay [14]. The method incorporates three dimensions which can be represented by the criteria. Each criterion is expressed in characteristic attributes.

Consequent, there are 48 (equal to 4 x 4 x 3) combinations of customer types. By means of simulation of these combinations, incorporating the results from field experiments, Müller-Kose [15] pointed out some examples which indicate different damage values for several combinations. The ranking of the customer type combinations regarding the damage values depends on the focused part and failure mode as well.

Table 1: Stress Influencing Criteria and Characteristic Attributes, c.f. [3,15]

Criteria	Description	Characteristic attributes
Road type	Proportion of total mileage on various types of roads such as motorway, rural road, urban traffic or mountain road	Motorway, rural road, urban traffic or mountain road
Loading	Percentage distribution of journeys with numbers of passengers, cargo and trailer weight	Light, average, heavy or extreme (trailer)
Driving style	Shifting frequency, gearshift engine speed, acceleration habits in town (moving-off from traffic lights), on rural roads (when leaving built up areas) and on the motorway (overtaking)	Sporting, average or moderate

As mentioned above, different vehicle configurations result in a different stress related to a certain part. In other words, a theoretical identical customer behavior leads to different stress due to different vehicle configurations. For instance, there are 64 different vehicle configurations when assuming each two car types, engines, moving-off elements, hybrid elements, main gearboxes and rear axle drives [16]. However, a certain part, e.g. a definite gear, is assembled in all configurations. This fact can be used to increase the database in reference to the field data approach.

3.2. Identification of the Customer Type Combination of Interest

Identification of the appropriate customer type combination enables inference about the applied stress. Required information is: a representative mapping of the behavior of all customers and on the other hand an assessment of the single customer whose product failed.

Müller-Kose collected a representative data set by means of comprehensive field experiments. The results given in Table 2 represent exemplary for each criterion a percentage allocation of the characteristic attributes. With this 11 customer types can be defined:

Table 2: Percentage Allocation of 11 Customer Types, Following [15]

[%]	Road type				[%]	Loading				[%]	Driving style		
Customer type	Urban traffic	Rural road	Mountain road	Motorway	Customer type	Light	Average	Heavy	Extreme (trailer)	Customer type	Sporting	Average	Moderate
Urban traffic driver	93	7	0	0	Light loading	94	6	0	0	Sporting driver	46	54	0
Rural road driver	4	92	0	4	Average loading	0	100	0	0	Average driver	6	86	8
Mountain road driver	10	50	30	10	Heavy loading	0	20	80	0	Moderate driver	5	10	85
Motorway driver	5	5	0	90	Extreme loading (trailer)	0	10	80	10				

If such a percentage allocation is known, this allocation can be matched with the customer behavior. In order to do this, the customer is encouraged to provide information if a failure occurred; e.g. to complete a questionnaire. The customer is then asked to estimate their preferred road type, loading and driving style. This results in an individual percentage allocation of the customer. For instance, customer "C" states, that the proportion of their total mileage is 20 % in urban traffic, 60 % on rural roads and 20 % on motorways. In the next step, based on this information the most proper customer type (see Table 2) is to be figured out. For this purpose, the sums of square errors are calculated (see Section 4). The customer type with the least square error represents the appropriate actual customer type. Repeating this analogously for loading as well as driving style yields the appropriate customer type combination of interest.

4. APPROACH

The following procedure is generally applicable for several parts and different failure modes. However, for the sake of simplicity it is exemplary introduced based on a gear pair of a rear axle drive. The observed failure mode is pitting due to fatigue. To transfer this on other parts and mechanisms, lifetime model and material data must be adapted among others. Here, the lifetime model is assumed to be valid for 50 % failure probability and the procedure is presented for identical products, i.e. an identical vehicle configuration.

Using this procedure, the following requirements must be taken as given: First, measurements are needed to establish a validated powertrain simulation for the vehicle longitudinal dynamics. Second, a geometry data set of the considered part is necessary. If the simulation does not provide the data directly, e.g. gears in a gearbox with just a simulated input or output rotation speed, this can be done indirectly by additional use of gear ratios. Third, material data and the underlying lifetime model are required, i.e. the Wöhler curve and its defining parameters respectively.

There are two main paths in the depicted algorithm (Figure 2):

- The determination of load values for each type. Lifetime model and material data is needed and damage values are determined (see I. simulation path).

- On the other hand, the given field data, its gathering procedure and Weibull analysis (see II. field path).

Both paths are combined in the algorithm. With this, the estimation of a stress-dependent Weibull shape parameter can be done and the optimization of the initial assumed lifetime exponent is enabled.

To simplify in the following remarks, “one” out of 48 customer type combinations is called “type *i*”. The implemented steps in the algorithm are as follows:

I. Simulation path

a) Simulate the load time function for each type i: The simulated variables are torque T_i and rotation speed n_i over time. The simulation is done for a given reference route. The reference route consists of a type-specific mixture of a representative road type course. The simulation ends, if the reference route length w_i is reached. In the automotive industry the use of shortened reference routes are standard in order to reduce simulation time significantly [3].

b) Transform the simulated torque T_i to stress σ_i by means of geometry data [3]: The stress of interest in case of pitting is the existing Hertzian stress. It can be calculated by using the standard ISO 6336 part 2 [17].

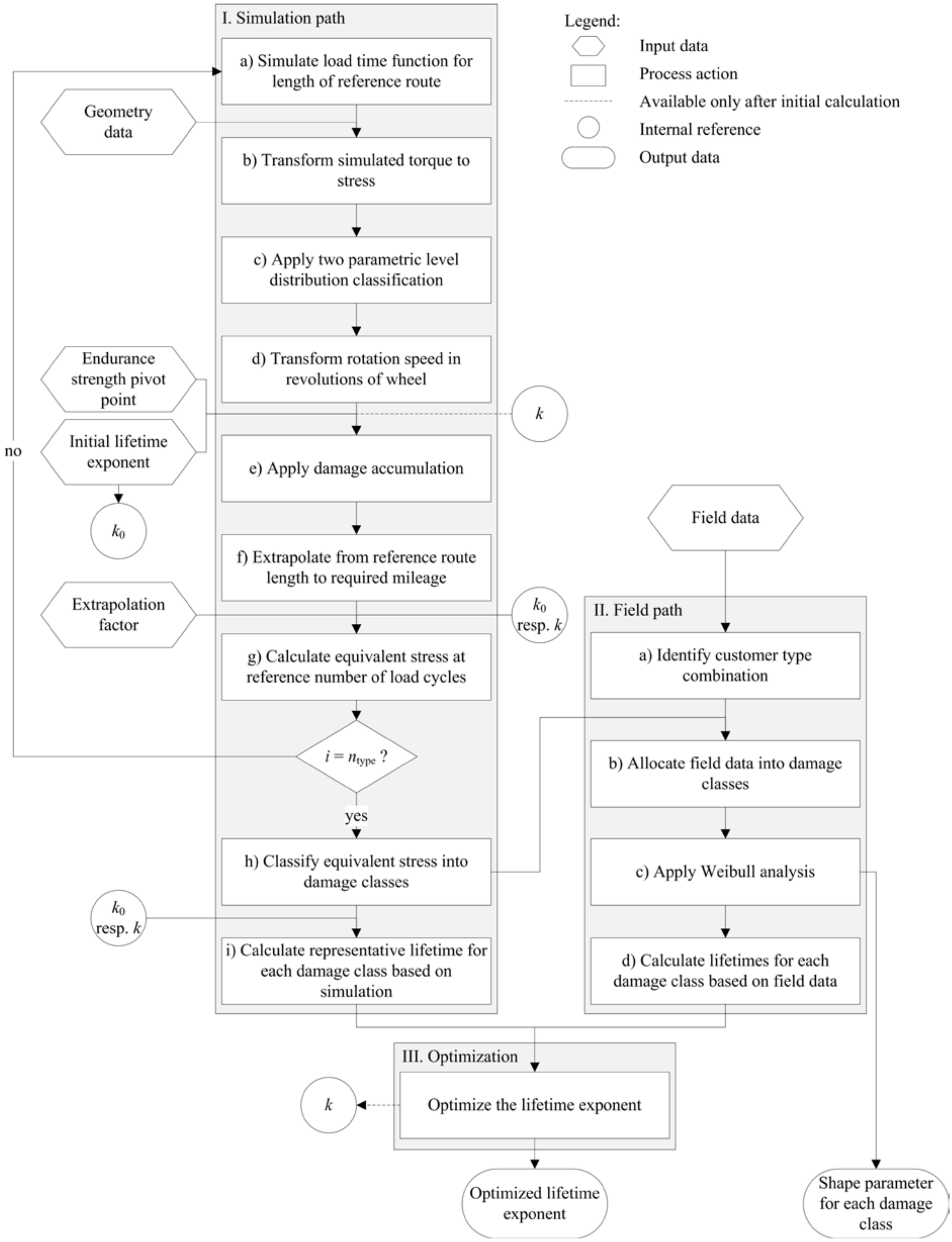
c) Apply the two parametric level distribution classification [1]: For each type i the results of this step are the dwell time $t_{i,j,v}$ in stress class j and rotation speed class v. The dwell times are a function of the reference route length w_i.

d) Transform the rotation speed $n_{i,j,v}$ [rpm] in revolutions of the wheel $r_{i,j}$ [absolute frequency], i.e. load cycles, per each stress class j by using equation

$$r_{i,j} = \sum_{v=1}^{n_v} \frac{n_{i,j,v}}{60\ \mathrm{s} \cdot \mathrm{rpm}} \cdot t_{i,j,v} \qquad (1)$$

with dwell time $t_{i,j,v}$ [s] for type i, stress class j and rotation speed class v [16].

Figure 2: Algorithm of the Approach



e) Apply the damage accumulation: In this case the Miner-Haibach damage accumulation hypothesis is assumed, due to experience in practice [3]. The specific Wöhler curve must be known. The slope of the fatigue strength zone is represented by the lifetime exponent k and the endurance strength pivot point is defined by the assumed number of endurance strength N_D

and the material specific endurance strength σ_D itself. First, an initial lifetime exponent k_0 is assumed. Afterward, for least square estimation purpose an optimized lifetime exponent k is used (see step III.). With this, the damage sum of the simulated reference route length $D_{i,w}$ is calculated by

$$D_{i,w} = \sum_{j=1}^{u} \frac{r_{i,j}}{N_D} \cdot \left(\frac{\sigma_{i,j}}{\sigma_D}\right)^{k} + \sum_{j=u+1}^{n_j} \frac{r_{i,j}}{N_D} \cdot \left(\frac{\sigma_{i,j}}{\sigma_D}\right)^{2k-1} \quad (2)$$

with $\sigma_{i,u} \geq \sigma_D$ and $\sigma_{i,u+1} < \sigma_D$.

f) Extrapolate from the reference route length w_i to the required mileage w_{req}: The required mileage is for instance assumed to 200,000 km. It is obvious that various customer type combinations lead to different damage values when this mileage is achieved. To compare these damage values from simulation, they are extrapolated to the required mileage. Each extrapolation factor EF_i is derived by the ratio w_{req}/w_i. Thus, the expected damage sum of type i, extrapolated up to the required mileage, is calculated by

$$D_i = EF_i \cdot D_{i,w} \,. \quad (3)$$

g) Calculate an equivalent stress level $\sigma_i(N_{ref})$ at a reference number of load cycles N_{ref} for each type i: Along with the assumption, that different types result in a different stress level about the required mileage, the underlying stress varies around a certain level if the stress is normalized at a definite reference number of load cycles. This normalization is done by equation

$$\sigma_i(N_{ref}) = \sigma_D \cdot \left(\frac{D_i \cdot N_D}{N_{ref}}\right)^{1/k} \quad (4)$$

(Miner elementary [4] is assumed for normalization).

Repeat steps a)-g) for all types "type i".

h) Classify the equivalent stress levels $\sigma_i(N_{ref})$ for $i = 1(1)n_{type}$ into damage classes: To increase the database it is recommended to classify the equivalent stress levels of all types into damage classes. Furthermore, the vagueness about the underlying stress in the field makes it complicated to clearly assign the failure time obtained from the field to a certain type. Thus a classification is beneficial. Depending on the number of types, a statistically proper amount of damage classes n_d is

$$n_d = \sqrt{n_{type}} \,. \quad (5)$$

Alternative approaches are given in [1]. If the amount of damage classes is determined the allocation can be done, e.g. by means of a linear approach. The class range is given by division of the stress range of all types by the number of damage classes n_d. With this upper class limits $\sigma_{UL,d}$ for each damage class $d = 1(1)(n_d\text{-}1)$ are defined by equation

$$\sigma_{UL,d} = \min(\sigma) + d \cdot \left(\frac{\max(\sigma) - \min(\sigma)}{n_d}\right). \quad (6)$$

The equivalent stress levels $\sigma_i(N_{ref})$ are allocated to the classes consequently. Finally the mean stress values $\sigma_{m,d}$ for all types in each damage class d for $d = 1(1)n_d$ are computed by arithmetic mean.

i) Calculate a representative lifetime for each damage class: At first identify an average damage class d_{ave}. Select the type i which is close to the damage sum $D_i \approx 1$ and set its mileage equal to B_q-lifetime (here: $q = 50$ %), i.e. the required mileage in case of $D_i = 1$. The selected type indicates the average damage class d_{ave}. With this the representative simulated lifetimes $B_{q,d,\text{sim}}$ for each damage class d is given by equation

$$B_{q,d,\text{sim}} = B_{q,d_{\text{ave}}} \cdot \left(\frac{\sigma_{d_{\text{ave}}}}{\sigma_{\text{m},d}}\right)^k . \tag{7}$$

II. Field path

a) Identify the customer type combination: If a failure occurs the associated type must be determined. The proposed method is a sum of square procedure that requires input from both statistics and the customer directly. More theoretical explanations are given in Section 3.2. With statistical percentage amount $p_{c,a,\text{stat}}$ of customer type c and characteristic attribute a (c.f. Table 2) as well as customer information $p_{c,a,\text{customer}}$ of customer type c and characteristic attribute a, the sum of square errors Φ regarding customer type c results in equation

$$\Phi_c = \sum_{a=1}^{n_a} \left(p_{c,a,\text{stat}} - p_{c,a,\text{customer}}\right)^2 . \tag{8}$$

The minimum of Φ_c indicates the appropriate customer type of each criterion. Replicate for all three criteria determines the appropriate 3F customer type combination of interest:

$$\min\left(\Phi_{\text{road type}}\right) \wedge \min\left(\Phi_{\text{loading}}\right) \wedge \min\left(\Phi_{\text{driving style}}\right) \Rightarrow \text{combination of interest} . \tag{9}$$

b) Allocate field data into damage classes: After the determination of the associated type, the failure can be allocated into the corresponding damage class d by using the results of step I. h). This is enabled, because of the link between the unknown customer stress causing the failure and the stress quantified by the simulation path. In other words, matching the determined type and the simulated type empowers the allocation. As a result of this, field failure data sets for n_{D} damage classes are obtained.

c) Apply Weibull analysis for each damage class: For analyzing field data the Sudden Death assessment is an appropriate method. Field data are usually multiple censored data. Thus additional information, as the delivered output and the amount of intact products in each damage class, is needed. To approximate the amount of intact products in each damage class d a normally distributed customer stress σ_{customer} is assumed. That means, the majority of customer represent types, which exhibit a stress allocated in a middle damage class. On the other hand, only a small proportion of customer represent types which exhibit a stress allocated in a lower or upper damage class. This assumption is qualitatively illustrated in Figure 3 exemplary for three damage classes. Hence, the amount of intact products $n_{\text{s},d}$ in damage class d is approximated by equation

$$n_{\text{s},d} = P\left(\sigma_{\text{UL},d-1} < \sigma_{\text{customer}} \leq \sigma_{\text{UL},d}\right) \cdot n_{\text{s}} = \int_{\sigma_{\text{UL},d-1}}^{\sigma_{\text{UL},d}} f\left(\sigma_{\text{customer}}\right) d\sigma \cdot n_{\text{s}} \tag{10}$$

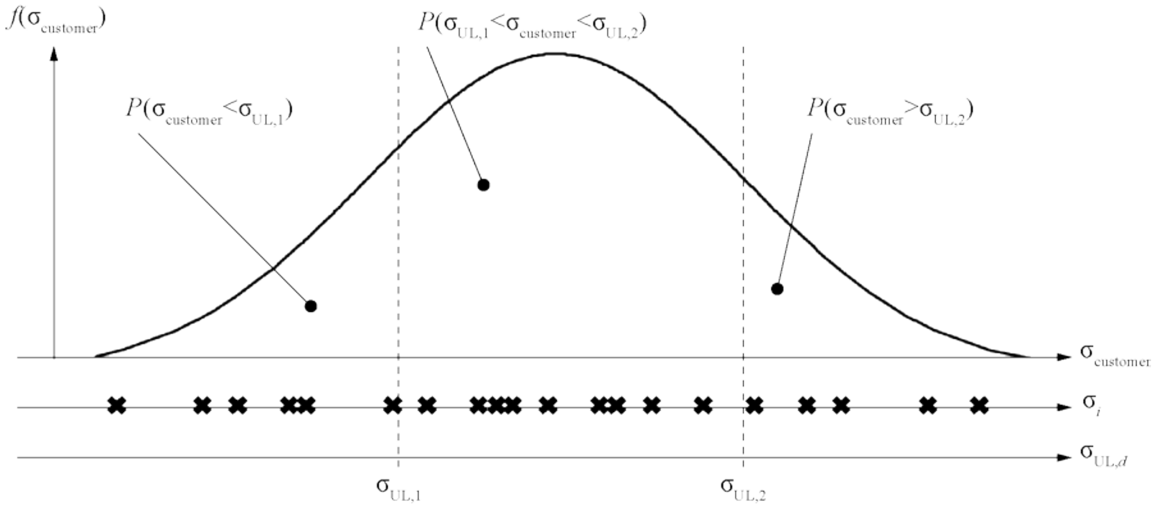
with the overall number of intact products n_{s}. For more explanation in Sudden Death, the reader is referred to [1]. Finally, this step leads directly to stress-dependent Weibull shape parameters.

d) Calculate the lifetimes for each damage class: Convert the characteristic life (or scale parameter) T to the $B_{q,d,\text{field}}$-lifetime based on field data, c.f. [1].

III. Optimize the lifetime exponent: The initial assumed lifetime exponent k_0 can be updated by using pre-processed field data (see II. above), e.g. by means of least square estimation (LSE) [18]. Finally, the lifetime exponent k is optimized by minimizing the sum of square errors Φ with

$$\Phi = \sum_{d=1}^{n_d} \left(B_{q,d,\mathrm{sim}} - B_{q,d,\mathrm{field}} \right)^2 \,. \tag{11}$$

Figure 3: Assumed Probability Density Function of the Customer Stress and Allocation in Damage Classes



Remark: The exact choice of the reference number of load cycles N_{ref} in step I. g) does not matter regarding the allocation in step II. b) as only relative ratios are considered.

5. EXAMPLE

The approach is illustrated by a simplified synthetic calculation example which is in line with the steps in Section 4. Some steps are omitted due to missing data, such as simulations and both identification and allocation of the customer type combinations. Table 3 depicts the obtained values after the optimization.

Eight customer type combinations are assumed. Each type i is simulated. The results of the two parametric level distribution classifications and the transformations in revolutions of the wheel $r_{i,j}$ per stress class j, are listed in the columns stress class 1 and 2. The next columns depict results of the steps I. e)-g). The mean stresses $\sigma_{\mathrm{m},d}$ are calculated for each damage class in step I. h). The next step identifies the average class d_{ave} and calculates representative lifetimes $B_{50,d,\mathrm{sim}}$ based on the simulation path. Next, field data are gathered and analyzed analogously to steps II. a)-c). The resulting Weibull shape parameters b_d as well as the characteristic lifetimes T_d are stated. Thus, the $B_{50,d,\mathrm{field}}$ -lifetimes based on field data are computed. Finally, a least square estimation regarding the lifetime exponent k is conducted (step III.).
Figure 4 depicts the Weibull shape parameter b as a function of an applied stress. By fitting the data, a convenient shape parameter can be derived from a specific stress level.

6. CONCLUSION

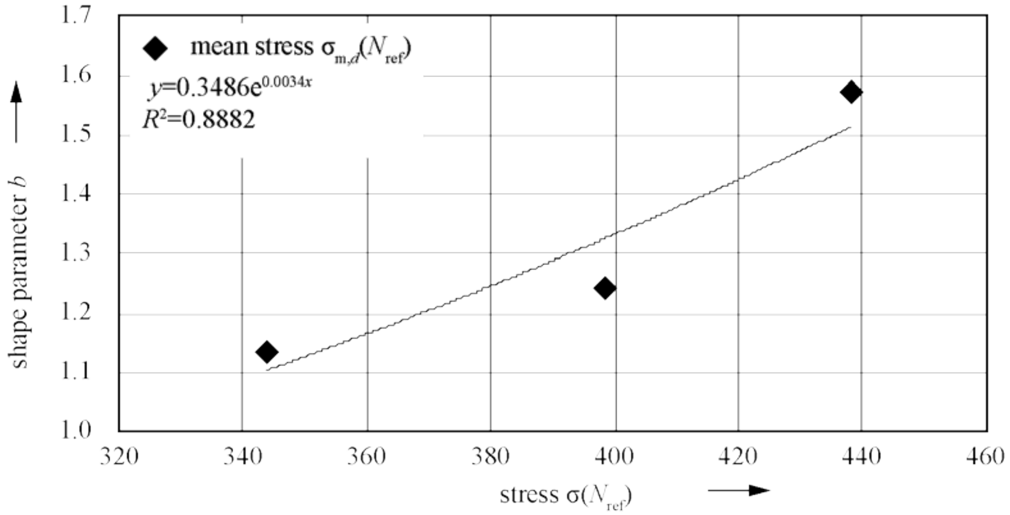
Products in the field provide an enormous amount of data. By means of linking the occurred failure event with a certain stress level, a stress-dependent analysis can be executed. The introduced approach empowers the decision-maker to gain a sharper understanding of a stress-dependent Weibull shape parameter based on field data. Using a more precisely determined stress-dependent Weibull shape parameter, results in a more realistic reliability prediction for future products. Thereby, drawbacks from product similarity related assumptions are avoided, as individual components' failure modes are the focal point. Better known Weibull shape parameters, e.g. stemming from both field and test stress levels, might be used to reduce the necessary sample size regarding reliability demonstration testing.

In addition, initially assumed lifetime model parameters can be optimized. Also, analyzing field data using this approach allows for identifying of stress-dependent components and failure modes, which are still unknown.

Table 3: Calculation Example

	k_0	5	Initially assumed							
	k	4.884	After optimization (step III.)							
	σ_D	400								
	N_D	5.0E+07							N_{ref}	5.0E+07
			stress class 1		**stress class 2**					
	type *i*	w_i	$\sigma_{i,1}$	$r_{i,1}$	$\sigma_{i,2}$	$r_{i,2}$	$D_{i,w}$	EF_i	D_i	$\sigma_i(N_{ref})$
I. e)-g)	1	8,000	420	300,000	500	250,000	0.022	25	0.562	355.5
	2	10,500	450	300,000	480	300,000	0.025	19	0.482	344.4
	3	10,000	420	300,000	450	350,000	0.020	20	0.401	331.8
	4	5,000	420	350,000	500	325,000	0.028	40	1.129	410.0
	5	5,500	450	350,000	480	300,000	0.027	36	0.984	398.7
	6	7,500	450	350,000	500	325,000	0.032	27	0.847	386.7
	7	6,000	500	300,000	550	400,000	0.056	33	1.858	454.1
	8	7,000	450	350,000	550	350,000	0.046	29	1.303	422.3
I. h)	n_d	3								
	d	1	2	3						
	$\sigma_{UL,d}$	372.5	413.3							
	$\sigma_{m,d}$	343.9	398.5	438.2						
I. i)	d_{ave}		2							
	$B_{50,d,sim}$	417,218	200,000	127,770						
II. c)	T_d	589,363	264,408	150,324	Estimated based on assumed field data					
	b_d	1.135	1.243	1.573						
II. d)	$B_{50,d,field}$	426,718	196,887	119,079						
III.	Φ	2.1E+08			Least square estimation					

Figure 4: Weibull Shape Parameter as a Function of Stress



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