Energy loss optimization in basic T-shaped water supply piping networks for probabilistic demands

KW Mui[a], LT Wong[a], and CT Cheung[a]
[a] Department of Building Service Engineering,
The Hong Kong Polytechnic University,
Hong Kong, China

Abstract: Minimization of energy loss of water supply networks is a major concern of pump power reduction for sustainable water systems in buildings. This paper presents a mathematical model for energy loss optimization in a common basic T-shaped water supply piping network that serves infinite probabilistic demands. Optimized designs based on proper network pipe sizes are analyzed. Optimal pipe radius ratios ($2^{1/7}$ to $2^{3/7}$) and their corresponding energy implications in the network are also discussed. The results show that existing piping designs are not optimized for probabilistic demands and there is potential for energy loss reduction.

Keywords: Buildings, Energy loss, Piping Network, Probabilistic Demands, Water Supply System

List of symbols

C, c	Constant and its matrix element
E, e	Energy and its matrix element (J)
F	Function as defined
f	Friction factor (-)
i, j, l	Dummy variables as defined
L	Length (m)
m	Number of independently operated appliance
n	State of a binary operated appliance
P, p	Pressure and its matrix element (Pa)
ϑ, φ	Demand probability and its matrix element (-)
r	Radius (m)
V, v	Volumetric demand flow rate and its matrix element(m^3s^{-1})
Δ	Change of
$[\ ], \{\ \}$	Matrix notation and matrix element group
ρ	Density (kg m^{-3})
λ	Dummy parameter group as defined
Superscript	
‘	Derivatives
Subscript	
0,1,2,3,4,…	Of conditions 0, 1, 2, 3, 4, … as defined
c	Of constant
i, j, l, m, n	Of i-th, j-th, l-th, m-th, n-th conditions as defined
max	Of maximum value
opt	Of optimum
R	Of pipe radius ratio
V, v	Of volumetric flow rate

1. INTRODUCTION

Potable water demands are unsteady, random and intermittent [1]. As survey studies indicated that the end-use water demand probability is generally less than 0.1 even during daily rush hours [2,3], demand overload is legitimate in the design of water supply pipe network inside the building. Statistical methods for estimating appliance usage patterns and associated instantaneous water demands have been developed to size mains water pipes [4]. To ensure supply certainty rather than optimality, the existing design approach to determine fixture units may overestimate the simultaneous water demand and oversize water pipes [5]. Alternatively, Monte Carlo simulations can be used to decide the failure probability density function of the water supply system, which is influenced by the occupant load profile, for meeting the demand and assessing the performance [6].

Distributing a supply of water as uniformly as possible over a territory through piping networks is a classical problem of optimization. An urban water supply study showed that 45% of the total pumping energy needed to deliver water from the treatment plants to households was consumed inside buildings [7]. Apart from building height, energy loss at supply pipes is another concern. Under steady conditions, Bejan's constructal law of the generation of flow configuration is a proven useful tool for optimizing the geometric layout of schemes by minimizing pumping power requirements for distributing water uniformly over an area [8,9]. However, with respect to the optimal operation of water supply networks in buildings where flow rates are unsteady, there are no existing models systematically optimizing energy efficiency and interrelated issues.

This paper dealt with the energy loss optimization problems of a common basic T-shaped water supply piping network that serves infinite probabilistic demands and established a mathematical model for the required minimum water pumping energy under a fixed pipe volume constraint. Energy loss reduction potential through proper pipe size was also investigated. The results were discussed in terms of their implications for theory and practice.

2. ENERGY LOSS

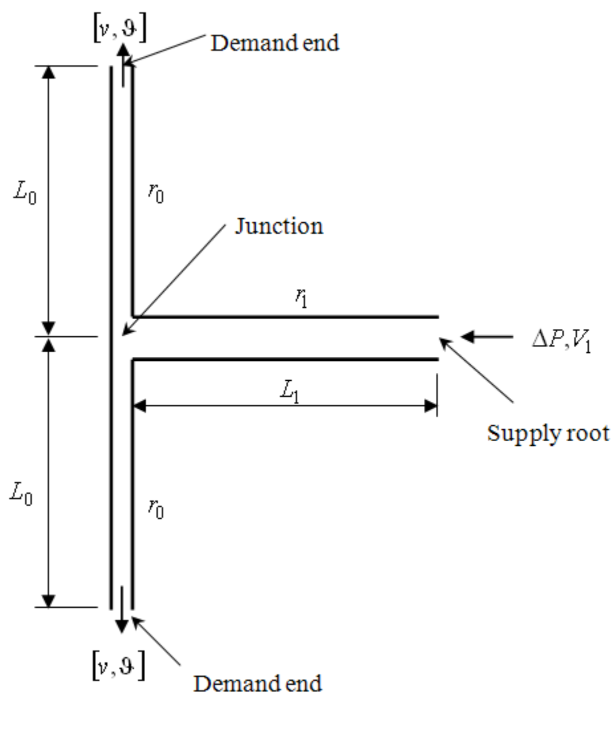


Figure 1. A basic T-shaped water supply piping network

Figure 1 shows a basic T-shape water supply piping network that a branch pipe of length L_0 and of radius r_0, fed by a centre main water pipe of length L_1 and radius r_1. The network consists a supply root and two demand ends as in a T-shape construct [8]. A demand in the network is due to a number of independently operated appliances $n_1, n_2, ... n_m$ at each of the two demand ends and each appliance demand is characterised by a constant flow rate $v = C_v$ and a probability $\varphi = \vartheta$. The flow rate between two consecutive demands is assumed zero with a probability $\varphi = 1 - \vartheta$. As an appliance is either 'in demand' (n=1) or 'not in demand' (n=0), an appliance demand can be described by,

$$n_i : \{0,1\};\ v_i : \{0, v_i\};\ \varphi_i : \{1 - \vartheta_i, \vartheta_i\};\ i = 1,2,...,m \tag{1}$$

Appliances are arranged according to their 'in demand' flow rates $[v_1, \vartheta_1]$, $[v_2, \vartheta_2]$, …, $[v_m, \vartheta_m]$ such that $v_1 \le v_2 \le ... \le v_m$ and expressed the flow rates in terms of $v_1 = C_v$ and $v = \left[C_v \left\{ 0, \frac{v_1}{v_1}, \frac{v_2}{v_1}, ..., \frac{v_{m-1}}{v_1}, \frac{v_m}{v_1} \right\} \right]$, then a demand at the demand end is given by,

$$v = C_v \left\{ 0, 1, C_{V_2}, ..., C_{V_{m-1}}, C_{V_m} \right\};\ C_{V_1} = 1 \le C_{V_2} \le ... \le C_{V_{m-1}} \le C_{V_m} \tag{2}$$

The demands $v_{i,j}$ at the two branches i, j fed by the m binarily operated appliances are probabilistic. There are 2^m combinations of demands in each branch pipe, and $(2^m)^2$ cases at the junction where two branches meet, throughout the centre pipe or at the root. For a pair of demand ends i, j, the demand probability at any instant is $\varphi_i \varphi_j$ (denoted as φ_{ij}) and the corresponding demand probabilities are expressed by,

$$\varphi = \left\{ \varphi_{11}, ..., \varphi_{(1)(2^m)}, ..., \varphi_{(2^m)(1)}, ..., \varphi_{(2^m)(2^m)} \right\}$$

$$\varphi_{ij} = \varphi_i \varphi_j;\ \varphi_i, \varphi_j = \prod_{l=1}^{m} \vartheta_l^{n_l} (1 - \vartheta_l)^{1 - n_l} \tag{3}$$

i and j are dependent on the number of end demands m,

$$i, j = 1 + \sum_{l=1}^{m} 2^{l-1} n_l;\ n_l = [0,1] \tag{4}$$

The centre pipe demand at the root is the sum of demands due to branch pipes, and given by an expression below, where a total of $(2^m)\,(2^m)$ combinations of demand pairs are encountered,

$$V_1 = V_1[v, \varphi];\ v = \left\{ v_{11}, ..., v_{1(2^m)}, ..., v_{(2^m)(1)}, ..., v_{(2^m)(2^m)} \right\};\ v_{ij} = v_i + v_j \tag{5}$$

Taking $C_0 = \frac{f \rho C_v^2}{\pi^2}$ as the unit pipe friction of turbulent flow, the total pressure loss of the network ΔP is determined by the following equations, where ΔP_0 and ΔP_1 are the pressure losses in the branch and centre pipes respectively,

$$\Delta P[p,\varphi] = \Delta P_0[p,\varphi] + \Delta P_1[p,\varphi]\ ;\ p = \left\{p_{11},\ldots,p_{1(2^m)},\ldots,p_{(2^m)(1)},\ldots,p_{(2^m)(2^m)}\right\}$$

$$\Delta P_0 : p_{ij} = \left\{ \begin{array}{ll} 0 & ; ij = 11 \\ \dfrac{C_0 L_0}{C_v^2 r_0^5} v_{i,j}^2 & ; ij \neq 11 ; v_{i,j} = \max(v_i, v_j) \end{array} \right\} ;\ \Delta P_1 ; p_{ij} = \frac{C_0}{C_v^2}\frac{L_1}{r_1^5} v_{ij}^2 \qquad (6)$$

It is noted for the above expression, the pressure required at the junction between i and j is maintained for the higher flow rate $\max(v_i, v_j)$.

As the pressure loss at the network is flow rate dependent and hence transient, the energy required for the pressure loss at the demands is chosen as the optimization parameter. The minimum energy E required for the water supply network is given by,

$$E[e,\varphi] = \Delta P V_1;\ e = \left\{ e_{11},\ldots,e_{1(2^m)},\ldots,e_{(2^m)(1)},\ldots,e_{(2^m)(2^m)} \right\};\ e_{ij} = p_{ij} v_{ij};$$
$$E = e_{11}\varphi_{11} + \ldots e_{1(2^m)}\varphi_{1(2^m)} + \ldots e_{(2^m)\,1}\varphi_{(2^m)\,1} + \ldots e_{(2^m)(2^m)}\varphi_{(2^m)(2^m)} = \sum_{ij} e_{ij}\varphi_{ij} \qquad (7)$$

For a constant pipe volume C, the energy in Eq. (7) can be minimized by choosing a proper pipe radius ratio $C_R = (r_1/r_0)$ for pressure terms p_{ij} in Eq. (6) such that,

$$2L_0 r_0^2 + L_1 r_1^2 = C \qquad (8)$$

$$r_1 = \left(\frac{C}{L_1} - \frac{2L_0}{L_1} r_0^2\right)^{1/2} \qquad (9)$$

Let F be the frictional losses in the branch and centre pipes, where λ is an arbitrary constant in the pressure terms in Eq. (6), the general solution for the optimal pipe radius ratios $C_{R,opt}$ can be determined by taking derivative $F' = 0$,

$$F = \lambda_0 L_0 r_0^{-5} + \lambda_1 L_1 r_1^{-5} = \lambda_0 L_0 r_0^{-5} + \lambda_1 L_1 \left(\frac{C}{L_1} - \frac{2L_0}{L_1} r_0^2\right)^{-5/2} \qquad (10)$$

$$F' = -5\lambda_0 L_0 r_0^{-6} + \lambda_1 L_1 \left(\frac{-5}{2}\right)\left(\frac{C}{L_1} - \frac{2L_0}{L_1} r_0^2\right)^{1/2(-7)}\left(-\frac{4L_0}{L_1} r_0\right) = 0 \qquad (11)$$

$$-5\lambda_0 L_0 r_0^{-7} + 5\left(\frac{2\lambda_1 L_1 L_0}{L_1}\right) r_1^{-7} = 0 \qquad (12)$$

$$C_{R,opt} = \frac{r_1}{r_0} = \left(\frac{\lambda_0}{2\lambda_1}\right)^{-1/7} = \left(\frac{2\lambda_1}{\lambda_0}\right)^{1/7} \qquad (13)$$

The optimal pipe radius ratios $C_{R,opt}$ for Eq. (6) with probabilities φ_{ij} expressed below can be determined by taking derivative $C'_{R,opt} = 0$ with an optimum demand probability ϑ_{opt},

$$C_R = [c,\varphi]\ ;\ c = \left\{ c_{11},\ldots,c_{1(2^m)},\ldots,c_{(2^m)(1)},\ldots c_{(2^m)(2^m)} \right\} \qquad (14)$$

$$C_{R,opt} = \sum_{ij} c_{ij}\varphi_{ij}\left(\vartheta_{opt}\right) \qquad (15)$$

3. OPTIMAL PIPE RADIUS RATIOS

Three cases for c_{ij} in calculating the pressure loss at pipe r_1 and r_0 are given below,

$$c_{ij} = \left(\frac{2\lambda_1}{\lambda_0}\right)_{ij}^{1/7} = \begin{cases} 2^{1/7} \\ 2^{1/7}\left(\dfrac{v_i + v_j}{max(v_i, v_j)}\right)^{2/7} \\ 2^{3/7} \end{cases}; \; v_{ij} = \begin{cases} v_i, v_j \to 0 \\ max\{v_i, v_j\} & ; v_i, v_j \neq 0 \\ v_i + v_j \end{cases} \tag{16}$$

It is noted for all positive demands, the term $1 \leq \left(\dfrac{v_i + v_j}{\max(v_i, v_j)}\right) \leq 2$ for all demand probabilities φ_{ij}, the optimal pipe radius ratio $C_{R,opt}$ exists only in a range between $2^{1/7}$ and $2^{3/7}$, i.e.

$$2^{1/7} \leq C_{R,opt} \leq 2^{3/7} \tag{17}$$

According to an earlier study, the case of demand probability $\vartheta = 1$ is actually a steady flow condition and the optimal pipe radius ratio determined for it is $2^{3/7}$ [8].

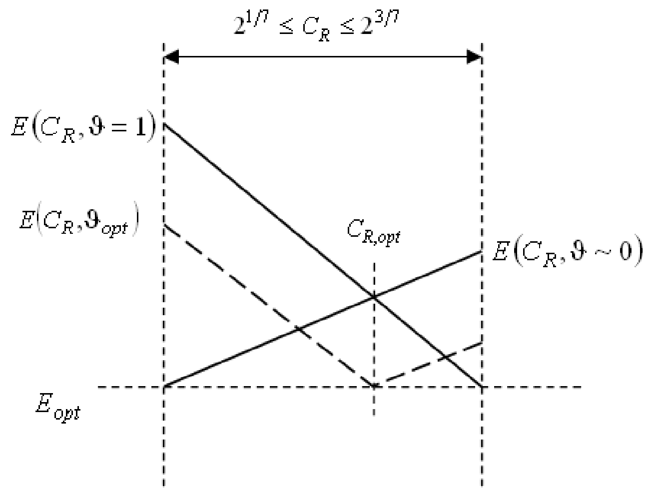


Figure 2. Relative pumping energy

Figure 2 illustrates the schematic relationships between the energy loss E for probabilistic demands and pipe radius ratio $2^{1/7} \leq C_R \leq 2^{3/7}$. Pipe radii at $C_R = 2^{3/7}$ optimized for the maximum demand (at the demand probabilities $\vartheta_{ij} = 1$) are not energy loss optimized for the cases of minimum demands (i.e. $\vartheta_{ij} \sim 0$) and vice versa. An optimized probability ϑ_{opt} exists as the energy loss at the boundaries of C_R.

The minimum energy loss E_{opt} can be derived by the two extreme demand cases, i.e. $\vartheta_{ij} \sim [0,1]$. The optimal probability, given by F below, is determined by taking such that,

$$F = c_{11}\varphi_{11} + c_{(2^m)(2^m)}\varphi_{(2^m)(2^m)} \tag{18}$$

$$F = 2^{3/7}(1 - \vartheta_{ij})^{2m} + 2^{3/7}\,\vartheta_{ij}^{2m} \tag{19}$$

$$F' = -2^{3/7}(2m)\left(1-\vartheta_{ij}\right)^{2m-1} + 2^{3/7}(2m)\vartheta_{ij}^{2m-1} = 0 \tag{20}$$

$$\vartheta_{opt} = 2^{-1} \tag{21}$$

$C_{R,opt}$ can be determined for the two extreme demand cases (i.e. $C_{R,opt}$) using the general equations of order $m=1$ and validated via the general equations of order $m=2$. It should be noted that Eq. (20) approximates the optimum without taking the influences of demand flow rates C_{V_i} in elements c_{ij}, where $ij \neq (1)(1),(2^m)(2^m),$ into account. This point shall be examined in the validation section below.

4. OPTIMAL PIPE RADIUS RATIOS (USING GENERAL EQUATIONS OF ORDER *m* = 1)

When $m=1$, water is supplied to two identical binary operated appliances $n:\{0,1\}$, one on each side of the T-shaped piping network. Hence, there are $2^m = 2$ demand combinations in each branch pipe and $\left(2^m\right)^2 = 4$ demand combinations at the junction, throughout the centre pipe or at the root of the network.

$$n_1 : \{0,1\} \ ; \ v : C_v\{0,1\} \ ; \ \varphi : \{1-\vartheta_1, \vartheta_1\} \tag{22}$$

Centre pipe demand V_1 is determined by $V_1 = V_1[v,\varphi]$,

$$v = \{v_{11}, v_{12}, v_{21}, v_{22}\} = \{v_1+v_1, v_1+v_2, v_2+v_1, v_2+v_2\} = C_v\{0,1,1,2\} \tag{23}$$

For the binary operated appliance n_1,

$$n_1 = \begin{cases} 0 & ;i,j=1 \\ 1 & ;i,j=2 \end{cases}; \ \varphi_1 = \left(1-\vartheta_1\right) ; \varphi_2 = \vartheta_1 \tag{24}$$

$$\begin{aligned} \varphi &= \{\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}\} = \{\varphi_1\varphi_1, \varphi_1\varphi_2, \varphi_2\varphi_2, \varphi_2\varphi_2\} \\ &= \left\{ \left(1-\vartheta_1\right)^2, \vartheta_1\left(1-\vartheta_1\right), \vartheta_1\left(1-\vartheta_1\right), {\vartheta_1}^2 \right\} \end{aligned} \tag{25}$$

Taking the total pressure loss of the network $C_0 = \dfrac{f\rho C_v^2}{\pi^2}$, the total pressure losses of the network, branch and centre pipes ΔP, ΔP_0, ΔP_1 are determined by the following equations,

$$\Delta P[p,\varphi] = \Delta P_0[p,\varphi] + \Delta P_1[p,\varphi] \ ; \ p = \{p_{11}, p_{12}, p_{21}, p_{22}\} \ ;$$

$$\Delta P_0 : p = \left\{0, \frac{C_0 L_0}{C_v^2 r_0^5} v_2^2, \frac{C_0 L_0}{C_v^2 r_0^5} v_2^2, \frac{C_0 L_0}{C_v^2 r_0^5} v_2^2 \right\} = C_0\left\{0, \frac{L_0}{r_0^5}, \frac{L_0}{r_0^5}, \frac{L_0}{r_0^5}\right\}$$

$$\Delta P_1 : p = \left\{0, \frac{C_0 L_1}{C_v^2 r_1^5} {v_2}^2, \frac{C_0 L_1}{C_v^2 r_1^5} {v_2}^2, \frac{C_0 L_1}{C_v^2 r_1^5}\left(2v_2\right)^2 \right\} = C_0\left\{0, \frac{L_1}{r_1^5}, \frac{L_1}{r_1^5}, \frac{4L_1}{r_1^5}\right\}$$

$$\Delta P : p = C_0\left\{0, \frac{L_0}{r_0^5}+\frac{L_1}{r_1^5}, \frac{L_0}{r_0^5}+\frac{L_1}{r_1^5}, \frac{L_0}{r_0^5}+4\frac{L_1}{r_1^5}\right\} \tag{26}$$

The energy loss E at the network is expressed by,

$$E : e = \{e_{11}, e_{12}, e_{21}, e_{22}\} = C_0 C_v \left\{0,\ \left(\frac{L_0}{r_0^5} + \frac{L_1}{r_1^5}\right),\ \left(\frac{L_0}{r_0^5} + \frac{L_1}{r_1^5}\right),\ 2\left(\frac{L_0}{r_0^5} + 4\frac{L_1}{r_1^5}\right)\right\}$$

$$E = C_0 C_v \left(0 + 2(1-\vartheta_1)\vartheta_1\left(\frac{L_0}{r_0^5} + \frac{L_1}{r_1^5}\right) + 2\vartheta_1^2\left(\frac{L_0}{r_0^5} + 4\frac{L_1}{r_1^5}\right)\right) \tag{27}$$

Energy loss E is optimized under a given pipe volume constraint C. Taking $\vartheta_{opt} = 2^{-1}$, the optimal pipe radius ratio $\vartheta_{opt} = 2^{-1}$, is given by,

$$C_R : c = \{\ 2^{3/7}, 2^{1/7}, 2^{1/7}, 2^{3/7}\} \tag{28}$$

$$C_R = 2^{3/7}(1-\vartheta_1)^2 + \left(2^{1/7}\right)\ 2(1-\vartheta_1)\vartheta_1 + 2^{3/7}\vartheta_1^2 \tag{29}$$

$$C_R' = 2^{3/7}(1-\vartheta_1)^2 + \left(2^{1/7}\right)\ 2(1-\vartheta_1)\vartheta_1 + 2^{3/7}\vartheta_1^2 = 0 \tag{30}$$

$$C_{R,opt} = 2^{3/7}\left(1-\vartheta_{opt}\right)^2 + \left(2^{1/7}\right)\ 2\left(1-\vartheta_{opt}\right)\vartheta_{opt} + 2^{3/7}\vartheta_{opt}^2 = 2^{2/7} \tag{31}$$

Figure 3 illustrates the relative energy loss E/E_{opt} required at a T-shaped water supply piping network when $m = 1$ and Figure 4 is the corresponding graph of optimal pipe radius ratio against demand probability. They confirm the range of optimal pipe radius ratios $C_{R,opt} \in \left[2^{1/7}, 2^{3/7}\right]$ at the boundary conditions illustrated in Figure 2, i.e. the minimum and maximum demand probabilities are $\vartheta \sim 0$ and $\vartheta \sim 0$ respectively.

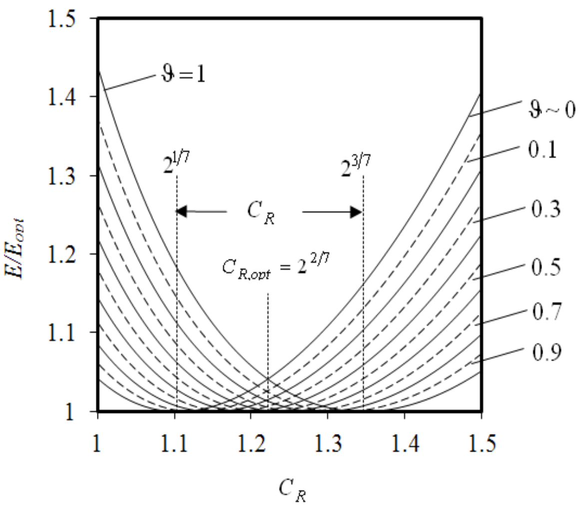


Figure 3. Relative energy loss at a T-shaped water supply piping network when *m* = 1

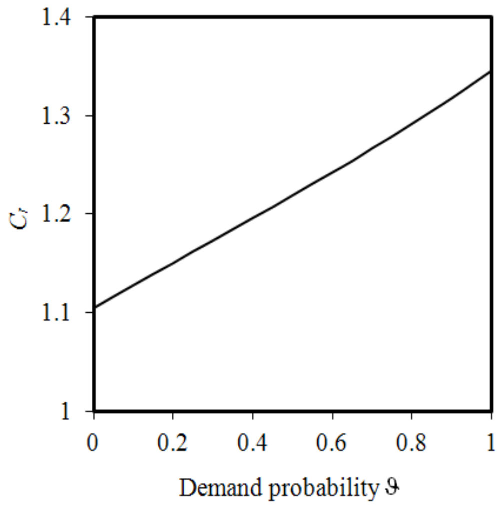


Figure 4. Optimal pipe radius ratios, *m* = 1

Eq. (30) gives $\vartheta_{opt} = 0.5$, which when substituted into Eq. (29) yields $\vartheta_{opt} = 0.5$, as shown in Figure 3. If $C_{R,opt} = 2^{2/7}$ is applied to all demand probabilities, up to 4.2% more energy loss can be produced as compared with any optimal cases with a single set of demand probabilities. If $C_{R,opt} = 2^{3/7}$ is optimized for the steady flow, the additional energy loss is 13-16% as compared with the optimal cases where demand probabilities are in between 0.001 and 0.1.

5. OPTIMAL PIPE RADIUS RATIO VALIDATION (USING GENERAL EQUATIONS OF ORDER *m* = 2)

Validity of the pipe radius ratio $C_{R,opt} = 2^{2/7}$ is tested in this case. When $m = 2$, two binary operated appliances (n_1, n_2) demand probabilities $(\vartheta_1, \vartheta_2)$ and thus two demand flow rates $(C_v, C_v C_V)$ exist at each demand end, i.e.

$$n_1 : \{0,1\} \; ; \; v : C_v \{0,1\} \; ; \; \varphi : \{ \; 1 - \vartheta_1, \vartheta_1 \} \tag{32}$$

$$n_2 : \{0,1\} \; ; \; v : C_v C_V \{0,1\} \; ; \; \varphi : \{ \; 1 - \vartheta_2, \vartheta_2 \} \tag{33}$$

There are $4 \left(= 2^m\right)$ demand combinations in each branch pipe,

$$v = \{ \; v_1, v_2, v_3, v_4 \} \; = \{ \; 0, 1, C_V, 1 + C_V \} \tag{34}$$

Correspondingly, there are 4×4=16$\left(= 2^{2m}\right)$ demand combinations at the junction, in the centre pipe and at the root.

For the binary operated appliances n_1, n_2, the centre pipe demand n_1, n_2, is, with matrix elements of v, φ are given in Table 1,

$$v = \{ \; v_{11}, v_{12}, v_{13}, v_{14}, v_{21}, \ldots, v_{44} \} \tag{35}$$

$$i, j = 1 + n_1 + 2n_2; \;\; n_{1,2} = [0,1] \tag{36}$$

$$\varphi = \{ \; \varphi_{11}, \varphi_{12}, \ldots, \varphi_{44} \} \; ; \;\; \varphi_{ij} = \varphi_i \varphi_j ; \;\; \varphi_{i,j} = \vartheta_1^{n_2} (1 - \vartheta_1)^{1 - n_2} \, \vartheta_2^{n_1} (1 - \vartheta_2)^{1 - n_1} \tag{37}$$

Table 1. Matrix elements for general equations of order *m* = 2

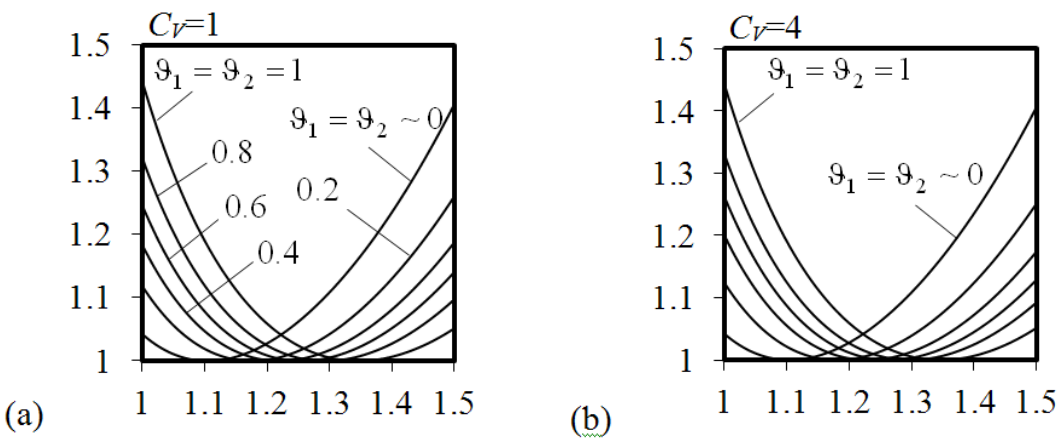
ij	$i : n_2, n_1$; $j : n_2, n_1$	$C_v^{-1}[v_i, v_j]$	$C_v^{-1} v_{ij}$	$\varphi_{ij} = \varphi_i \varphi_j$	$\Delta P_0 : C_v^{-1} C_0^{-1} p_{ij}$	$\Delta P_1 : C_v^{-1} C_0^{-1} p_{ij}$	$C_v^{-1} C_0^{-1} e_{ij}$	c_{ij}
11	0000	0,0	0	$(1-\vartheta_1)^2 (1-\vartheta_2)^2$	0	0	0	$2^{3/7}$
12	0001	0,1	1	$(1-\vartheta_1)^2 \vartheta_2 (1-\vartheta_2)$	$L_0 r_0^{-5}$	$L_1 r_1^{-5}$	$L_0 r_0^{-5} + L_1 r_1^{-5}$	$2^{1/7}$
13	0010	0, C_V	C_V	$(1-\vartheta_1) \vartheta_1 (1-\vartheta_2)^2$	$C_V^2 L_0 r_0^{-5}$	$C_V^2 L_1 r_1^{-5}$	$C_V^3 \left(L_0 r_0^{-5} + L_1 r_1^{-5} \right)$	$2^{1/7}$
14	0011	0, $1 + C_V$	$1 + C_V$	$\vartheta_1 (1-\vartheta_1) \vartheta_2 (1-\vartheta_2)$	$(1 + C_V)^2 L_0 r_0^{-5}$	$(1 + C_V)^2 L_1 r_1^{-5}$	$(1 + C_V)^3 \left(L_0 r_0^{-5} + L_1 r_1^{-5} \right)$	$2^{1/7}$
21	0100	1,0	1	same with $ij = 12$				
22	0101	1,1	2	$(1-\vartheta_1)^2 \vartheta_2^2$	$L_0 r_0^{-5}$	$4 L_1 r_1^{-5}$	$2 \left(L_0 r_0^{-5} + 4 L_1 r_1^{-5} \right)$	$2^{3/7}$

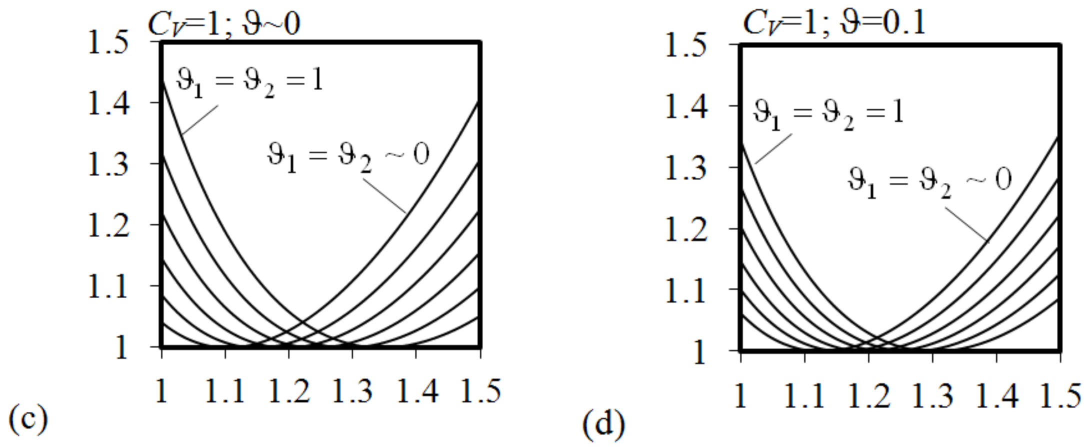
23	0110	$1,\ C_V$	$1+C_V$	$\vartheta_1(1-\vartheta_1)$ $\vartheta_2(1-\vartheta_2)$	$C_V^2 L_0 r_0^{-5}$	$(1+C_V)^2 L_1 r_1^{-5}$	$(1+C_V)\begin{pmatrix} C_V^2 L_0 r_0^{-5} \\ +(1+C_V)^2 L_1 r_1^{-5}\end{pmatrix}$	$\frac{2^{1/7}(1+C_V)^{2/7}}{C_V^{2/7}}$
24	0111	$1,\ C_V$	$2+C_V$	$\vartheta_1(1-\vartheta_1)\vartheta_2^2$	$(1+C_V)^2 L_0 r_0^{-5}$	$(2+C_V)^2 L_1 r_1^{-5}$	$(2+C_V)\begin{pmatrix} (1+C_V)^2 L_0 r_0^{-5} \\ +(2+C_V)^2 L_1 r_1^{-5}\end{pmatrix}$	$\frac{2^{1/7}(2+C_V)^{2/7}}{(1+C_V)^{2/7}}$
31	1000	$C_V,\ 0$	C_V	same with $ij=13$				
32	1001	$C_V,\ 1$	$1+C_V$	same with $ij=23$				
33	1010	$C_V,\ C_V$	$2C_V$	$\vartheta_1^2(1-\vartheta_2)^2$	$C_V^2 L_0 r_0^{-5}$	$4C_V^2 L_1 r_1^{-5}$	$2C_V^3\begin{pmatrix} L_0 r_0^{-5} \\ +4L_1 r_1^{-5}\end{pmatrix}$	$2^{3/7}$
34	1011	$C_V,$ $1+C_V$	$1+2C_V$	$\vartheta_1^2\vartheta_2(1-\vartheta_2)$	$(1+C_V)^2 L_0 r_0^{-5}$	$(1+2C_V)^2 L_1 r_1^{-5}$	$(1+2C_V)\begin{pmatrix} (1+C_V)^2 L_0 r_0^{-5} \\ +(1+2C_V)^2 L_1 r_1^{-5}\end{pmatrix}$	$\frac{2^{1/7}(1+2C_V)^{2/7}}{(1+C_V)^{2/7}}$
41	1100	$1+C_V,\ 0$	$1+C_V$	same with $ij=14$				
42	1101	$1+C_V,\ 1$	$2+C_V$	same with $ij=24$				
43	1110	$1+C_V,$ C_V	$1+2C_V$	same with $ij=34$				
44	1111	$1+C_V,$ $1+C_V$	$2(1+C_V)$	$\vartheta_1^2\vartheta_2^2$	$(1+C_V)^2 L_0 r_0^{-5}$	$4(1+C_V)^2 L_1 r_1^{-5}$	$2(1+C_V)^3\left(L_0 r_0^{-5}+4L_1 r_1^{-5}\right)$	$2^{3/7}$

The pressure loss required at the network ΔP and the corresponding pressure loss are given by, where the matrix elements p_{ij}, e_{ij} are summarized in Table 1 for easy reference,

$$\Delta P[p,\varphi] = \Delta P_0[p,\varphi] + \Delta P_1[p,\varphi]\ ;\ p = \{p_{11}, p_{12}, p_{13}, p_{14}, p_{21}, \ldots, p_{44}\} \tag{38}$$

$$E : e = \{\ e_{11}, e_{12}, \ldots, e_{44}\}\ ;\ e_{ij} = \left(\Delta P : p_{ij}\right)\left(V_1 : v_{ij}\right) \tag{39}$$



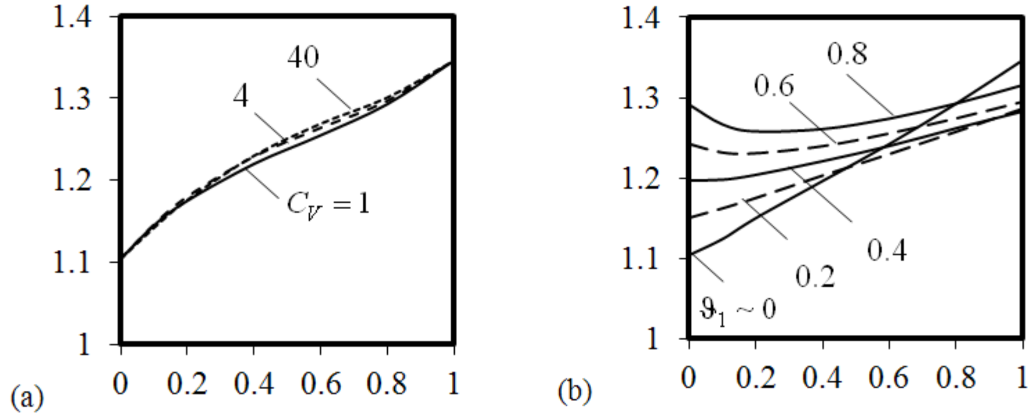


x-axis: Pipe radius ratio C_R

y-axis: Relative pumping energy E/E_{opt}

Figure 5. Relative energy loss for a T-shaped water supply piping network when *m* = 2

Figure 5 presents the relative energy loss for some demand cases while Figure 6 exhibits the optimal pipe radius ratios for various sets of demand flow rates and demand probability combinations. Again, they confirm the validity of the $C_{R,opt}$ range and boundary conditions as in case $m = 1$. It can be seen that demand flow rates $(C_V > 1)$ have some influences on the middle range of demand probabilities (e.g. $\vartheta = 0.1$ to 0.9), but not on $\vartheta > 0.9$ (almost steady flow) or $\vartheta < 0.1$ (almost minimal flow).



x-axis: Probability ϑ_2

y-axis: Optimum pipe radius ratio $C_{R,opt}$

Figure 6. Optimum pipe radius ratios, *m* = 2

In Figure 6(a), the optimal pipe radius ratios for $\vartheta = 0.5$ are 1.247 and 1.251 when C_V = 4 and 40 respectively. It is noted that typical appliance flow rates are in the range 0.08 Ls^{-1} (shower) to 0.3 Ls^{-1} (kitchen sink) corresponding to a C_V < 4 [10]. In Figure 6(b), the $C_{R,opt}$ values for fixed demand probabilities φ_1 = 0.2, 0.4, 0.6 and 0.8 are 1.218, 1.239, 1.268 and 1.304 respectively if $\frac{\partial C_{R,opt}(\varphi_1, \varphi_2)}{\partial \varphi_2} = 0$. As expected, more frequent demands (i.e. 'larger' flow rates) lead to higher $C_{R,opt}$ values. The findings suggest that $C_R = 2^{2/7} \approx 1.22$ should be the optimal choice for the design of water supply piping networks that serve probabilistic demands (at uniformly distributed probabilities).

6. ENERGY IMPLICATIONS OF EXISTING PIPING NETWORKS

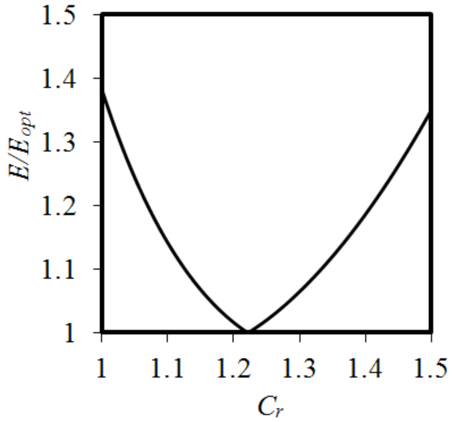


Figure 7. Maximum relative pumping energy at $C_{R,opt} = 2^{2/7}$

Figure 7 shows the maximum relative energy loss for probabilistic demands with $C_R = 1$ to 1.5, illustrating a potential reduction of energy loss up to 38% at $C_{R,opt} = 2^{2/7}$. Table 2 exhibits the common pipe sizes available for water supply systems in buildings [10]. The pipe radius ratios C_R shown are in the range of 1.13-1.47 and many of them are very close to the optimal ratio value $\left(C_{R,opt} = 2^{2/7} \approx 1.22\right)$ proposed in this work. If the optimal value 1.22 substitutes 1.13 or 1.47, 9-30% savings in energy loss are achievable. In view of the fact that the smallest pipe is practically employed in end-use appliances, one more pipe size in between 15 mm and 22 mm, i.e. 15×1.22 mm or 22÷1.22 mm = 18 mm, is required for energy loss optimization.

Table 2. Common pipe sizes for water supply systems in buildings[10]

Copper and stainless steel pipes		Plastic pipe	
Diameter (mm)	C_R	**Diameter (mm)**	C_R
15		16	
22	1.47	20	1.25
28	1.27	25	1.25
35	1.25	32	1.28
42	1.2	40	1.25
54	1.28	50	1.25
67	1.24	63	1.26
76	1.13	75	1.19
108	1.42	90	1.20
133	1.23	110	1.22
159	1.2	160	1.45

It should be noted that for a wide range of sanitation appliances operated at a demand probability typically lower than 0.1 [11,12], a pipe radius ratio based on steady flow conditions $\left(C_{R,opt} = 2^{3/7}\right)$ leads to an additional energy loss of 14% and 10% respectively, and thus the choice is not optimized for many water supply systems in buildings. Sizing pipes with $C_R = 2^{2/7}$ will be a better choice corresponding to a less energy loss of 2.6% and 1.5% as compared with the cases of known single set of demand probabilities.

Typical water supply systems are designed to cope with a design condition of the probable maximum demand that sufficient pressure is available at all appliance outlets at its design flow rate. The outlet pressure control is achieved by the user through regulating the flow control valve of the appliance.

However, this over-provided pressure relates to energy wastage. The significance of this paper is to understand the required pressure of probabilistic demands. With proper demand control on the appliance outlet pressure at probabilistic demands, potential pumping energy savings for water supply networks can be studied.

7. CONCLUSION

This paper presented the general equations for solving energy loss optimization problems associated with a common basic T-shaped water supply piping network that serves infinite probabilistic demands, and established a mathematical model for energy loss optimization under a fixed pipe volume constraint. Potential reduction of energy loss through proper pipe size was investigated and the optimal pipe radius ratios were found to be in between $2^{1/7}$ and $2^{3/7}$. The findings suggested that $2^{2/7}$ should be the optimal choice for the design of water supply pipe networks that serve probabilistic demands. They also showed that existing piping designs are not optimized for probabilistic demands and reduction of energy loss up to 38% can be made at supply networks, with proper demand control of pumping system.

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