Probabilistic Analysis of Geological Properties to Support Equipment Selection for a Deepwater Subsea Oil Project

Christopher J. Jablonowski[a], Edward E. Shumilak[a], Kenneth F. Tyler[a], Arash Haghshenas[b]
[a]Shell Exploration and Production Company, Houston, TX, U.S.A.
[b]Boots & Coots Services LLC, Houston, TX, U.S.A.

Abstract: This paper describes the method and results of a probabilistic risk analysis that was used to provide a quantitative basis for a complex and high-stakes design decision for a deepwater subsea oil project. The analysis specified probabilistic simulations of geologic properties based on information from a small number of exploration and appraisal wells. Each iteration of the simulated data was then fed into a deterministic engineering model to simulate various operational scenarios. Conventional probabilistic sampling and a more efficient experimental design approach were both employed. The key results are cumulative density functions for critical operational variables that drive design decisions.

Keywords: Risk Analysis, Probabilistic Analysis, Experimental Design, Oil and Gas.

1. INTRODUCTION

In almost all oil and gas projects, incomplete information about the geologic properties of the asset (e.g. rock and fluid properties) leads to uncertainties in derivative computations that are used for design decisions. Options are available to collect additional information by drilling additional exploration and appraisal wells, by completing additional modeling and analysis, etc., but this information typically comes at a significant cost in time and/or expenditures. Therefore, at some point in the project maturation process, the cost of additional information destroys project value, and decisions of all kinds must be made giving consideration to the residual uncertainty. There are numerous decision analysis frameworks and quantitative methods that can be applied depending on the decision setting, for example, stochastic and/or deterministic optimization. There is a rich literature that demonstrates the application of these concepts and others for oil and gas problems [3,4,5,6,8,9,11,12,13,15,17, 18, 19,22,25,27].

In the deepwater oil and gas industry, wells routinely cost over $100 million each, and this expenditure only adds one additional data point for analysis. For the project examined in this study, the exploration and appraisal drilling phase is complete, and no additional geologic information was going to be obtained prior to most of the major design decisions for the project. Therefore, the design decisions are based on assumptions about the probability density functions (PDFs) of the geologic properties.

The design decision examined in this study was the specification of the pressure rating of the wellheads for a deepwater subsea oil project, the “Project.” Specifically, the design decision was whether the wellheads should be specified for 15,000 psi or 20,000 psi. The 15,000 psi equipment provides less operational flexibility under certain geologic and operational outcomes and could cause a loss of reserves, or at least a delay in production. Specifying 20,000 psi equipment would eliminate almost all of the risks. While the 15,000 psi equipment is readily available, the 20,000 psi equipment does not exist and would impose a three to four year delay in the project to allow the equipment to be designed, tested, and certified. From a risk analysis perspective, the question is “What is the likelihood of a loss or delay if 15,000 psi equipment is specified, given the current assumptions (PDFs) regarding the geologic uncertainty?”

2. WELL ENGINEERING BASICS

2.1. Wellbore, Wellhead, and Access

A typical wellbore schematic is provided in Figure 1. In a conventional oil well, steel casings are cemented in place as the well is drilled deeper and deeper until the oil reservoir is penetrated. At the top of the wellbore is a wellhead that is appropriately pressure-rated to contain the maximum reservoir pressure, and to enable monitoring and control of fluids. In a subsea setting, the wellhead sits close to the seafloor. Figure 2 depicts a typical subsea wellhead. After a well is drilled and the wellhead installed, the well is put on production and flows back to a gathering facility through a subsea flow line.

During the life of a well, it is probable that some form of intervention will be required. Interventions are required to repair damage, to re-complete the well in a different reservoir, to plug a depleted reservoir, and for other reasons. During an intervention, the wellhead is accessed by a floating drilling rig or similarly capable vessel, as depicted in Figure 3. During an intervention, it is possible that a process called "bullheading" will be required. In a bullheading operation, a high-density fluid is pumped down the well, displacing the fluid in the well back into the reservoir. After a high-density fluid is in the well, intervention operations can proceed in an efficient and safe manner.

Figure 1. Generic Wellbore Schematic
(figure courtesy of the EPA)

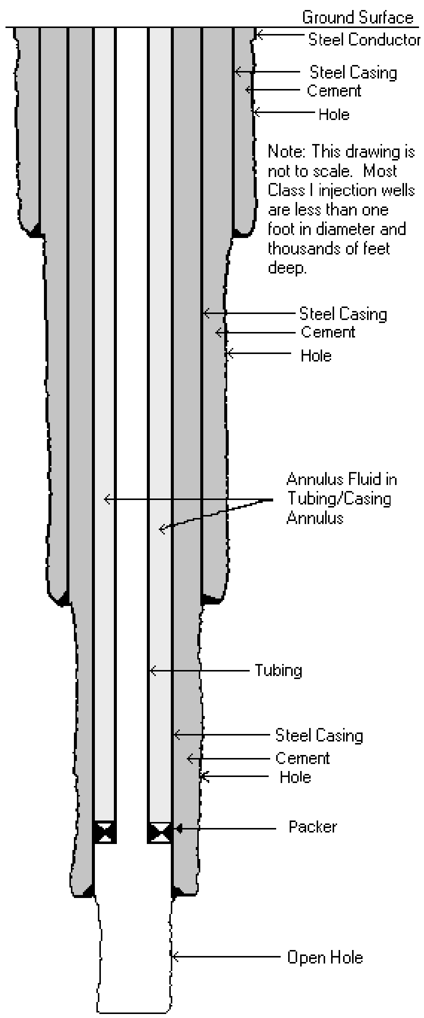
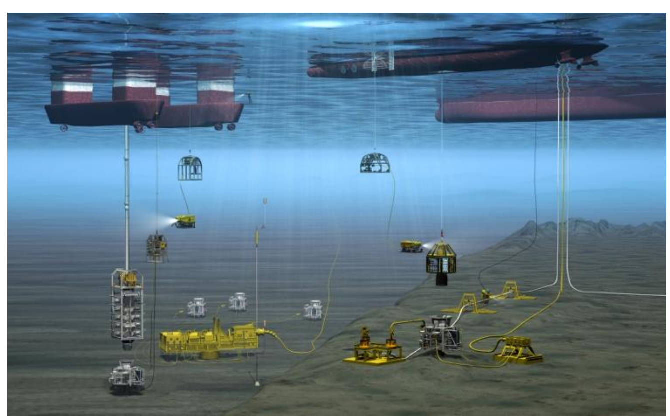


Figure 2. Subsea Wellhead (typical)
(figure courtesy of FMC)

Figure 3. Accessing Subsea Wellheads and Equipment
(figure courtesy of Oceaneering)



2.2. Specification of System Model

When a well is shut-in, the pressure at the wellhead builds up to the reservoir pressure less the hydrostatic gradient of the fluid in the well, and this is called the shut-in pressure. If the well is to be bullheaded, the shut-in pressure must be increased to overcome the pressure losses in the system, most notably the pressure loss incurred when pumping into a permeable reservoir (per Darcy's Law), or in some cases the pressure required to fracture the formation. This pressure is called the bullhead pressure. The expected shut-in and bullhead pressures are important inputs into the specification of the pressure rating of the wellhead.

To estimate the expected bullhead pressure, a common first step is to conduct a simple hydrostatic analysis. If the resulting estimate of bullhead pressure leads to an obvious and economic choice, then a more detailed analysis probably is not warranted. However, if the resulting estimate of bullhead pressure is close to the cross-over point between a lower and higher pressure rating of the wellhead, then a more detailed analysis is warranted, especially if the incremental cost of the higher-rated wellhead is significant. In the extreme case where the higher-rated wellhead does not exist, the analysis of bullhead pressure may be central to the economic viability of the project.

A more detailed analysis of bullhead pressures requires a shift from a simple static hydrostatic analysis to a more complex dynamic analysis. Modeling a dynamic bullheading operation is not a trivial exercise. The information requirements are significant: reservoir rock properties, fluid properties (reservoir fluids and bullhead fluids), reservoir pressures, geothermal gradients, mechanical properties of the hydraulic flow path, bullhead rates, completion efficiency, and the depletion plan.

A physics-based deterministic model of the system was specified for the Project that explicitly accounts for all of these inputs. The system model serves as the computational core of the subsequent risk analysis.

3. WORKFLOW AND RESULTS

The system model is deterministic and solves one case at a time. But as described above, many of the geologic variables are defined only as PDFs. Therefore, a workflow was specified that uses the deterministic system model in a probabilistic manner. Two approaches were employed. In the following descriptions, a *scenario* is defined as the collection of the PDFs of the uncertain variables. Because there may be uncertainty in the properties of the PDFs, it may be desirable to investigate different scenarios. A *sample* is defined as one random observation from each of the uncertain input PDFs for a given scenario. An *iteration* is one run of the system model using one sample. A *simulation* is the collection of multiple iterations for one scenario.

Full Probabilistic. A scenario is defined and a simulation is run. Because of the large number of uncertain variables, a somewhat large simulation size of 2000 iterations was used. The combination of system model complexity and sample size entails significant resource requirements for each simulation. This resource requirement increases linearly with the number of scenarios. After the results from a simulation are available, it is possible to specify regression models that relate variables of interest, e.g. shut-in pressures and bullhead pressures, to the uncertain variables. The resulting models can be used as fast surrogates for the system model for future probabilistic analysis or other analytical needs. The surrogates can also be used to make point predictions and associated probability statements. This approach has been employed in various oil and gas settings and is well-documented in the literature [1,2,10,14,20,21,23,24].

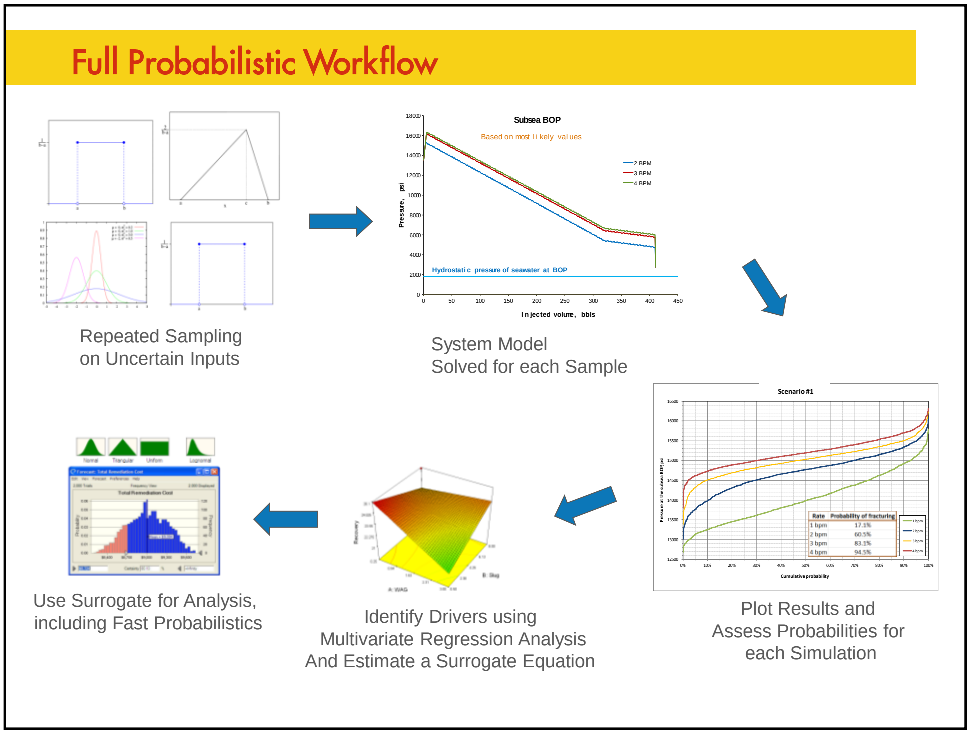
Experimental Design. In contrast to the large simulation size used in the full probabilistic analysis, one can specify a reduced number of iterations for each simulation. That is, the samples are not random, but rather are *designed* to explore the range of uncertainty in the variables. Again, after the results from a simulation are available, it is possible to specify regression models that relate variables of interest to the uncertain variables, and to use the resulting model as a fast surrogate for the system model. This approach is also known to the oil and gas literature [7,16,26,28,29,30,31].

The experimental design, if properly constructed, should yield a regression model of similar explanatory power as that from the full probabilistic analysis. So why do it? First, it was desired to demonstrate that the experimental design approach produces such an equivalent result. In the future on this project, it may be necessary to update the system model and/or run many different scenarios, and the experimental design will be significantly more efficient than reproducing the full probabilistic analysis. Also, for other projects in the future, it is desired to use experimental design only, and this comparison can be referenced to demonstrate their equivalence.

3.1. Probabilistic Analysis

The full probabilistic analysis workflow is depicted in Figure 4. Its major steps include sampling, computations using the system model, analysis of the cumulative distribution, estimation of the surrogate equation, and finally use of the surrogate equation in place of the system model.

Figure 4: Probabilistic Workflow

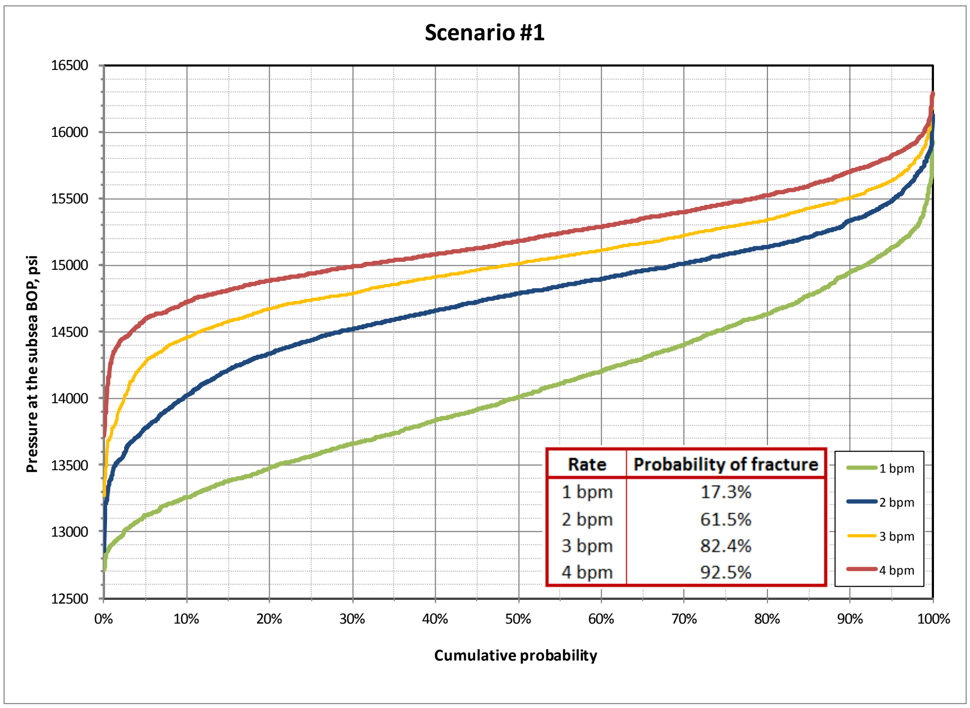


A probabilistic model was specified for three scenarios representing different reservoir pressure regimes (different PDFs): initial conditions, 6 months, and 1 year after initiation of production. This is intended to generate information regarding the rate at which the risk is reduced. In each of these scenarios, a simulation was run for each of four bullhead flow rates: 1, 2, 3, and 4 bpm. Each simulation consisted of 2,000 iterations. This setup results in 3 x 4 x 2,000 = 24,000 model runs or observations.

The shut-in pressure never exceeds 13,200 psi and thus does not impact the wellhead pressure specification decision except as an input into the bullhead pressure computation. The cumulative distribution functions for the initial condition scenario and each of the four bull-head rates are depicted in Figure 5. As can be observed in Figure 5, the probability that the internal pressure will exceed 15,000 psi during a 1 bpm bullhead operation initiated at initial reservoir conditions (worst case) is about 8%. After 6 months of production and pressure decline, this same probability decreases to less than 1%, and after 1 year it approaches 0%. As expected, the probabilities of exceeding 15,000 psi increase as a function of flow rate. At 2, 3, and 4 bpm bullhead rates the probabilities of exceeding 15,000 psi are 30%, 50%, and 70%, respectively. If the 15,000 psi wellhead pressure rating is specified, then prevailing conditions will dictate the maximum bullhead rate. These results indicate that it is very likely that a 1-2 bpm rate will be attainable without exceeding the wellhead pressure rating.

The next step is to specify regression models that relate variables of interest, e.g. shut-in pressures and bullhead pressures, to the uncertain variables. The resulting models can be used as fast surrogates for the system model for future probabilistic analysis or other analytical needs. For example, if it is desired to analyze a new scenario for one or more of the uncertain variables, the surrogate can be used to generate a probabilistic simulation in minutes with the caveat that the range of the revised PDFs are not dissimilar from the original scenario.

Figure 5: Cumulative Distribution of Maximum Bullhead Pressure for Scenario 1 (Initial Reservoir Pressure)



Regression models were specified where the dependent variable was specified as the maximum bullhead pressure, and the independent variables were defined as the uncertain variables. Because the system model is physics-based and deterministic, it is known that there will be two distinct cases. One case is governed by the Darcy equation where the fluid is radially displaced into the pore space of the reservoir. A second case , where the Darcy differential pressures are large, is governed by the formation fracture gradient where fluid is displaced into the fracture. Note, the Darcy differential is defined as follows: Darcy differential (psi) $\approx \frac{203328 q B_o \mu_o}{kh}\left(ln\frac{r_w}{r_r} + s\right)$.

3.1.1. Surrogate Model Using the Full Probabilistic Results: The "No-Fracture" Model

In this regression model, the Darcy differential is small and thus no fracture occurs. Initial analysis showed that results could be pooled across all three (*i*) pore pressure scenarios and all four (*q*) bullhead rates, and the regression was specified as $\boldsymbol{y_{iq} = c + x_{iq}\beta + \varepsilon_{iq}}$. Based on knowledge of the design of the system model, the independent variables are defined using the uncertain variables: $\boldsymbol{x_1 = \frac{B_o \mu_{oil} q}{kh}}$, $\boldsymbol{x_2 = \frac{B_o \mu_{oil} q Skin}{kh}}$, shut-in pressure, oil compressibility, q, and q^2. The linear and quadratic q terms are to account for friction losses. The radius terms are constant for all observations and can be ignored. The results of this regression are provided in Table 1, and a plot of the regression model predictions (x-axis) versus the system model output (y-axis) is depicted in Figure 6.

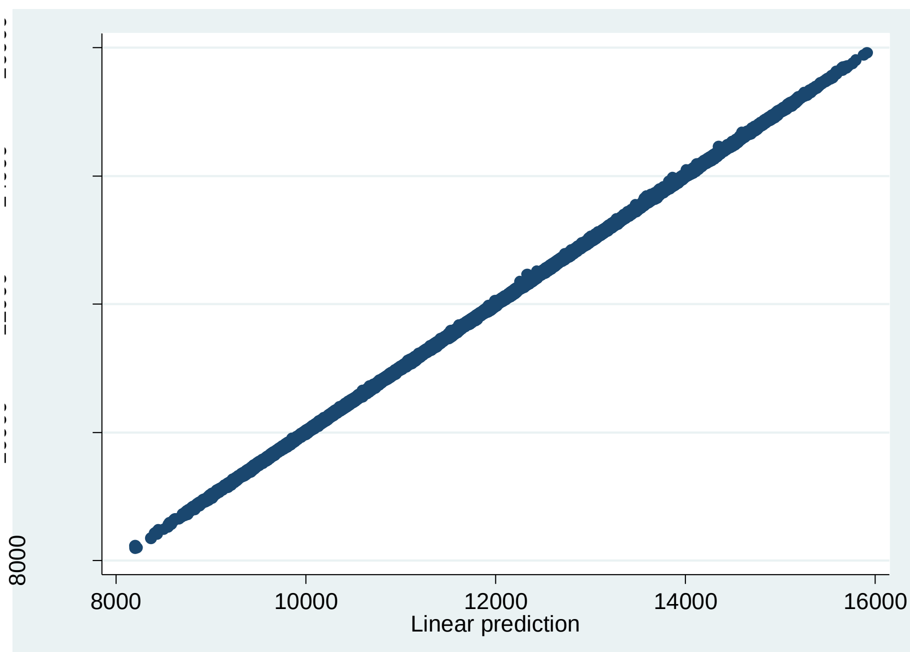
Table 1: The “No Fracture” Surrogate Model (full probabilistic)

```
. reg  maxsspresspsi x1 x2 shutinpresssspsi avgcomppsi q qsq if frac10==0

      Source |       SS       df       MS              Number of obs =   13457
-------------+------------------------------           F(  6, 13450) =       .
       Model |  3.1840e+10     6  5.3067e+09           Prob > F      =  0.0000
    Residual |   4657456.3 13450  346.279279           R-squared     =  0.9999
-------------+------------------------------           Adj R-squared =  0.9999
       Total |  3.1845e+10 13456  2366590.35           Root MSE      =  18.609

------------------------------------------------------------------------------
maxsspress~i |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |    1520749    1074.29  1415.58   0.000      1518643     1522855
          x2 |     196632   45.71866  4300.91   0.000     196542.4    196721.6
shutinpres~i |   .9995036   .0003444  2902.04   0.000     .9988285    1.000179
  avgcomppsi |  -1.11e+08    8891966   -12.49   0.000    -1.28e+08   -9.36e+07
           q |   11.51706   .8594087    13.40   0.000     9.832496    13.20162
         qsq |   17.01699   .1727986    98.48   0.000     16.67828     17.3557
       _cons |   400.5776   35.76365    11.20   0.000     330.4758    470.6793
------------------------------------------------------------------------------
```

Figure 6: The “No Fracture” Surrogate Model Predictions versus the System Model (full probabilistic)



3.1.2. Surrogate Model Using Full Probabilistic Results: The "Fracture" Model

In this regression model, the Darcy differential is large and fracture occurs. Again, initial analysis showed that results could be pooled across all three (*i*) pore pressure scenarios and all four (*q*) bullhead rates. The independent variables are defined using the uncertain variables: shut-in pressure, fracture pressure minus reservoir pressure, oil compressibility, q, and q^2. The results of this regression are provided in Table 2, and a plot of the regression model predictions (x-axis) versus the system model output (y-axis) is depicted in Figure 7.

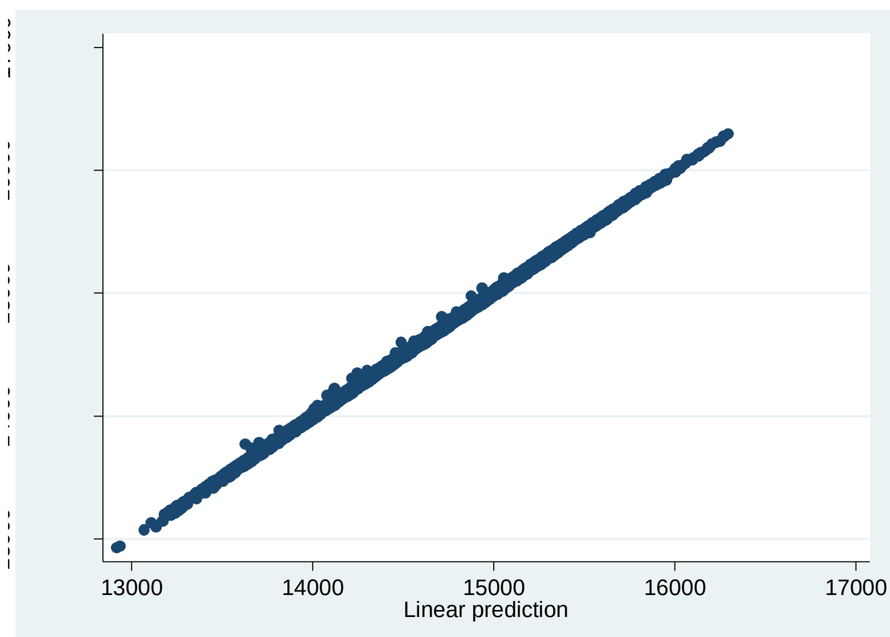
Table 2: The "Fracture" Surrogate Model (full probabilistic)

```
. reg  maxsspresspsi shutinpressssspsi  deltap avgcomppsi q qsq if frac10==1

      Source |       SS       df       MS              Number of obs =   10543
-------------+------------------------------           F(  5, 10537) =       .
       Model |  2.7271e+09     5   545429627           Prob > F      =  0.0000
    Residual |  2527230.05 10537  239.843414           R-squared     =  0.9991
-------------+------------------------------           Adj R-squared =  0.9991
       Total |  2.7297e+09 10542  258933.349           Root MSE      =  15.487

------------------------------------------------------------------------------
maxsspress~i |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
shutinpres~i |   .9996581   .0005427  1841.98   0.000     .9985942    1.000722
      deltap |   .9692905   .0004745  2042.90   0.000     .9683604    .9702205
  avgcomppsi |  -1.41e+08    7790680   -18.15   0.000    -1.57e+08   -1.26e+08
           q |   14.91068   1.096346    13.60   0.000     12.76163    17.05972
         qsq |   16.79008   .1905019    88.14   0.000     16.41666     17.1635
       _cons |   488.0527   33.86037    14.41   0.000       421.68    554.4254
------------------------------------------------------------------------------
```

Figure 6: The "Fracture" Surrogate Model Predictions versus the System Model (full probabilistic)



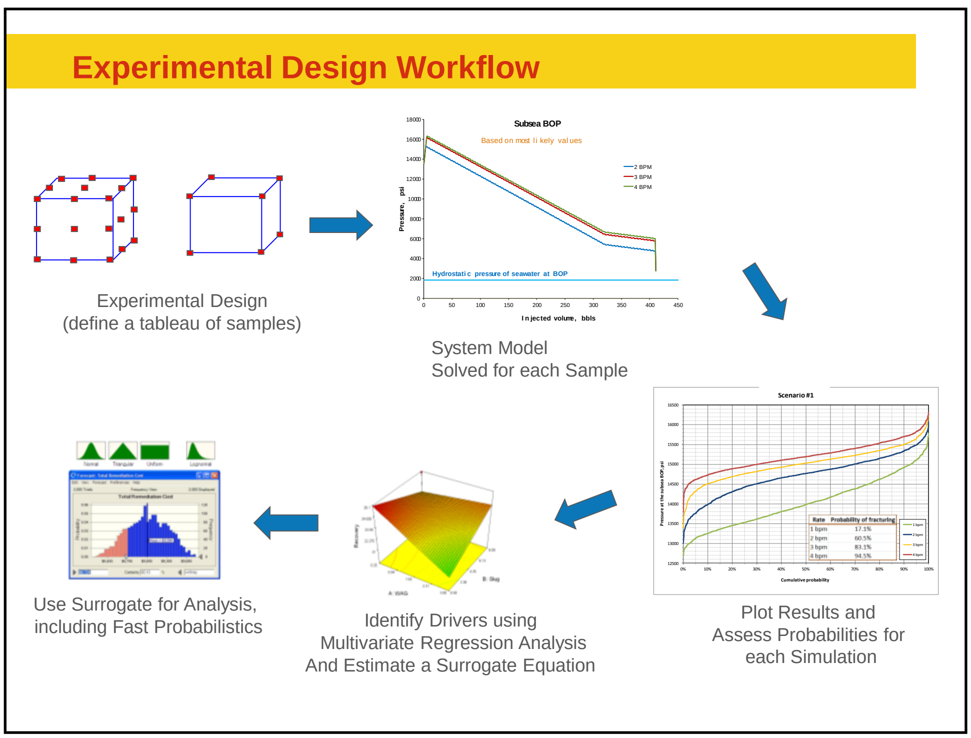
Both the "No Fracture" and "Fracture" surrogate models accurately replicate the system model and are judged to be acceptable surrogates.

3.2. Experimental Design

The experimental design workflow is depicted in Figure 7. It is identical to the probabilistic workflow except for the first step. Instead of repeated sampling, a smaller set of samples is specified for simulation in the system model. That is, the samples are not random, but rather are designed to explore the range of uncertainty in the variables. The smaller simulation size reduces the time required for the computations, and specification of the PDFs for the uncertain variables is not required. The results of the simulation are used to estimate regression models that relate variables of interest to the uncertain variables, and to use the resulting model as a fast surrogate for the system model. Of course, when the surrogate equation is used to conduct a probabilistic simulation, the uncertain variables would need to be fully specified, and these results could be used to create the desired cumulative distribution plots for maximum bullhead pressure as depicted in Figure 5.

Experimental design was used to specify 72 samples for each of the four bullhead rates. Whereas the probabilistic model results in 24,000 observations, the experimental design only requires 72 x 4 = 288 observations (the full range of reservoir pressure can be sampled rather than sampling the three distinct regimes as was done is Section 3.1.). If the surrogate equation from the experimental design is judged to be sufficiently accurate when compared to the surrogate from the full probabilistic model, the full probabilistic model does not need to be repeated in the future.

Figure 7: Design of Experiments Workflow



3.2.1. Surrogate Model Using Experimental Design Results: The "No-Fracture" Model

The identical specification of the "No Fracture" model from Section 3.1.1. was specified and estimated on the appropriate subset of the 288 experimental design observations. The results of this regression are provided in

Table 3, and a plot of model predictions versus the system model are depicted in Figure 8.

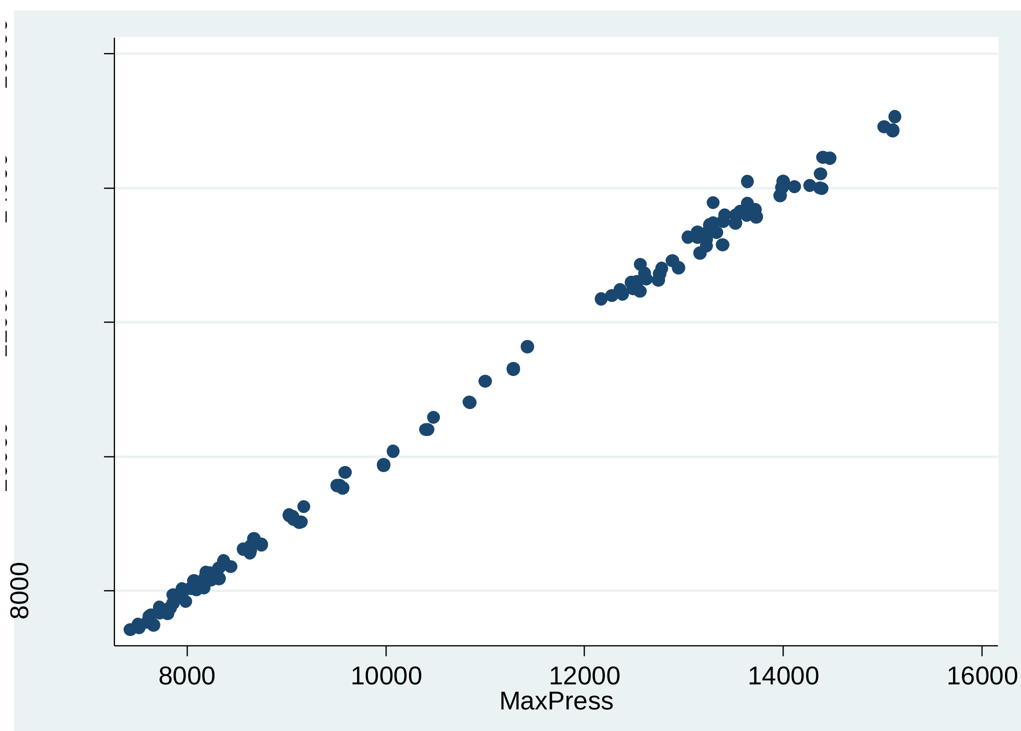
Table 3: The Experimental Design "No Fracture" Surrogate Model

```
. reg  maxpress x1 x2 sitp avgcomppsi q qsq if  simplefrac==0

      Source |       SS       df       MS              Number of obs =     146
-------------+------------------------------           F(  6,   139) = 9575.98
       Model |   942783887     6   157130648           Prob > F      =  0.0000
    Residual |  2280827.92   139  16408.8339           R-squared     =  0.9976
-------------+------------------------------           Adj R-squared =  0.9975
       Total |   945064715   145  6517687.69           Root MSE      =   128.1

------------------------------------------------------------------------------
    maxpress |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |    1252225   18128.68    69.07   0.000      1216381     1288068
          x2 |   162545.8   2587.263    62.83   0.000     157430.3    167661.3
shutinpres~i |   .9771403   .0041214   237.09   0.000     .9689915    .9852892
   avgcomppsi |  -2.00e+08   3.27e+08    -0.61   0.541    -8.47e+08    4.46e+08
           q |   8.622247   53.36682     0.16   0.872    -96.89344    114.1379
         qsq |    14.2422   10.67501     1.33   0.184    -6.864193    35.34859
       _cons |   910.5656    1164.52     0.78   0.436    -1391.898     3213.03
------------------------------------------------------------------------------
```

Figure 8: The Experimental Design "No Fracture" Surrogate Model Predictions versus the System Model



3.2.2. Surrogate Model Using Experimental Design Results: The "Fracture" Model

The regression process was repeated for the appropriate "Fracture" subset of the 288 experimental design observations. The results of this regression are provided in Table 4, and a plot of model predictions versus the system model are depicted in Figure 9.

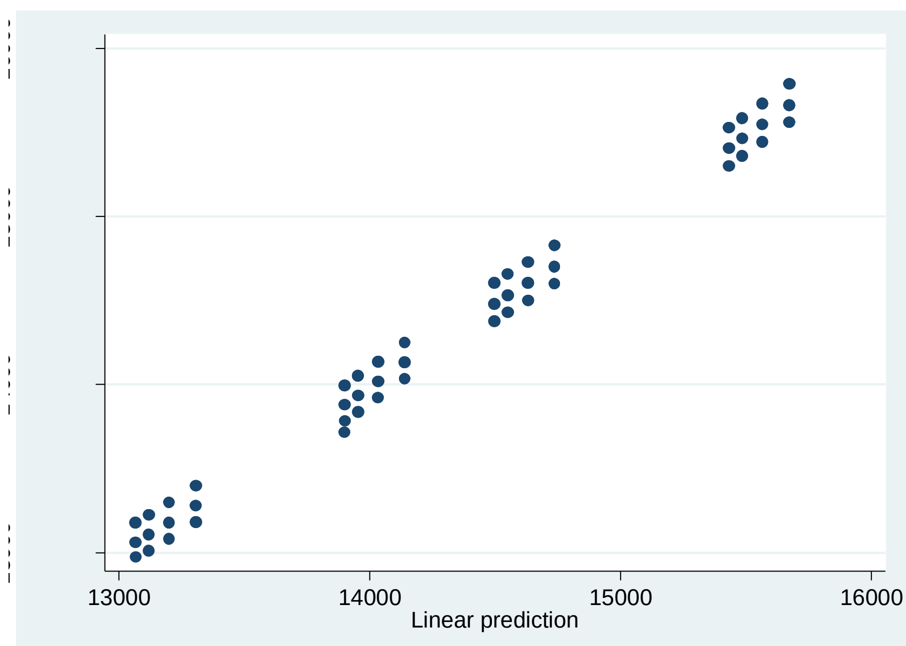
Table 4: The Experimental Design "Fracture" Surrogate Model

```
. reg  maxpress sitp deltap avgcomppsi qsq if  simplefrac==1

      Source |       SS       df       MS              Number of obs =     142
-------------+------------------------------           F(  4,   137) = 1462.83
       Model |  109718984      4  27429745.9           Prob > F      =  0.0000
    Residual |  2568899.96   137  18751.0946           R-squared     =  0.9771
-------------+------------------------------           Adj R-squared =  0.9765
       Total |  112287884    141  796367.969           Root MSE      =  136.93

------------------------------------------------------------------------------
    maxpress |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        sitp |   .8861487   .0205793    43.06   0.000     .8454544    .9268429
      deltap |   .8370611   .0290006    28.86   0.000     .7797144    .8944079
  avgcomppsi |  -1.70e+08   3.68e+08    -0.46   0.646    -8.98e+08    5.59e+08
         qsq |   16.72607   2.052675     8.15   0.000     12.66705    20.78509
       _cons |   2129.825   1351.013     1.58   0.117    -541.7104    4801.361
------------------------------------------------------------------------------
```

Figure 9: The Experimental Design "Fracture" Surrogate Model Predictions versus the System Model



The surrogate models that are based on the experimental design yield very good fits to the system model results. There is more variance in the prediction when compared to the full probabilistic surrogate because of the smaller number of observations in the experimental design. However, for this decision-setting, these small differences are not decision-relevant, and the experimental design approach is judged to be adequate for analyzing different scenarios in the future.

4. NOMENCLATURE

bpm = barrels per minute

B_o = formation volume factor (rb/stb)
h = reservoir thickness (feet)
k = permeability (md)
μ = viscosity (cp)
q = flow rate (bpm)
r_r = radius of drainage (feet)
r_w = radius of well (feet)
s = skin factor

5. REFERENCES

[1] Adams, A.J., Gibson, C., Smith, R. 2009. Probabilistic Well Time Estimation Revisited. Paper SPE 119287 presented at the SPE/IADC Drilling Conference and Exhibition, Amsterdam, 17–19 March. doi: 10.2118/119287-MS.
[2] Akins, W.M., Abell, M.P., and Diggins, E.M. 2005. Enhancing Drilling Risk and Performance Management Through the Use of Probabilistic Time and Cost Estimating. Paper SPE 92340 presented at the SPE/IADC Drilling Conference, Amsterdam, 23–25 February. doi: 10.2118/92340-MS.
[3] Barnes, R.J., Linke, P. and Kokossis, A. 2002. Optimization of oil-field development production capacity. European Symposium on Computer Aided Process Engineering, **12**: 631.
[4] Begg, S.H., Bratvold, R.B. and Campbell, J.M. 2001. Improving Investment Decisions Using a Stochastic Integrated Asset Model. Paper SPE-71414 presented at the SPE Annual Technical Conference and Exhibition, New Orleans, Louisiana, 30 September-3 October.
[5] Cullick, A.S., Heath, D., Narayanan, K., April, J. and Kelly, J. 2003. Optimizing Multiple-field Scheduling and Production Strategy with Reduced Risk. Paper SPE 84239 presented at the SPE Annual Technical Conference and Exhibition, Denver, Colorado, October 4-6.
[6] Cullick, A.S., Cude, R. and Tarman, M. 2007. Optimizing Field Development Concepts for Complex Offshore Production Systems. Paper SPE 108562 presented at the SPE Offshore Europe, Aberdeen, 4-7 September.
[7] Damsleth E., Hage A., Volden R. 1992. Maximum Information at Minimum Cost: A North Sea Field Development Study with an Experimental Design. *J. Pet Tech* (December): 1350-1356.
[8] Ettehad, A., Jablonowski, C.J. and Lake, L.W. 2010. Gas Storage Facility Design under Uncertainty. *SPE Proj Fac & Const* **5** (3): 155-165. SPE-123987-PA.
[9] Goel, V. and Grossmann, I.E. 2004. A Stochastic Programming Approach to Planning of Offshore Gas Field Developments under Uncertainty in Reserves. *Journal of Computers and Chemical Engineering* **28**: 1409–1429.
[10] Hariharan, P.R., Judge, R.A., and Nguyen, D.M. 2006. The Use of Probabilistic Analysis for Estimating Drilling Time and Costs While Evaluating Economic Benefits of New Technologies. Paper SPE 98695 presented at the IADC/SPE Drilling Conference in Miami, Florida, USA, 21–23 February. doi: 10.2118/98695-MS.

[11] Haugen, K. 1996. A Stochastic Dynamic Programming Model for Scheduling of Offshore Petroleum Fields with Resource Uncertainty. *European Journal of Operational Research* **88:** 88-100.
[12] Kabir, C.S., Chawathe, A., Jenkins, S.D., Olayomi, A.J., Aigbe, C. and Faparusi, D.B. 2004. Developing New Fields Using Probabilistic Reservoir Forecasting. *SPE Reservoir Evaluation & Engineering*, **7**(1): 15-23. SPE-87643-PA.
[13] Kabir, C.S., Gorell, S.B., Portillo, M.E. and Cullick, A.S. 2007. Decision Making With Uncertainty While Developing Multiple Gas/Condensate Reservoirs: Well Count and Pipeline Optimization. *SPE Res Eval & Eng* **10** (3): 251-259. SPE-95528-PA.
[14] Kitchel, B.G., Moore, S.O., Banks, W.H., and Borland, B.M. 1997. Probabilistic Drilling-Cost Estimating. *SPE Comp App* **12** (4): 121–125. SPE-35990-PA. doi: 10.2118/35990-PA.
[15] Kosmidis, V.D., Perkins, J. D. and Pistikopoulos, E.N. 2002. A Mixed Integer Optimization Strategy for Integrated Gas/Oil Production. European Symposium on Computer Aided Process Engineering, **12**: 697.
[16] Li, B., Friedmann, F. 2005. Novel Multiple Resolutions Design of Experiment/Response Surface Methodology for Uncertainty Analysis of Reservoir Simulation Forecasts. Paper SPE 92853 presented at the SPE Reservoir Simulation Symposium, Houston, TX, 31 January-2 February.
[17] Lund, M.W. 2000. Valuing Flexibility in Offshore Petroleum Projects. *Annals of Operations Research* **99** (1-4): 325-349.
[18] McFarland, J.W., Lasdon L. and Loose V. 1984. Development Planning and Management of Petroleum Reservoirs Using Tank Models and Nonlinear Programming. *Operations Research* **32** (2): 270-289.
[19] Murray, J.E. and Edgar, T.F. 1978. Optimal Scheduling of Production and Compression in Gas Fields. *J Pet Technol* **30** (1): 109-116. SPE-6033-PA.
[20] Murtha, J. 1997. Monte Carlo Simulation: Its Status and Future. Distinguished Author Series, *J Pet Technol* **49** (4): 361–370. SPE-37932-MS. doi: 10.2118/37932-MS.
[21] Noerager, J.A., Norge, E., White, J.P., Floetra, A., and Dawson, R. 1987. Drilling Time Predictions From Statistical Analysis. Paper SPE 16164 presented at the SPE/IADC Drilling Conference, New Orleans, 15–18 March. doi: 10.2118/16164-MS.
[22] Ortiz-Gomez, A., Rico-Ramirez, V. and Hernandez-Castro, S. 2002. Mixed-integer Multi-period Model for the Planning of Oil-field Production. *Computers and Chemical Engineering* **26** (4–5): 703.
[23] Peterson, S.K., Murtha, J.A., and Roberts, R.W. 1995. Drilling Performance Predictions: Case Studies Illustrating the Use of Risk Analysis. Paper SPE 29364 presented at the SPE/IADC Drilling Conference, Amsterdam, 28 February –2 March. doi: 10.2118/29364-MS.
[24] Peterson, S.K., Murtha, J.A., and Schneider, F.F. 1993. Risk Analysis and Monte Carlo Simulation Applied to the Generation of Drilling AFE Estimates. Paper SPE 26339 presented at the SPE Annual Technical Conference, Houston, 3–6 October. doi: 10.2118/26339-MS.
[25] Purwar, S., Jablonowski, C., Nguyen, Q. "Development Optimization Using Reservoir Response Surfaces: Methods for Integrating Facility and Operational Options," *Natural Resources Research*, 20 (1): 1-9. March, 2011.
[26] Schiozer, D.J., Ligero, E.L., Maschio, C., Risso, F.V.A. 2008. Risk Assessment of Petroleum Fields—Use of Numerical Simulation and Proxy Models. *Petroleum Science and Technology*, 26 (10): 1247-1266.
[27] Van den Heever, S., Grossmann, I.E., Vasantharajan, S. and Edwards, K.L. 2001. A Lagrangean Decomposition Heuristic for the Design and Planning of Offshore Hydrocarbon Field Infrastructure with Complex Economic Objectives. *I&EC Research* **40** (13): 2857.
[28] Vanegas Prada, J.W., Cunha, L.B. 2008. Assessment of Optimal Operating Conditions in a SAGD Project by Design of Experiments and Response Surface Methodology. *Petroleum Science and Technology*, 26 (17): 2095-2107.
[29] White, C.D., Willis, B.J., Narayanan, K., Dutton, S.P. 2001. Identifying and Estimating Significant Geologic Parameters with Experimental Design. *SPE Journal*, 6 (3): 311-324.
[30] White, C.D., Royer S.A. 2003. Experimental Design as a Framework for Reservoir Studies. Paper SPE 79676 presented at the SPE Reservoir Simulation Symposium, Houston, Texas, 3-5 February.

[31] Yamali, N., Nguyen, Q.P., Srinivasan, S. 2007. Optimum Control of Unwanted Water Production in Stratified Gas Reservoirs. Paper SPE 106640 presented at the SPE Production and Operations Symposium, Oklahoma City, Oklahoma, 31 March-3 April.