A Usage-Informed Preventive Maintenance Policy to Optimize the Maintenance Free Operating Period for Multi-Component Systems

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Abstract: This paper deals with the concept of Maintenance Free Operating Period (MFOP). This MFOP is defined as a period of operation during which the system should be able with a given level of confidence to carry out all its assigned missions without system fault or performance limitation. Based on this concept, a dynamic maintenance policy for a multi-component system is implemented. The main objective of this paper is to propose a method to integrate the usage information of the system components in order to optimize the implemented policy. The method is evaluated considering the Total Maintenance Cost (TMC) value.

Keywords: Maintenance, Reliability, MFOP, Usage

1. INTRODUCTION

Nowadays even if the vehicle configuration is important for any customer, the development of an efficient maintenance management system appears as another key of success. Aware of this opportunity, the commercial heavy vehicle industry propose, to its customers, service contracts in order to manage the vehicle maintenance.

These contracts are built from information on the vehicle configuration and on the estimation of vehicle operating conditions provided by the customer. Based on this information, a maintenance planning is created to inform the customer on the planned service operations during the maintenance contract period.

Currently the maintenance planning is static. It means that the maintenance intervals defined at the vehicle purchase date aren't updated during all the vehicle life. Moreover this planning is based on a component perspective in which the interactions at the system level are not taken into account. As a consequence, the total maintenance cost is impacted by unplanned maintenances generating high immobilization costs.

In this framework, a dynamic maintenance policy for a multi-component system integrating the possibilities offered by new information and communication technology solutions can be investigated. To increase the operational reliability of the system and decrease downtime and maintenance costs, a reliability based maintenance policy can be used. Note that most of the time, the optimization of these policies aims to define the best moments to perform maintenance tasks or inspections in order to find the best balance between preventive and corrective maintenance. The problem and the constraints are different in the heavy vehicle industry. Indeed the maintenance can be performed in a preventive way

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exclusively when the vehicle returns in the workshop. Out of these occasions, in period of operations, the maintenance is almost impossible or generates high immobilization costs. To overcome this problem, the developed maintenance policy should be able to ensure failure free operation on given period with a high confidence level and select maintenance operations to be performed during stops at the workshop.

Aware of this issue, the Royal Air Force proposed in 1996 the concept of the Maintenance Free Operating Period (MFOP) [1,2] with the objective to obtain better operational planning capability, improved operational availability and reduce running costs.

The main contribution of this paper is to propose a dynamic maintenance policy for a multicomponent system. This policy, based on MFOP concept, integrates the usage information of the system components to optimize the total maintenance costs.

The remainder of the article is organized as follows. Section 2 defines the MFOP concept and the implemented maintenance policy. Section 3 illustrates the impact of usage information on the maintenance decision. Section 4 develops the use of mixture models to support the usage-informed maintenance policy. Section 5 deals with the total maintenance cost definition and the maintenance strategy optimization. The last section illustrates the method on a numerical example.

2. MAINTENANCE POLICY BASED ON MFOP CONCEPT

2.1. MFOP Concept Definition

The MFOP is defined as a period of operation during which the equipment must be able to carry out all its assigned missions without any maintenance action and without the operator being restricted in any way due to system faults or limitations [3]. The MFOP measure assumes that success is attainable and that failures can be accurately forecast [4].

According to its definition, the main objective is to avoid unplanned maintenance operations in moving all upcoming corrective maintenances to a schedule period of time of preventive maintenance. Based on this objective, the concept appears as a method to group maintenance operations at the end of MFOP (or cycle of MFOP) during stop at the workshop.

In [5], Tinga and al. argue that in this form of grouping, called time-driven clustering, the moment of maintenance is not driven by the failure of one of the components but must be planned carefully. Thereby, contrary to other forms like block replacement policy or opportunity-based maintenance, this clustering method could be very interesting for systems with high immobilization costs and where the number of maintenance opportunities is quite limited such as transport systems.

To ensure this MFOP, maintenance policies based on this concept have been introduced [6]. Nevertheless these policies are developed for single component system and do not integrate the possibility to take into account the available information on component usage in the maintenance decision process.

2.2. Dynamic Maintenance Policy

The dynamic maintenance policy implemented (see Fig. 1) consists in estimating at each end of MFOP or when a failure occurs, the probability that the multi-component system survives for the duration of MFOP given the available information [7].

If the reliability requirement is a MFOP of t_{MFOP} life units for the *i*th cycle of MFOP, this probability called Maintenance Free Operating Period Survivability (*MFOPS*) is given by:

$$MFOPS(t_{MFOP}) = \frac{R_{syst}(i * t_{MFOP})}{R_{syst}((i-1) * t_{MFOP})}$$
(1)

where $R_{syst}(t_{MFOP})$ is the system reliability after t_{MFOP} life units.

Thereby if the *MFOPS* at the end of a MFOP is higher than a specified confidence level, no maintenance operation is necessary and the system can be deployed for the next period. In the opposite case, where the *MFOPS* is lower than a specified confidence level, maintenance occasion is needed to reach again the confidence level. Consequently, the *MFOPS* allows to define if a maintenance occasion is needed or not.





When a maintenance occasion is needed, a maintenance decision rule to select the maintenance operations to be performed during this occasion should be defined. In this paper a maintenance decision rule based on the cost minimization on the MFOP horizon is introduced [8]. In this case the problem can be mathematically formulated as follows:

$$\min_{\{x_i\}} \sum_{i=1}^{n} x_i * C_i$$
(2)

s.t MFOPS > CL

where *n* is the number of system components, C_i is the operation cost including labor and spare part cost of component *i*, x_i is a binary variable which indicates the selection of a maintenance operation on the component *i* and *CL* is the specified confidence level. Further the following assumptions are made to solve this optimization problem. Assumption 1: After each maintenance operation where one or several components are replaced, their reliability performances are considered "as good as new". Assumption 2: The reliability performance of the other components is considered unchanged or "as bad as old".

The interesting feature of the MFOPS is its update with the reliability of the components at the end of each period. Based on this feature, the uncertainty of the MFOPS strongly depends on the available monitoring information. In a previous paper [8], the impact of different information levels on the components state has been illustrated. The impact of usage information will be investigated in the next section.

3. IMPACT OF USAGE INFORMATION ON THE MAINTENANCE DECISION

3.1. Lifetime Models and Usage Information

According to the operating condition in which a component is used, its degradation mechanism will be different, more or less variable. Note that disregarding this usage information, especially for systems

which operate in variable conditions, generates a large uncertainty in the lifetime models and an efficiency decrease of the maintenance policy [9].



Figure 2: Usage Uncertainty and the Consequences on the Lifetime Models

The lifetime models are obtained from Volvo databases where maintenance and repair events have been recorded on a per-vehicle basis. These models are built per component for a given vehicle range and purchase year. In these models a significant statistical variance appears that makes them economically inefficient and unprofitable on larger scales.

The recorded failures come from components used in variable usage conditions. Currently in the databases, no direct link is available to correlate the usage information and the failure date. In this framework, considering a unique lifetime model for the component datasets could be inappropriate and can explain the significant variance. To avoid this kind of problems, Tinga [9] mentions that removing the uncertainty in usage reduces the width of distributions and increases the reliability models accuracy (see Fig. 2). The proposed method to define the various lifetime models correlated with the component usage will be presented in the next section.

3.2. Information Levels Definition

In order to demonstrate the impact of the usage information on the maintenance policy, three information levels are considered per component (see Tab. 1). For the first information level, no usage information is available. For the components under this information level, a unimodal lifetime model will be considered.

Information Level	Usage Information	Component Lifetime Models		
Level 1	No available information	Unimodal lifetime model		
Level 2	Past usage information	Lifetime model selected according to the usage information		
Level 3	Past usage information + Future usage estimation on the next MFOP	Lifetime model selected according to the usage information		

Table 1: Information Levels Definition

For the second information level, it is assumed that different lifetime models are defined per component according to the operating conditions and that the usage information is known in a

continuous way. Thereby the selection of component lifetime model may be updated at the end of each MFOP cycle thanks to the exact knowledge on the past operating conditions.

For the last level, the previous assumptions made for the second level are considered and additionally it is assumed that the predicted usage information for the next MFOP is available. This prevision of future operating conditions can be obtained thanks to close relationship with customers. The component lifetime model can be updated at the end of each MFOP cycle according to the knowledge on the past operating conditions and on the next MFOP.

4. MIXTURE MODELS TO CONNECT USAGES AND COMPONENT LIFETIMES

As mentioned previously, the main objective is to implement a usage-informed maintenance policy which selects, at the end of each MFOP cycle, the component lifetime models according to the usage information and uses it in the maintenance decision rule. To achieve this objective, it is necessary to be able to connect the usage profiles and the component life consumption [5].

In this section, an experience-based method is proposed in order to define this connection. Note that the component failure behavior is based on failure data collected in the past. Under the assumption that the operating conditions affect the component reliability, a mixture models identification can be applied on these failure data to identify the lifetime models conditional to the different usages.

4.1. Mixture Model Definition

Considering a mixture of lifetime distributions consists of assuming that failure data come from several sub-populations. Each sub-population can be modeled, in a separate manner, by a unique lifetime distribution. The total dataset is thus a mixture of these sub-populations. Each sub-population is assumed to represent a type of usage for the specified component.

Zaman and al. [10] explained that a mixture model of distributions is a weighted average of probability distributions with positive weights that sum to one. The density function of mixture distribution is given by:

$$g(x) = \sum_{j=1}^{k} w_j f_{j(x)}$$
(3)

where k represents the assumed number of sub-population in the mixture under study, w_j is the proportion of the *j*th sub-population in the mixture and $f_{j(x)}$ the density function of the *j*th sub-population.

4.2. Parameter Estimation

According to Razali and al. [11], a number of methods for estimating the parameters for mixture distribution represented by Eq. 3 including Maximum likelihood (MLE), moment method, Bayesian method and least square method can be investigated. Currently, MLE became more popular for parameter estimation in mixture model. Therefore in this paper, the MLE method will be considered.

The MLE parameter estimators can be obtained by finding the log likelihood function of Eq. 3 as follows:

$$LL = \sum_{l=1}^{m} \log(\sum_{j=1}^{k} w_j f_{j(x_l)})$$
(4)

where m represents the number of data observed in the total population. The maximum of Eq. 4 can be obtained by taking the first derivative of LL with respect to all parameters and set it to be zero [12].

To solve this mathematical problem, the most popular method is the Expectation-Maximization algorithm (EM algorithm). In practice, this method needs a parameters initialization to start the iterative process. In this paper, the K-means method will be considered to increase the EM-algorithm efficiency [13]. This method aims to divide the initial population into K clusters in which each observation belongs to the cluster with the nearest mean.

The following process allows the parameters estimation for mixture distribution assuming a given number of sub-populations. Nevertheless, this sub-populations number is most of the time unknown. Only expert statements can be used to define the maximum possible number of sub-populations. Thereby a criterion based on the coefficient of determination R^2 will be used to determine the best number of sub-populations for a specified mixture distribution. R^2 is given by:

$$R^{2} = 1 - \frac{\sum_{l=1}^{m} (P_{l} - \hat{P}_{l})^{2}}{\sum_{l=1}^{m} (P_{l} - \bar{P})^{2}}$$
(5)

where *m* represents the number of data observed in the total population, P_l are the values of relative frequencies of observed data, \hat{P}_l are the forecast values using the mixture distribution function and \bar{P} is the mean of relative frequencies of observed data. Usually the coefficient of determination R^2 , which is a measure of goodness fit, increases with the number of sub-populations. In order to avoid selecting always the maximum number of sub-population, a threshold equals to 0.99 is introduced on the R^2 value from which the number of sub-populations is validated.

4.3. Allocation Method

According to the methods presented in the two previous sub-sections, the number of sub-populations and the lifetime models for each sub-population can be determined from the initial mixed dataset. Then from these results, an allocation method should be implemented in order to classify the initial failure time values in each sub-population.

The natural idea is to allocate the failure data in the sub-population from which it is most likely to be seen from the observed value and characteristics of sub-populations. The probability that the observed failure a belongs to the sub-population k given the value of x_a is given by:

$$\tau_{ak} = \frac{w_k f_k(x_a)}{\sum_{j=1}^k w_j f_j(x_a)} \tag{6}$$

Based on this probability, a maximum a posteriori classification method can be used. This allocation method imposes to compute for each observed failure *a* the probabilities τ_{ak} relative to each sub-population and to allocate the failure in the sub-population with the maximum probability τ_{ak} .

As mentioned previously, no direct link is available in the current databases to correlate the usage information and the failure date. In this framework, this allocation method is able to cluster the initial failures in various sub-populations facilitating the highlighting of covariates explaining the emergence of these sub-populations. These covariates could be determined thanks to vehicle signals analysis for the different sub-populations. Thereby with the monitoring of these covariates, the current operating conditions can be correlated with the identified sub-populations and the lifetime model can be updated. Note that in this paper the covariates will be assumed to be known.

5. MAINTENANCE STRATEGY OPTIMIZATON BASED ON TOTAL MAINTENANCE COST

In order to evaluate the alternative maintenance strategies and to optimize the usage-informed maintenance policy based on MFOP concept, the Total Maintenance Cost (TMC) could be evaluated over five years which represents the nominal contract duration.

The TMC is expressed as:

$$TMC = C_{repl} + C_{cor} + C_{diag} \tag{7}$$

where C_{repl} is the replacement cost, C_{cor} is the corrective cost and C_{diag} is the diagnosis cost. The C_{repl} can be defined as:

$$C_{repl} = \sum_{i=1}^{n} C_i * (N_{i,prev} + N_{i,cor}) + C_{setup} * N_{MS}$$
(8)

where *n* is the number of system components, C_i is the operation cost including labor and spare part cost of component *i*, $N_{i,prev}$ is the number of replacements of component *i* during a system preventive stop, $N_{i,cor}$ is the number of replacements of component *i* during a system corrective stop, C_{setup} is the setup cost and N_{MS} is the total number of maintenance stops.

Then the
$$C_{cor}$$
 is given by:

$$C_{cor} = \sum_{i=1}^{n} D_i * N_{i,cor} * \tau_{immo} + ((D_{setup} + D_{tow}) * \tau_{immo} * N_{sfailure}) + (C_{tow} * N_{sfailure})$$
(9)

where D_i is the replacement duration of component *i* in hour, τ_{immo} is the hourly rate for a system immobilization, D_{setup} is the setup activities duration, D_{tow} is the tow duration, $N_{sfailure}$ is the number of system failures and C_{tow} is the tow cost. Thereby a failure at the system level is considered to impact the customer by the tow cost but also by the total stop duration which leads to a loss of production.

Finally the
$$C_{diag}$$
 is expressed as:

$$C_{diag} = (C_{udiag} * n * N_{sfailure}) + (D_{udiag} * n * N_{sfailure} * \tau_{immo})$$
(10)

where C_{udiag} is the unitary diagnosis cost and D_{udiag} is the unitary diagnosis duration. Indeed when the system failed, a diagnosis for each system components is considered as mandatory to repair the system.

By Monte Carlo simulation, various maintenance strategies can be examined. The optimal solution is the strategy corresponding to the lowest value of *TMC*.

6. NUMERICAL EXAMPLE

6.1. Initial Database Implementation and Mixture Models Application

In the real databases of the company, no direct link is currently available to correlate the usage information and the failure date. To overcome this problem, a simulated database is built to be able to highlight the covariates effects responsible of the possible sub-populations.

In this sub-section, the aim is to define a method to build an initial failure database per component. The mixture models method will then be applied on these failure databases in order to connect lifetime models and assumed usages. Note that the way to build the initial failure databases is totally independent of the mixture models method.

Consider a deteriorating component subject to a failure mechanism due to an excessive deterioration level L. A Gamma process is considered to describe its evolution. Assume that the component operates in variable conditions and consider that the operating environment influences the speed and the variance of the degradation process [14].

In order to build the failure database relative to each component, assume that the component operates under a two-stages environment: "normal" and "stressed", and the deterioration follows a homogeneous Gamma process for each of the environment states (see Fig. 3). Let $(\alpha_n. t, \beta_n)$ and $(\alpha_s. t, \beta_s)$ denote the couples of parameters respectively for the "normal" and "stressed" environments [15]. Note that these couples of parameters will be different for each considered component.



Figure 3: Degradation Process in a Dynamic Environment

Consider the component life as a succession of running period during which its environment evolves between the "normal" and "stressed" state. A Normal distribution N(10000, 1000) will be defined to simulate each running period length in kilometer. A probability of being in a "stressed" state between 0% and 60%, considered as the minimum and the maximum threshold, is affected at each component history. This probability will be used at each running period to define the environment state.





Based on this process, 10000 histories are simulated for each system component and for each history the failure date in kilometer and the kilometer ratio in the "stressed" environment are computed. Note that the kilometer ratio in the "stressed" environment is defined as the covariate. Further the Gamma processes used to build the initial failure database per component will be thereafter assumed unknown.

To illustrate the mixture models application assume that a component follows a Gamma process $Ga(7e^{-4} * t, 20)$ in a "normal" environment and $Ga(1.4e^{-3} * t, 20)$ in a "stressed" environment and that the degradation threshold is fixed at L = 12. The histogram given in Fig. 4 represents the obtained failure database for a component according to the implemented methodology.



The main objective consists then to determine if the initial failure database comes from one or several sub-populations. The first step is to consider only the case with k = 1 sub-population. A Weibull distribution is used to model the component lifetime. Note that the Weibull distribution is the most widely used distribution for modeling failure datasets. For this first step, the Weibull lifetime model W(2.9e5, 6.7) is obtained and the measure of goodness gives $R_{k=1}^2 = 0.984$. In general this value is sufficient to validate the unique model nevertheless the variance obtained in this case is very width (see Fig. 5) and the use of mixture models seems to be appropriate.



Figure 6: Initial Failure Allocation and Limit Definition on the Covariate

The second step is to consider the case with k = 2 sub-populations. In applying the defined process, the Weibull lifetime models W(2.5e5, 12.2) and W(3.1e5, 8.2) are obtained and the measure of goodness of fit gives $R_{k=2}^2 = 0.999$. According to the rule previously mentioned, $R_{k=2}^2 > 0.99$ thus the case with k = 2 sub-populations is validated.

Once the parameters and the number of sub-populations are defined for the mixture model, the next step is to allocate the initial failure data in each sub-population thanks to the maximum a posteriori classification method. Assume that the first sub-population represents the model W(2.5e5, 12.2) and that the second sub-population represents the model W(3.1e5, 8.2).

In the Fig. 6, each classified failure is associated with the kilometer ratio in the "stressed" environment. This measure defined as the covariate is assumed to be able to explain the emergence of these two sub-populations. To select, at the end of each MFOP cycle, the lifetime model according to its operating conditions, a limit to distinguish the two sub-populations is fixed on this covariate. To determine this limit, the aim is to minimize the allocation errors on the total population. Based on this requirement, the limit of 30% is fixed. Thereby at the end of each MFOP cycle, if the kilometer ratio in the "stressed" environment is inferior at 30% for the specified component, the lifetime model W(3.1e5, 8.2) is selected and in the other case W(2.5e5, 12.2).

6.2. System Definition



In order to illustrate the usage-informed preventive maintenance policy based on MFOP concept, the following multi-component system is defined (see Fig. 7).

For this system, the hourly rate for a system immobilization is fixed at $\tau_{immo} = 100 \in$, the unitary diagnosis cost and duration are respectively fixed at $C_{udiag} = 20 \in$ and $D_{udiag} = 5$ min, the tow cost and duration are respectively fixed at $C_{tow} = 1500 \in$ and $D_{tow} = 5$ h and finally the setup cost and duration are respectively fixed at $C_{setup} = 100 \in$ and $D_{setup} = 30$ min.

	C1	C2	C3 C4		C5	C6	
Gamma processes	G(7e-4,20) G(1.4e-3,20) L=12	G(7e-4,20) G(1.4e-3,20) L=12	G(8e-4,20) G(1.6e-3,20) L=12	G(8e-4,20) G(1.6e-3,20) L=12	G(6e-4,20) G(1e-3,20) L=11	G(6.4e-4,20) G(1.4e-3,20) L=11	
Lifetime models and limit k = 2	W(2.5e5,12.2) W(3.1e5,8.2) Limit = 30%	W(2.5e5,12.2) W(3.1e5,8.2) Limit = 30%	W(2.2e5,12.1) W(2.7e5,8.2) Limit = 31%	W(2.2e5,12.1) W(2.7e5,8.2) Limit = 31%	W(2.9e5,15.1) W(3.5e5,9.9) Limit = 32%	W(2.4e5,10.8) W(3.1e5,7.4) Limit = 31%	
Lifetime model k = 1	W(2.9e5,6.7)	W(2.9e5,6.7)	W(2.6e5,6.7)	W(2.6e5,6.7)	W(3.3e5,8.6)	W(2.9e5,5.9)	
C _i	458	458	407	407 842		1268	
D _i	1	1	1.5	1.5	3.7	1.8	

Table 2: S	System	Parameters
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Tab. 2 describes the Gamma processes used to build the initial failure database per component, the lifetime models obtained for k = 1 and k = 2 sub-populations as well as the defined limit on the covariate and the specific maintenance cost and duration per component. Note that for each system component, the mixture model for k = 2 sub-populations has been validated.

6.3. Cost-Optimized Maintenance Policy Based on MFOP and Usage Information

A maintenance model is developed in order to calculate the *TMC* index over five years based on Monte Carlo simulation. Assume that, first at the end of each MFOP cycle, only the information at the system level is available and the components state inside the system is unknown. No implementation cost, including for example sensors costs or technology solutions costs, are considered.

In order to demonstrate the impact of the usage information on the TMC, the computation of TMC is realized for the same MFOP and confidence level values in integrating the first, the second and the third information level on usage for each system component.

Firstly, the Fig. 8 represents the *TMC* index for different MFOP between 20000 km and 60000 km by step of 10000 km and confidence levels between 75% and 95% by step of 5% when no usage information is available for each system component. In this case, the *TMC* is minimal when MFOP and the confidence level are respectively equal to 60 000 km and 90%. This cost-optimized solution provides the best balance between corrective and preventive maintenance operations.

Note that for some configurations, the TMC increases with the confidence level. This behavior is explained by the fact that the additional preventive maintenance cost can be higher than the gain saved by the immobilization costs reduction.



Figure 8: The TMC (Euro) with Information Level 1 on Usage

Secondly a comparison between the three information levels is performed in Tab. 3. These results justify the positive impact created by the increase of information level on usage. If the second information level is implemented for each system component, the saved cost in considering each *TMC* value is in average of 15.6% comparatively with the first level. For the third information level the saved cost is in average of 16.2% comparatively with the first level.

Information Level	Optimal TMC (Euro)	Saved costs on the optimal TMC comparatively with Level 1 (%)	Mean Saved Costs comparatively with Level 1 (%)
Level 1	6387€	-	-
Level 2	5808€	9.1%	15.6%
Level 3	5724€	10.4%	16.2%

Table 3:	TMC	with 1	the	Three	Information	Levels	on l	Usage
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Note that the cost-optimized solution decreases of 9.1% with the second information level and of 10.4% with the third information level. Thereby this example illustrates how the usage information can be used to optimize the dynamic maintenance policy based on MFOP concept.

7. CONCLUSION

In this article, a usage-informed preventive maintenance policy based on MFOP concept has been proposed. This dynamic maintenance policy is able to take into account, at the end of each MFOP, the usage information of each system component to update the maintenance decision process. The connection between the usage information and the component life consumption is performed thanks to an experience-based method named mixture models. The alternative maintenance strategies in considering various information levels on usage are evaluated based on TMC value. The results presented on a specified system allow illustrating the positive impact of usage information to define the cost-optimized maintenance strategy.

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