Analyzing system changes with importance measure pairs: Risk increase factor and Fussell-Vesely compared to Birnbaum and failure probability

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Abstract: Importance measures are used to rank components of a system according to a selected criterion depending on the decision problem. Sometimes, more than one importance measure may be used. In risk-informed decision making, a component that is critical to safety is usually prioritized higher in allocating activities, e.g. maintenance or inspection. One desired effect of this prioritization is to improve the reliability of the critical components. Changes in the system or component reliability affect their importance measures. If these feedbacks are taken into account, new ranking for the components may be obtained.

This paper examines the properties of risk importance measure pairs in analyzing system changes with fault tree analysis. A common approach is to use risk increase factor (or risk achievement worth) and Fussell-Vesely importance measure. This approach is compared to an alternative method which utilizes Birnbaum importance measure and the failure probability of a basic event. It is shown that the first approach may lead to difficulties in understanding the effect of system changes whereas the latter seems to provide simpler and more robust alternative. The paper includes examples to show and compare the differences between the two methods. The key advantages of the alternative method are that it reflects the absolute instead of relative change, the variables are independent, and that the interpretation of the importance measures is straightforward, reflecting risk in terms of safety margin and failure probability.

Keywords: Importance measure, basic event ranking, PRA, PSA

1. INTRODUCTION

Risk-informed decision making is becoming more and more common in many industrial sectors [1]. In nuclear industry, it is used by both licensees and regulatory bodies (see, e.g., [2]). For instance, in Finland the foundation for risk-informed licensing, regulation and safety management of a nuclear power plant is laid in the nuclear safety legislation and the requirements on the use of risk-informed methods are included in about ten nuclear regulatory guides (see, e.g., nuclear regulatory guide YVL A.7 [3]). Typical subjects for decision making are ranking and prioritizing systems, structures and components (SSC) for enhanced maintenance, inspection, testing or safety categorization [1]. Sometimes, more than one importance measure may be used in decision making. For example, NUMARC 93-01 provides the implementation guidelines for maintenance rule: in determining the risk-significant SSCs risk reduction worth (RRW), risk achievement worth (RAW) and the top 90% of the cut sets contributing to core damage frequency are to be applied (for more information see, e.g., [4], [5], [6]).

In addition to RRW and RAW, Fussell-Vesely (FV) importance is commonly used, also in combination with RAW [7] which is also called risk increase factor (RIF, which is used in this paper). But even as appealing as it might be, using several importance measures in risk-informed decision making may be more complex than at first one might assume. This has been recognized also in [7] where the authors advice to use Birnbaum importance instead of RAW in combination with FV. This paper tries to contribute on this discussion by studying the use of importance measure pairs: risk increase factor and Fussell-Vesely are compared to Birnbaum and failure probability in analyzing system changes.

First, the importance measures used are introduced. Then, the dependence between RIF and FV is shown which is also given in [1] although the discussion in [8], [9] and [10] states that there would be no functional relation or direct relation between FV importances and RAWs. The influence of the dependence is briefly discussed and illustrated graphically with FV-RIF -map. After this, an alternative method utilizing independent variables failure probability and Birnbaum is discussed briefly. Finally, the two methods are compared with a simple pump line system which goes through various changes including both structural and component reliability improvements. Since living PRA, system modifications or even safety classification (one goal may be to improve reliability by increasing tests, maintenance and inspections) all introduce feedback and updates to PRA model, the authors consider relevant to study which method should be used in analyzing them.

2. IMPORTACE MEASURES

It is assumed that the reliability model is presented in the form of minimal cut sets. The following discussion does not deal with initiating events since they have different dimension compared to failure probabilities and therefore their Birnbaums cannot be compared (see, e.g., [11], [12]). The total risk, R, is represented as a (multi)linear function $R(X_i) = aX_i + b$ where aX_i represents all the cut sets containing the event i with failure probability X_i and b all other cut sets. Using this notation risk increase factor (or risk achievement worth, RAW) is defined as

$$RIF(X_i) = \frac{a+b}{aX_i + b} \tag{1}$$

Fussell-Vesely is defined as

$$FV(X_i) = \frac{aX_i}{aX_i + b} \tag{2}$$

Birnbaum is defined as

$$B = \frac{\partial R(X_i)}{\partial X_i} = a \tag{3}$$

Risk increase factor is a weak (almost independent) function of X_i and Fussell-Vesely is proportional to X_i when $aX_i \ll b$ (see, e.g., [7] and [13]). The independence of Birnbaum from X_i is evident. In addition, the following relation holds between risk reduction worth and Fussell-Vesely: $RRW = 1/(1 - FV)$ [7]. Hence, although the following discussion deals with RIF, FV, Birnbaum and failure probability, similar treatment for RAW and RRW is possible.

It must be noted that PRA codes may approximate RIF by setting the corresponding failure probability X_i equal to unity without reducing the Boolean minimal cut set expression (i.e., reminimization). Hence, the result may not be consistent with the value that is obtained if the Boolean value of the event i is set to 'true' [11], [13]. For simplification and clarity, it is assumed in this paper that RIF is approximated without reminimization. This means that a and b do not change whether we calculate RIF, FV or Birnbaum.

3. RIF AND FV MAPPING

Using equations (1) and (2) (and assuming that RIF is approximated without reminimization) one obtains $RIF(X_i) \approx FV(X_i)(a + b)/aX_i$ and noticing that $FV(X_i)b/aX_i = 1 - FV(X_i)$ one yields to

$$RIF(X_i) \approx 1 + FV(X_i)\left(\frac{1}{X_i} - 1\right) \tag{4}$$

Thus, for any X_i, possible combinations of RIF and FV are defined by equation (4) above. Similar relation is also presented in [1]. The equation shows that for a constant failure probability, RIF and FV have an (almost) exact relation, i.e., in FV-RIF -map the plotted basic events can only appear on the curves defined by equation (4) regardless of the system. Let us call these curves of equal failure probability equiprobability curves which are shown in figure 1 for different failure probabilities on FV-RIF -map. The failure probabilities X_i for each equiprobability curve are given on the right. The dashed lines illustrate arbitrary criteria by which the basic events can be categorized into high, medium and low risk areas, i.e., upper right corner (FV > 1.00E-2 and RIF > 1.00E+2) is considered high risk area, lower right and upper left corners medium risk areas, and lower left corner (FV < 1.00E-2 and RIF < 1.00E+2) low risk area.

The dependence of RIF and FV may lead to difficulties in understanding the effect of system changes in, e.g., risk-informed decision making. Since RIF, FV and failure probability are tied by equation (4) it is clear that if two of these variables are known, the third can be calculated regardless of the failure logic of the system. Since failure probabilities are needed for calculating any risk importance measure, either FV or RIF can be detemined by the corresponding equiprobability curve. Hence, the third variable can be explained by the other two and therefore does not add any information about the importance of the basic event.

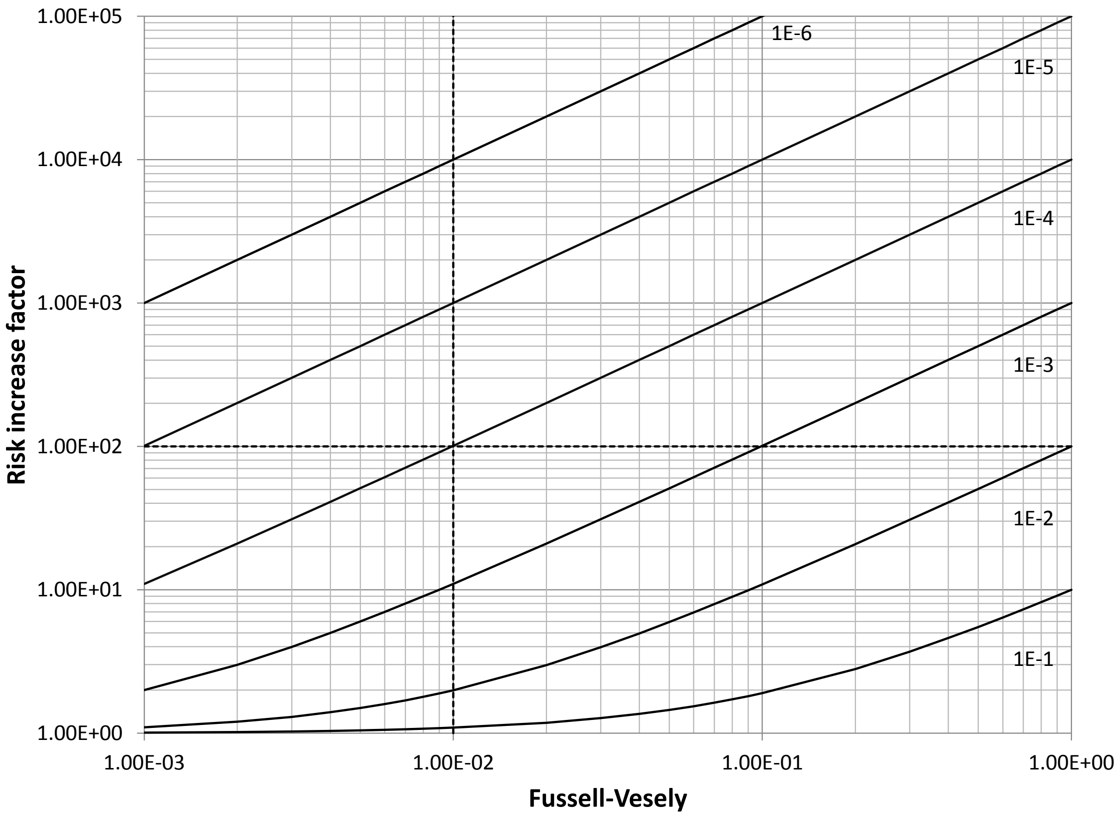


Figure 1: The black curves (called equiprobability curves in this paper) illustrate the dependence of FV and RIF as defined by equation (4). The failure probabilities X_i for each equiprobability curve are given on the right. For example, basic event whose failure probability is 1.00E-4 can only appear on the corresponding curve regardless of the system. The dashed lines illustrate arbitrary criteria by which the basic events can be categorized into high, medium and low risk areas.

4. BIRNBAUM AND FAILURE PROBABILITY MAPPING

Risk inherent to a component can be expressed in terms of failure probability of the component and the remaining safety margin if the component is to fail. This is reflected by choosing the importance measures failure probability X_i and Birnbaum as the two attributes for analyzing system changes or ranking components according to their criticality to safety. Birnbaum, see equation (3) above, is completely dependent on the structure of the system and independent of the corresponding failure probability X_i [7]. Therefore, this choice avoids the problematic issues of the dependence between FV and RIF that were discussed above and shown in equation (4). The interpretation of both importance measures is also quite straightforward or even intuitive since failure probability is a measure of reliability or quality characteristic to the component and Birnbaum measures the safety margin inherent to the remaining structure if the component fails.

These findings motivate to consider the use of failure probability X_i and Birnbaum instead of Fussell-Vesely and risk increase factor for analyzing system changes or ranking components. Figure 2 presents the X_i-B -map. The black lines represent contours of equal risk and are defined as a product of failure probability X_i and Birnbaum. Let us call these lines equirisk lines. The numerical value of risk is given next to the corresponding equirisk line. The lines divide the space into separate risk zones, e.g., high, medium and low risk zones that may be used in classification of the components. The components (or basic events) can move only horizontally or vertically, both of which have a practical explanation: Horizontal movement naturally means change in the component reliability and vertical movement changing the defense-in-depth of the system.

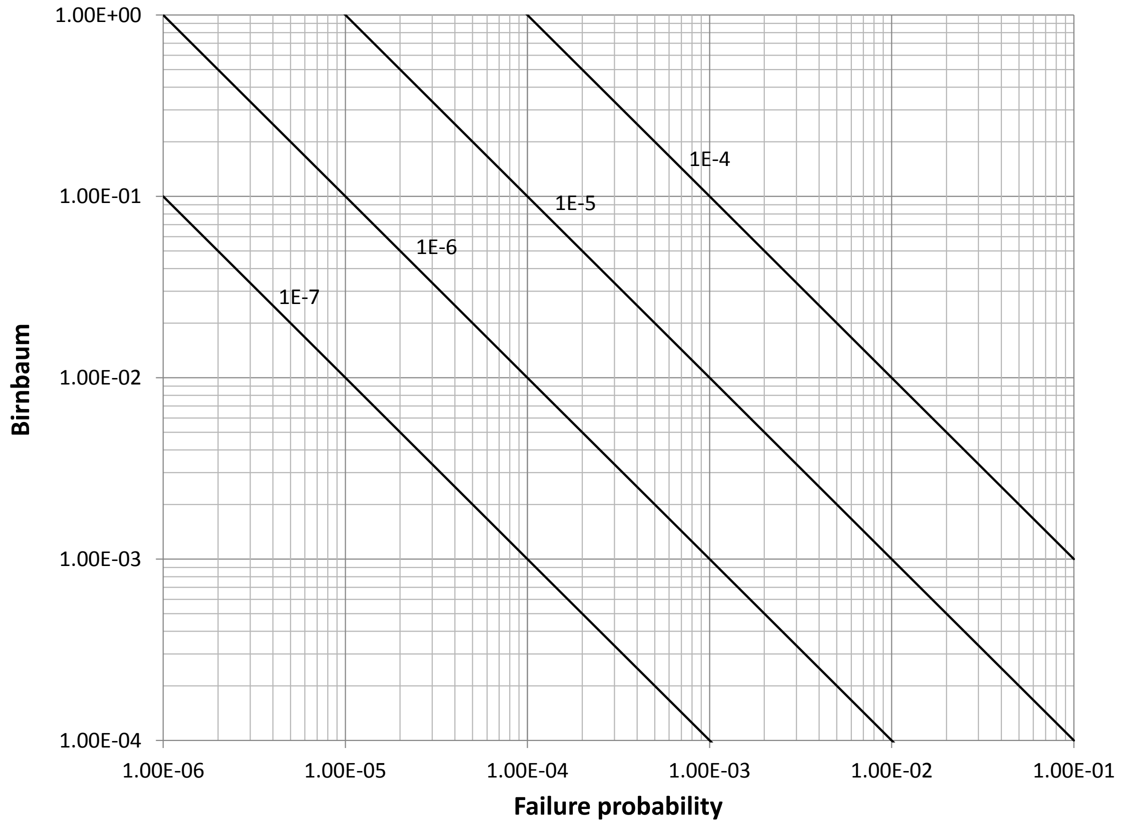


Figure 2: X_i-B -map. The black lines represent contours of equal risk (called equirisk lines in this paper) and the numerical value is given next to the corresponding equirisk line. The lines divide the space into separate risk zones, e.g., high, medium and low risk zones. The components (or basic events) can move only horizontally or vertically since X_i and B are independent on each other.

5. COMPARISON BETWEEN (FV, RIF) AND (X_I, B) IN ANALYZING SYSTEM CHANGES

The following simple example compares the two methods discussed above, i.e., utilizing importance measure pair (FV, RIF) or (X_i, B), and illustrates some potential problems that may arise when FV and RIF are used in analyzing system changes.

Consider a simple pump line system shown in figure 3 containing basic events L1 (line fails), V1 (valve fails) and P1 (pump fails). The Boolean logic of the system is TOP = L1 + V1 + P1. The basic event failure probabilities and the discussed importance measures are presented in table 1. The total failure probability for the system is 1.01E-2.

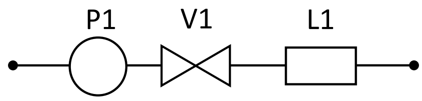


Figure 3: Simple pump line system with one pump and valve.

Table 1: Basic event (BE) failure probabilities X_i and the discussed importance measures Fussell-Vesely (FV), risk increase factor (RIF) and Birnbaum (B) for a single pump line system shown in figure 3.

BE	X_i	FV	RIF	B
L1	1E-5	9.79E-4	9.89E+1	1.00
V1	1E-4	9.79E-3	9.89E+1	1.00
P1	1E-2	9.89E-1	9.89E+1	1.00

Now, in order to increase the reliability of the system, a redundant pump P2 and valve V2 are installed in parallel with the originals. Notice from table 1 that the Fussell-Vesely values are the highest for P1 and V1, they are also the most unreliable components. The new system is shown in figure 4. The Boolean logic of the system is now TOP = L1 + (V1 + P1)(V2 + P2). The basic event failure probabilities and the discussed importance measures are presented in table 2 (it is assumed that the probability for line failure does not increase substantially although the length of the pipeline increases). The total failure probability for the system has now decreased to 2.11E-5.

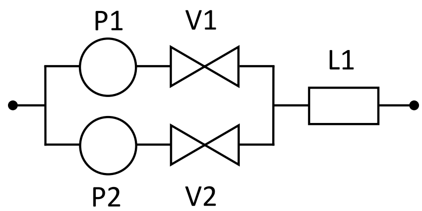
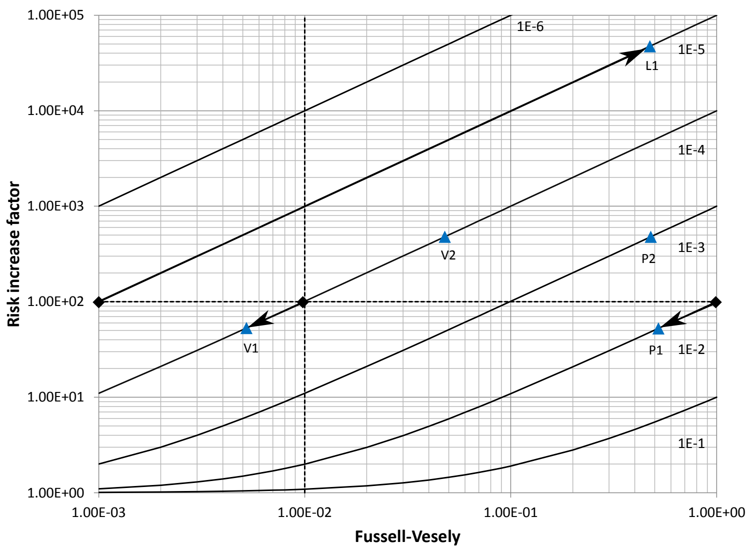


Figure 4: Simple pump line system with two redundant pumps and valves.

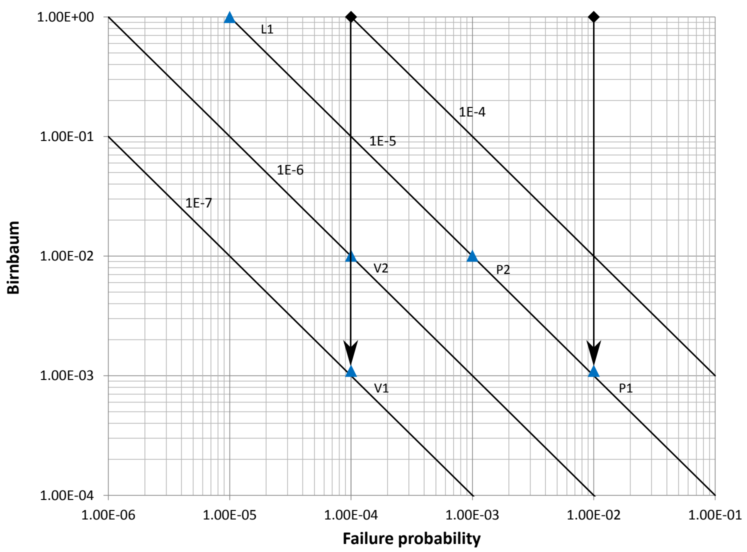
Table 2: Basic event (BE) failure probabilities X_i and the discussed importance measures Fussell-Vesely (FV), risk increase factor (RIF) and Birnbaum (B) for a simple pump line system with two redundant pumps and valves shown in figure 4.

BE	X_i	FV	RIF	B
L1	1E-5	4.74E-1	4.74E+4	1.00
V1	1E-4	5.21E-3	5.31E+1	1.10E-3
P1	1E-2	5.21E-1	5.26E+1	1.10E-3
V2	1E-4	4.78E-2	4.79E+2	1.01E-2
P2	1E-3	4.78E-1	4.79E+2	1.01E-2

This system change is illustrated in figures 5(a) and 5(b). In figure 5(a) each basic event is plotted on the FV-RIF -map. The black diamonds represent the original system with only one pipeline and the blue triangles the new system with two redundant lines. The black arrows show how the basic events have moved on the map due to the system change. Notice that all three basic events move along the equiprobability curves since their failure probabilities have not changed. As seen, the FV and RIF importances of basic event L1 (line fails) increase dramatically although the total failure probability for the system has changed from 1.01E-2 to 2.11E-5. The FV and RIF importances of basic events V1 and P1 decrease. In figure 5(b) each basic event is plotted on the X_i-B -map. Again the black diamonds represent the original system and blue triangles the new one. Basic events V1 and P1 move down since their Birnbaum importance decreases due to introducing redundant valve and pump. It can be seen that the change has improved the reliability of the system.



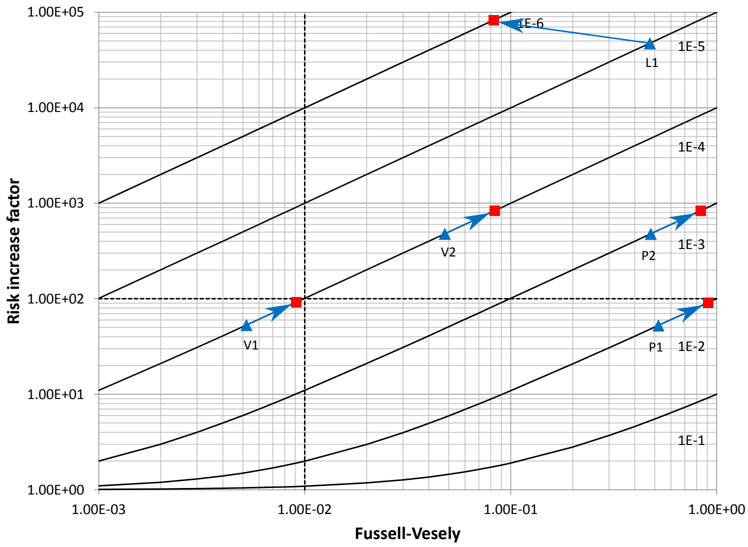
(a) FV-RIF -map. The importance measures of V1 and P1 decrease but the importance measures of L1 increase due to the system improvement. The basic events are located on and move along the equiprobability curves defined by equation (4) since FV and RIF are not independent.



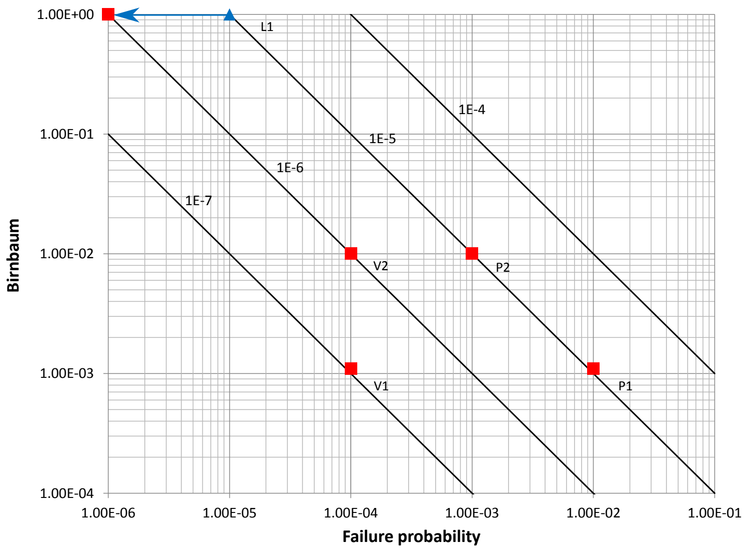
(b) X_i-B -map. The Birnbaum of V1 and P1 decreases due to the system improvement. It can be seen that the system has been improved. X_i and B are independent. After the system improvement L1, P1 and P2 are located on the same equirisk line.

Figure 5: The system is changed and improved by introducing redundant valve and pump. The black diamonds represent the original system with only one pipeline (see fig. 3) and the blue triangles the new system with two redundant lines (see fig. 4).

By using the figure 5(a) one might now consider the basic event L1 the most important but this conclusion can be misleading since L1, P1 and P2 all contain the same risk as seen in figure 5(b) (the blue triagles are located on the same equirisk line). Let us assume that L1 can be made ten times more reliable, i.e., now L1 has a failure probability of 1E-6. The total failure probability for the system decreases from 2.11E-5 to 1.21E-5 and affects the values of the importance measures. This change is illustrated in figures 6(a) and 6(b). The new values for importance measures are shown in red squares. The blue arrows show how the basic events have moved due to the reliability improvement. As seen in figure 6(a), basic events V1, P1, V2 and P2 move along the equipropability curves since their failure probability has not changed whereas L1 changes the curve due to the reliability improvement. On the other hand, it is fairly straightforward to observe the reliability improvement of L1 as well as of the system from figure 6(b).



(a) FV-RIF -map. The reliability improvement of L1 increases the FV and RIF importances of the other basic events and, e.g., V1 almost moves from low risk area to high risk area.

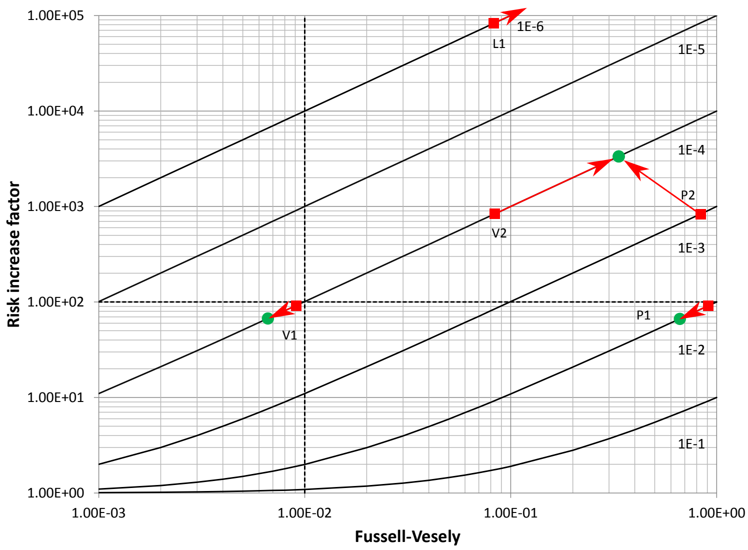


(b) X_i-B -map.The reliability improvement of L1 does not affect the other basic events.

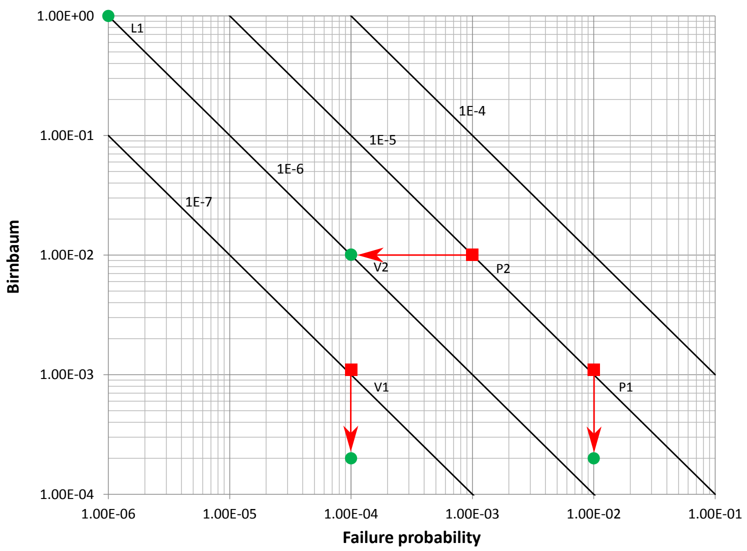
Figure 6: The system is changed and improved by making L1 ten times more reliable. The old values for importance measures are shown in blue triangles and the new values in red squares.

Last, let us study how making P2 ten times more reliable affects the importance measures and their graphical illustration. Thus, the failure probability of P2 is changed from 1E-3 to 1E-4. The total failure probability for the system is further decreased from 1.21E-5 to 3.02E-6. Figures 7(a) and 7(b) illustrate the change. The new values for importance measures are shown in green circles and the red arrows show how the basic events move due to the reliability improvement. Figure 7(a) shows that if the criteria for the high risk area were, e.g., FV > 1.00E-1 and RIF > 1.00E+3 (instead of FV

> 1.00E-2 and RIF > 1.00E+2 as illustrated in the figure 7(a) and discussed before), the reliability improvement of P2 would actually move it from medium risk area to high risk area and V2 from low risk area directly to high risk area. On the other hand, figure 7(b) clearly shows that the decrease in the failure probability of P2 affects the Birnbaums of V1 and P1 via improving their safety margin. The importance of L1 and V2 is not influenced by the system improvement because neither their failure probability nor safety margin is changed.



(a) FV-RIF -map. It is rather challenging to see that the system has been improved.



(b) X_i-B -map. Seeing the effects of reliability improvement is fairly straightforward compared to FV-RIF -map.

Figure 7: The system is changed and improved by making P2 ten times more reliable. The old values for importance measures are shown in red squares and the new values in green circles.

The simple example above demonstrated some of the ambiguities of using FV and RIF that the authors have found problematic. Utilizing FV and RIF in analyzing system changes (both component reliability improvement and changing the system failure logic) may lead to complex behavior and difficulties in interpreting the results if the feedback is taken into account in the reliability model or PRA. In particular, the equiprobability curves defined by equation (4), which shows that FV and RIF are not independent, along with the division of FV-RIF -space into four quadrants cause that some basic events may move from low risk area to high risk area even with a relatively small change.

Living PRA, system modifications or even safety classification (one goal may be to improve reliability by increasing tests, maintenance and inspections) all introduce feedback and updates to PRA model. Hence, it is relevant to consider which method should be used in analyzing them.

6. CONCLUSION

In this paper analyzing system changes, including both system failure logic changes and basic event reliability improvements, with importance measure pairs was discussed. The importance measure pairs were Fussell-Vesely and risk increase factor versus failure probability and Birnbaum. The dependence between risk increase factor and Fussell-Vesely was shown. Utilizing the two importance measure pairs was compared with a simple illustrative example.

Using failure probability and Birnbaum instead of Fussell-Vesely and risk increase factor in analyzing system changes was found to have several advantages: First, failure probability and Birnbaum are independent on each other and therefore can be illustrated orthogonally. Second, the failure probability and Birnbaum measures are absolute which means that comparison of different system configurations is evident. Another consequence of absoluteness is that the change on basic event importances expresses real change and only those basic events move on the graphical illustration that are really affected by the modification. Third, the axes of the graphical illustration have clear interpretations, failure probability and safety margin, so understanding the location of the basic events is straightforward. These features make using failure probability and Birnbaum map a practical tool for decision maker to assess wheter the system under consideration should be improved by increasing component reliability or safety margin.

The authors recognize that each importance measure contains different information and has its own use but the findings made in this paper hopefully shed some light on the complexity that lies behind using two or more importance measures in component ranking.

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